ARCHIVES OF ACOUSTICS
34, 2, 157–168 (2009)

Computer Modelling of Coupled Spaces: Variations of Eigenmodes Frequency Due to a Change in Coupling Area

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(*received September 26, 2008; accepted March 2, 2009*)

Acoustically coupled spaces have recently been drawing more and more attention in the architectural acoustics community, thus a determination of shapes and frequencies of eigenmodes in these room systems from computer-based models has become increasingly significant. In this investigation, an eigenvalue problem was solved numerically for a simple room system consisting of two connected rectangular spaces. In a numerical procedure, the forced oscillator method with a finite difference algorithm was applied. In order to determine the influence of irregularity of system shape on eigenmodes frequency, a modal behaviour in the coupled spaces was studied for several sizes of coupling area. Calculation results have shown that with the exception of a fundamental mode, the changes in resonant frequencies were relatively small. However, an increase in a system irregularity led to a coincidence of frequencies of neighbouring modes and variations in a sequence of modes on a frequency axis, which both contributed to a degeneration of eigenmodes.

Keywords: coupled spaces, modal analysis, degenerate modes, mode localization.

1. Introduction

A large amount of studies of vibration behaviours of acoustic field in enclosed spaces was concerned with rectangular and cylindrical enclosures. The shapes and frequencies of rigid-walled modes of these regular enclosures are well defined analytically [1, 2], and have been used for predicting the acoustical characteristics of lightly damped enclosures, which include a vibration spectrum, a pressure response to acoustical or structural excitations and modal decay times [3, 4]. However, irregularly shaped enclosures are often encountered in practice. They usually consist of several partial rooms which are connected to each other, consequently, they constitute room systems with acoustically coupled spaces. Examples of such systems are theatres, churches or large-size halls with an irregular geometry, thus acoustic properties of coupled spaces have been studied intensively in the past [5–13]. Unlike regular rooms, the shapes and frequencies of rigid-walled modes of irregular enclosures are not definable analytically, therefore an application of modal analysis to such systems was possible through numerical methods [14, 15].

In a room system consisting of two connected spaces, spatial distributions and frequencies of modes depend on dimensions and shapes of partial enclosures and the size of a coupling area. The consequence of a complex room geometry is the phenomenon of mode localization which characterizes an irregular distribution of mode amplitude. This effect is similar to that observed in resonators with fractal boundaries [16] or irregularly shaped cavities [17]. The mode localization strongly affects the sound pressure distribution in a steady-state and may cause the double-sloped sound decay when one of enclosures contains a large amount of acoustic absorption and the second one is more reverberant [18, 19].

As fundamental characteristics of acoustic fields in coupled spaces have an enormous practical significance, there is a necessity to investigate how the mode shapes and frequencies are modified with a change in the irregularity of enclosure. A study of this problem is the subject of the present paper and it was realized for a room system which consisted of two rectangular spaces coupled acoustically. A diagram of this system is shown in Fig. 1, where the dimension d_1 is a variable and its change modifies a size of coupling area.

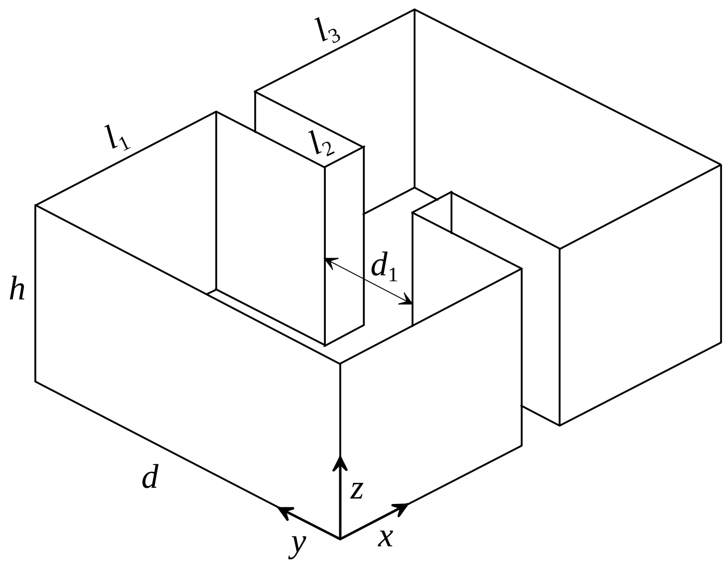

Fig. 1. Irregularly shaped room in a form of two connected rectangular spaces.

2. Numerical method

A theoretical modelling of acoustic field in enclosures, having dimensions comparable with a length of sound wave, is based on a solution of the wave equation with specified initial and boundary conditions [1]. From this point of view, the irregularly shaped room may be treated as a resonance system with characteristic acoustic normal modes determined by eigenfunctions $\Phi_{mn}(x, y, z)$ with corresponding eigenfrequencies ω_{mn} $(m, n = 0, 1, 2...)$. Eigenfunctions Φ_{mn} depend on a room shape and boundary conditions, and are mutually coupled through the impedance condition on room walls, but in a low-frequency range, where typical materials are characterized by a small sound absorption, a distribution of eigenmodes amplitude is well approximated by uncoupled eigenfunctions satisfying the Neumann boundary condition (rigid-walled modes) [3]. For a room geometry shown in Fig. 1, normalized eigenfunctions satisfying this condition are given by

$$\Phi_{mn} = \begin{cases} \Psi_n(x, y)/\sqrt{h}, & m = 0, \\ \sqrt{2/h}\cos(m\pi z/h)\Psi_n(x, y), & m > 0, \end{cases} \tag{1}$$

where h is a room height and Ψ_n are eigenfunctions which are normalized over a surface S of room horizontal cross-section and $\Psi_0 = 1/\sqrt{S}$. For the Helmholtz mode $(m, n = 0)$ the eigenfrequency ω_{mn} is equal to zero and for remaining modes, the eigenfrequencies normalized by the frequency $\omega_r = \pi c/l$ are given by

$$\Omega_{mn} = \sqrt{(ml/h)^2 + \Omega_n^2}, \tag{2}$$

where Ω_n is a non-dimensional eigenfrequency corresponding to the function Ψ_n and $\Omega_0 = 0$, and ω_r is the fundamental resonance frequency for a rectangular room with dimensions d, h and $l = l_1 + l_2 + l_3$, where c is the sound speed and l is equal or greater than d and h. Distributions of eigenfunctions Ψ_n in (x, y) plane were computed numerically via application of the forced oscillator method [20] with a finite difference algorithm. Non-dimensional eigenfrequencies Ω_n were calculated from the expression

$$\Omega_n = \frac{l}{\pi}\sqrt{-\int\limits_S \Psi_n \nabla^2 \Psi_n \,\mathrm{d}x\,\mathrm{d}y} \tag{3}$$

derived directly from two-dimensional eigenvalue equation: $\nabla^2\Psi_n + (\pi\Omega_n/l)^2\Psi_n = 0$. Calculations of eigenfunctions Ψ_n were carried out for a room having the following proportions: $l_1/l = 0.5$, $l_2/l = 0.1$, $l_3/l = 0.4$ and $d/l = 0.8$. A modification of the coupling area was realized by a variation of the dimension d_1 of a room what is equivalent to a change in the non-dimensional parameter d_1/d. In a numerical study, this parameter was assumed to vary from 0.05 to unity with an increment $\delta = 0.05$. Dependences of eigenfrequencies Ω_n on d_1/d were determined for the first thirty eigenmodes. Values of Ω_n for $d_1/d = 1$ were calculated for a rectangle with dimensions l and d.

3. Analysis of calculation data

Calculation data presented in Fig. 2 depict changes in the eigenfrequency Ω_n with the parameter d_1/d for the first eight eigenmodes. Numbers at each curve denote mode numbers n, respectively. A substantial variation of the eigenfrequency is noted for the fundamental resonant mode ($n = 1$), for which the non-dimensional frequency Ω_n becomes nearly three times greater with an increase of d_1/d from 0.05 to 1 (Fig. 2a). Significant changes in Ω_n are also observed for mode 8 (Fig. 2b). It is easy to see that these modes for $d_1/d = 1$ correspond to the first and third lengthwise axial modes (modes excited in a rectangle due to an acoustic resonance along a higher dimension, in our case in the x-direction). For the remaining modes a variation of the ratio d_1/d results in relatively small changes in the eigenfrequency.

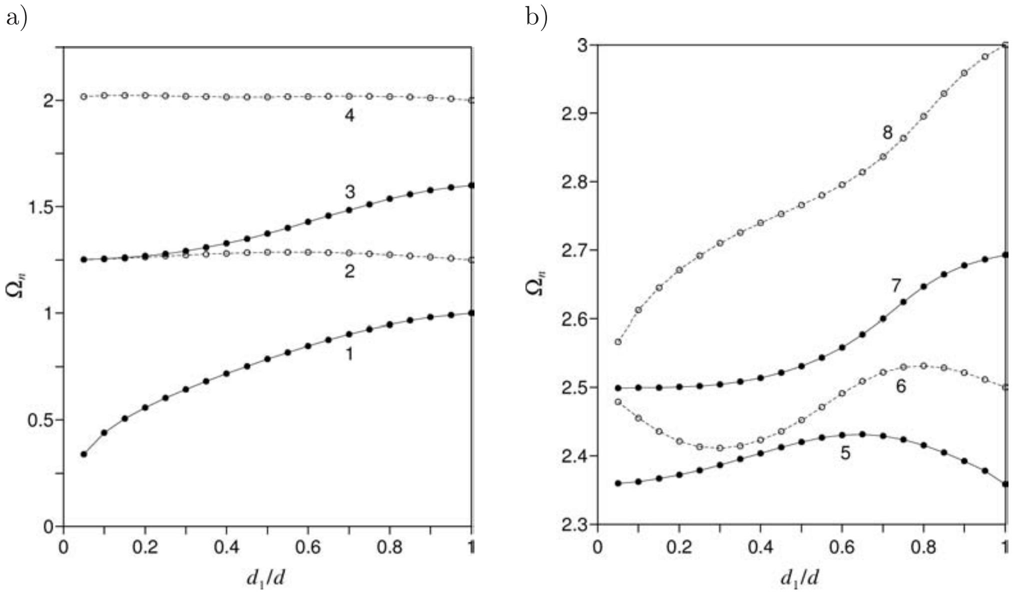

Fig. 2. Non-dimensional eigenfrequency Ω_n versus parameter d_1/d for mode numbers n: a) 1–4, b) 5–8.

For modes 2 and 3 one can observe a very interesting behaviour of eigenmode frequency, namely a coincidence of frequencies of neighbouring modes with a decrease of coupling area (Fig. 2a). Explanation of this phenomenon can be found in Fig. 3 exhibiting the shapes of eigenfunctions Ψ_n for modes 2 and 3 for three different values of the parameter d_1/d. The plots in Fig. 3 are in a form of filled contour maps, which are a two-dimensional representation of three-dimensional data, where contours define lines of constant value of Ψ_n. For $d_1/d = 1$, modes 2 and 3 represent the first widthwise axial mode (modes excited in a rectangle due to an acoustic resonance along a smaller dimension, in our case in the y-direction) and the first oblique mode, respectively. As d_1/d decreases, frequencies of these modes approach each other and finally, for value of d_1/d close to zero they become a pair of degenerate eigenmodes. Since an energy of these modes is concentrated in the first or the second part of a room, they represent also a pair of localized eigenmodes.

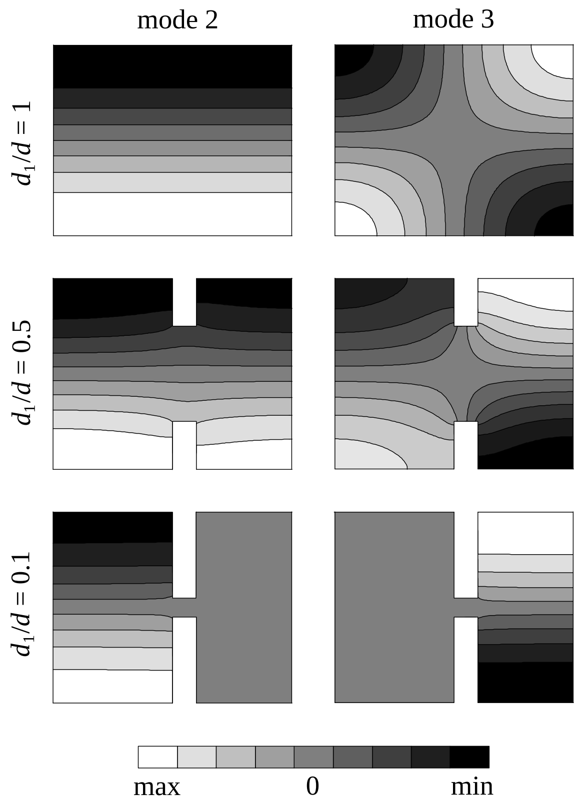

Fig. 3. Shapes of eigenfunctions Ψ_n for modes 2 and 3 for d_1/d equal to 0.1, 0.5 and 1.

To analyse theoretically an acoustic motion associated with the fundamental resonant mode we assume that the system of coupled rooms, shown in Fig. 1, represents a duct, where a plane wave motion parallel to the x-direction occurs only, and the velocity potential ϕ is given by

$$\phi(x,t) = [(A\sin(kx) + B\cos(kx)]e^{j\omega t}, \tag{4}$$

where A and B are unknown amplitudes and $k = \omega/c$ is a wave number. Using Eq. (4) a general expression for the acoustic impedance can be found

$$Z(x) = -j\frac{\rho c}{S_0}\frac{A\sin(kx) + B\cos(kx)}{A\cos(kx) - B\sin(kx)}, \tag{5}$$

where ρ is the air density and S_0 is the surface of a duct cross-section. For waves travelling along x-axis the room system represents a connection of ducts with different cross-sections, thus at junctions the following conditions of continuity of acoustic impedance must be satisfied:

$$\begin{aligned} Z(l_1^-) &= Z(l_1^+), \\ Z[(l_1+l_2)^-] &= Z[(l_1+l_2)^+], \end{aligned} \tag{6}$$

where the notations a^- and a^+ respectively denote the values of x immediately smaller and immediately greater than a. At positions $x = 0$ and $x = l$ the duct is terminated by rigid walls, thus using Eq. (5) it is easy to find that

$$\begin{aligned} Z(l_1^-) &= j\frac{\rho c}{S_1}\cot(kl_1), \\ Z[(l_1+l_2)^+] &= -j\frac{\rho c}{S_1}\cot(kl_3), \end{aligned} \tag{7}$$

where $S_1 = dh$. A resonant condition for lengthwise axial modes can be derived from Eq. (6) after inserting Eq. (7) and applying Eq. (5). The result is

$$S_1 S_2 \cot(kl_2)\left[\cot(kl_1) + \cot(kl_3)\right] + S_2^2 \cot(kl_1)\cot(kl_3) = S_1^2, \tag{8}$$

where $S_2 = d_1 h$. Frequencies of modes 1 and 8 calculated numerically and resonant frequencies computed from Eq. (8) are compared in Fig. 4. The analytical model accurately predicts a tendency of eigenfrequency variations with a growth of the ratio d_1/d and evaluates well the values of Ω_n for d_1/d close to zero or unity. This can be explained by the fact that an acoustic motion inside a room system at extreme values of d_1/d is nearly parallel to the x-axis. Larger discrepancies between the numerical and analytical data are observed in a middle range of d_1/d values. It is due to the fact that in this case, eigenmodes have a more visible two-dimensional structure, therefore the assumption that a distribution of acoustic field inside a room system is one-dimensional, results in larger errors in resonant frequency calculations.

Calculation results obtained for the next set of eigenmodes are shown in Fig. 5. Since for some values of d_1/d the frequencies of neighbouring eigenmodes are very similar, a dependence of Ω_n on the ratio d_1/d for respective eigenmodes, was determined using the similarity criterion

$$\int_S \Psi_n^{(1)} \Psi_n^{(2)} \,\mathrm{d}x\,\mathrm{d}y \approx 1, \tag{9}$$

where $\Psi_n^{(1)}$ and $\Psi_n^{(2)}$ are eigenfunctions computed for values of d_1/d differing from each other by δ. A coincidence of mode frequencies with increasing the room irregularity, appearing previously for modes 2 and 3, one may note for modes 12 and 13 (Fig. 5a). The other interesting thing is that the curves obtained for some neighbouring eigenmodes intersect. Consequently, a sequence of eigenmodes on a frequency axis may change with a variation of coupling area, and for values of d_1/d corresponding to intersection points there exist pairs of degenerate eigenmodes.

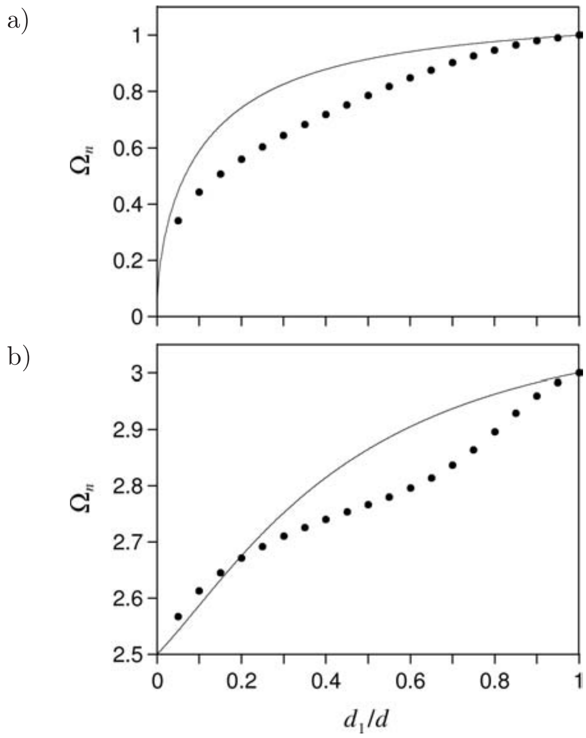

Fig. 4. Non-dimensional eigenfrequency Ω_n versus parameter d_1/d for mode numbers n: a) 1 and b) 8. Points: numerical data. Solid lines: calculation results from Eq. (8).

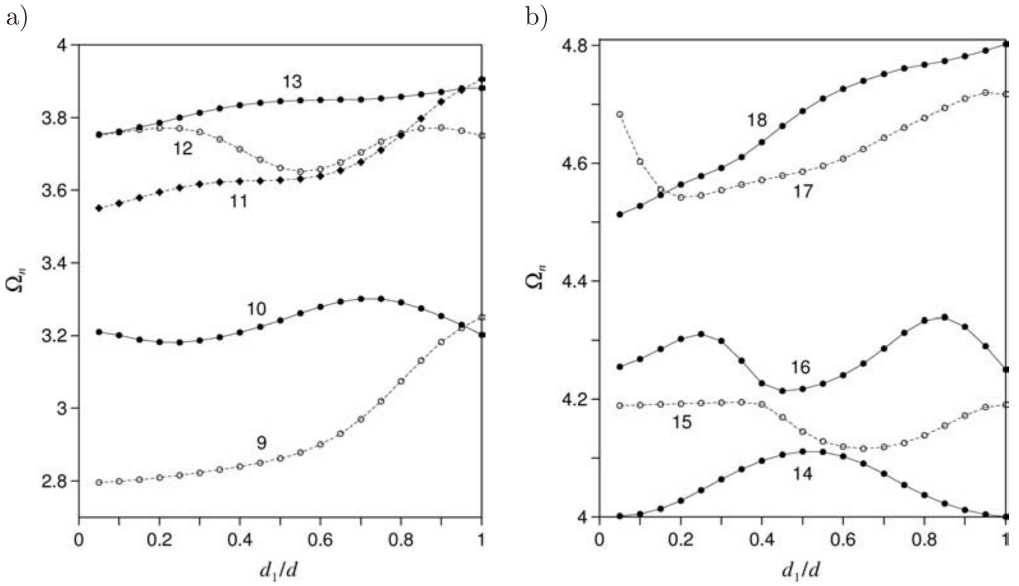

Fig. 5. Non-dimensional eigenfrequency Ω_n versus parameter d_1/d for mode numbers n: a) 9–13, b) 14–18.

Figure 6 presents calculation data for the last twelve eigenmodes. The graphs obtained for modes 20–22 indicate that a change in a coupling area can force such variations of eigenmode frequency that multiple changes in a sequence of neighbouring modes on a frequency axis are observed (Fig. 6a). Another thing of special interest, which needs a wider explanation, is untypical behaviour of eigenmode frequency for two pairs of modes: 19, 20 and 29, 30, because curves corresponding to each pair are joined for $d_1/d = 1$. In these particular cases a conversion of two separate eigenmodes into degenerate modes is caused by a special proportion between dimensions d and l of a room.

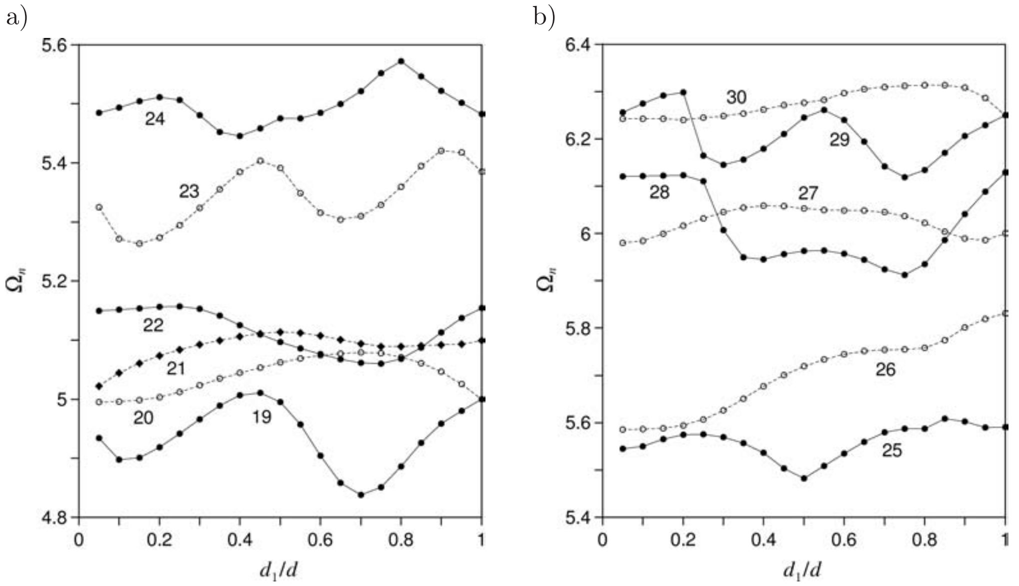

Fig. 6. Non-dimensional eigenfrequency Ω_n versus parameter d_1/d for mode numbers n: a) 19–24, b) 25–30.

A horizontal room cross-section for d_1/d equal to unity has a form of rectangle with normalized eigenfrequencies given by the expression

$$\Omega_n = \sqrt{m_1^2 + m_2^2 \left(\frac{l}{d}\right)^2}, \tag{10}$$

where $m_1 = 0, 1, 2...$ and $m_2 = 0, 1, 2...$ are mode indices for lengthwise and widthwise resonances, respectively. Since $d/l = 0.8$, the first pair of degenerate eigenmodes occurs for $m_1 = 0$, $m_2 = 4$ and $m_1 = 5$, $m_2 = 0$ and it corresponds to the value of Ω_n equal to 5. The second pair is formed by eigenmodes for which $m_1 = 0$, $m_2 = 5$ and $m_1 = 5$, $m_2 = 3$ and in this case $\Omega_n = 6.25$. Shapes of eigenfunctions Ψ_n^d for the first pair of degenerate eigenmodes, together with shapes of eigenfunctions Ψ_n for separate eigenmodes ($d_1/d = 0.95$), are shown in Fig. 7. These graphs imply that functions Ψ_n for modes 19 and 20 just after a separation are a sum of eigenfunctions Ψ_n^d with a certain proportion. This regularity can be written as

$$\begin{aligned} \Psi_n &\approx a_n \Psi_n^d + b_n \Psi_{n+1}^d, \\ \Psi_{n+1} &\approx a_{n+1} \Psi_n^d + b_{n+1} \Psi_{n+1}^d, \end{aligned} \tag{11}$$

where a_n, b_n and a_{n+1}, b_{n+1} are unknown coefficients. Since eigenfunctions are normalized over a room horizontal cross-section, expressions for these coefficients are

$$\begin{aligned} a_m &\approx \int_S \Psi_m \Psi_n^d \,\mathrm{d}x\,\mathrm{d}y, \\ b_m &\approx \int_S \Psi_m \Psi_{n+1}^d \,\mathrm{d}x\,\mathrm{d}y, \end{aligned} \tag{12}$$

where m is equal to n or $n+1$. From Eq. (11) it results that a_m and b_m must satisfy the condition

$$a_m^2 + b_m^2 \approx 1. \tag{13}$$

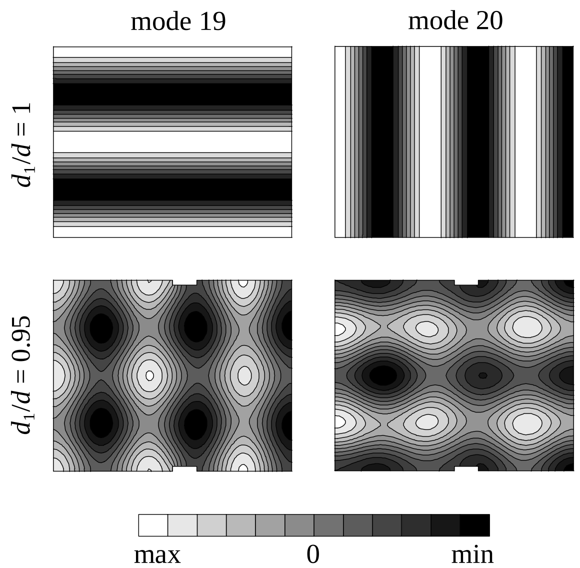

Fig. 7. Shapes of eigenfunctions Ψ_n^d ($d_1/d = 1$) and eigenfunctions Ψ_n ($d_1/d = 0.95$) for modes 19 and 20.

Values of a_m and b_m calculated from Eq. (12) for modes 19, 20, 29 and 30 are collected in Table 1. As it may be seen, when $d_1/d = 0.95$ the condition (13) is not fulfilled with a good precision for some cases and it is satisfied much more accurately for the value of d_1/d somewhat closer to unity ($d_1/d = 0.975$).

Table 1. Coefficients a_m and b_m calculated from Eq. (12) for modes 19, 20, 29, 30 and d_1/d equal to 0.95 and 0.975.

d_1/d	m	a_m	b_m	$a_m^2 + b_m^2$
0.95	19	0.4983	0.8632	0.9935
0.95	20	−0.8841	0.4258	0.9630
0.95	29	0.6617	0.7418	0.9880
0.95	30	−0.7478	0.5721	0.8865
0.975	19	0.5682	0.8216	0.9979
0.975	20	−0.9114	0.3992	0.9901
0.975	29	0.7240	0.6862	0.9950
0.975	30	−0.7867	0.5858	0.9621

4. Conclusions

Enclosed spaces which are coupled together by an open area are known to exhibit some interesting phenomena like a confinement of an acoustic vibration in a restricted part of enclosure, commonly known as the mode localization, and variations of initial and late decay times with a distribution of absorbing material on room walls, often referred to as the double-sloped sound decay. These effects are strongly frequency-dependent because fundamental acoustic characteristics of coupled spaces are shapes and frequencies of eigenmodes generated inside the enclosure as a response to acoustical or structural excitations.

In a simple system of coupled spaces consisting of two connected rectangular enclosures, the shapes of eigenmodes and resonant frequencies depend on dimensions of partial enclosures and a size of a coupling area, which is a measure of the room irregularity. In this study it was found that a change in the coupling area contributes to relatively small variations of eigenfrequencies. An exception is the fundamental resonant mode for which the eigenfrequency decreases approximately three times in the considered range of a coupling area. As was shown for lengthwise axial modes, the ranges of eigenfrequency variations are fairly well predicted by the theory based on a plane wave transmission within the room system. Larger differences between numerical and analytical data are noted for a moderate room irregularity. In this case eigenmodes have a more visible two-dimensional structure, therefore the assumption that a distribution of acoustic field inside a room system is one-dimensional, results in larger errors in frequency calculations.

Along with a change in a coupling area one might observe the effect of a mode degeneration and it was found that there are two main reasons for this. Firstly, the degeneracy is connected with a coincidence of frequencies of neighbouring modes with an increase of a room irregularity and it appears for a coupling area close to zero, where an energy of modes is concentrated inside the first or the second part of a room. Thus, this kind of a mode degeneration is associated directly with a phenomenon of mode localization. Another reason for a mode degeneration are variations in a sequence of modes on a frequency axis, with a modification of a coupling area. In this case, frequencies of modes are equal to each other just before a change in a sequence of modes.

Owing to a particular geometry of coupled spaces, the mode degeneracy was also present in a room system without a shape irregularity, i.e. for enclosed spaces forming a rectangular enclosure. When a system geometry deviated slightly from a regular shape, a conversion of degenerate eigenmodes into two separate modes occurred. It was demonstrated that eigenfunctions corresponding to the separate modes are a sum of eigenfunctions of degenerate eigenmodes with certain proportions.

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