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Calibration of EM and acoustic antisniper systems

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Abstract. Antisniper systems exploit numerous sensors (acoustic or electromagnetic, EM) which spatial coordinates should be known with great accuracy, otherwise the system performance can be significantly deteriorated. Triangulation of many sensors is not, however, an easy task, particularly under possible enemy fire. Here, we propose a method exploiting a round of calibration shooting over the system, which measured by the sensors provide data for evaluation of the sensors' spatial positions. The method can be applied for calibration of the Doppler radar antisniper sensors presented here in certain basic arrangement, and the acoustic microphone system measuring the shock wave generated by supersonic bullet.
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Introduction

Most antisniper systems exploit acoustic phenomena for localization of sniper position, either by measuring the muzzle blast or the shock wave generated by supersonic bullet passing by, or both, by set of microphones sewn in the field in front of the protected area [1]. Here, we present yet another system that utilizes the known Doppler radar measurements. This is perhaps the only feasible method of continuous observation of large angular sector of expected enemy fire. Without the constraint of detecting only supersonic bullets, the Doppler radar system can find wider applications (including the case of mortar fire, Fig. 1) than acoustic ones.

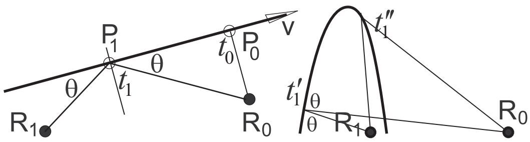

Fig. 1. Left: Doppler radar principles: Active sensor detects Doppler frequency shift at t_0 when $(\mathbf{P}_0 - \mathbf{R}_0) \perp \mathbf{v}$, while the passive sensor $\mathbf{R}_1$ does that at t_1 when both $\mathbf{R}_0$ and $\mathbf{R}_1$ reside on the opposite side of the cone tip $\mathbf{P}_1$ on the cone axis $\mathbf{v}$. Generally, $\mathbf{R}_0$, $\mathbf{R}_1$, $\mathbf{P}_1$ and $\mathbf{v}$ are not coplanar. Right: In the case of mortar fire (note its parabolic path), passive sensor detects Doppler frequency shift twice, at t' and t''

1. Doppler measurements

Electromagnetic signal of the frequency f_o scattered by real moving object with the velocity $v \ll c$ (c — the light speed) is frequency-shifted by:

$$f_D = f_o \frac{v}{c} \cos\theta, \quad \theta = \theta(t), \tag{1}$$

where θ is the angle between the velocity vector and the direction from the observation point $\mathbf{R}$ to the moving bullet $\mathbf{P}$ (both vectors in certain cartesian coordinates) depending on the time t (Fig. 2). For simplicity reason, we assume the constant bullet velocity, hence its position is:

$$\mathbf{P} = \mathbf{S} + \mathbf{v}t, \quad v = |\mathbf{v}|. \tag{2}$$

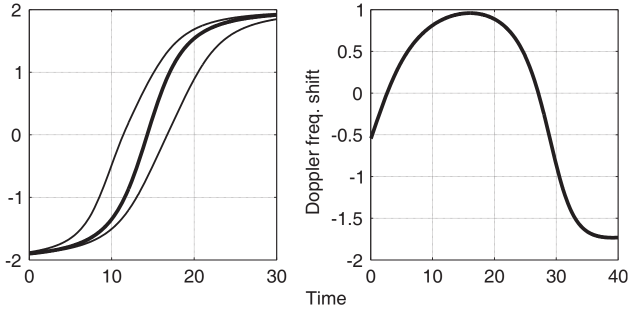

Fig. 2. Doppler frequency shift for active sensor (thick line) and passive ones positioned differently (left figure) and mortar fire observed by passive sensor (right figure, note its double pass through zero)

The only measurement considered here is the time t_R for which $f_D = 0$:

$$f_D = 0 \ \text{at} \ t = t_R, \tag{3}$$

which t_R depends on $\mathbf{S}$, $\mathbf{v}$, and $\mathbf{R}$.

In an antisniper system, we need to evaluate at least the direction of $\mathbf{v}$. It is shown below that this requires at least four independent measurements by four Doppler sensors placed at the positions $\mathbf{R}_i$, $i = 1,\ldots,4$ (Fig. 3), yielding four corresponding values t_i when their measured Doppler frequency shifts f_D pass by zero. This naturally happens when the corresponding angles $\theta = \pi / 2$, that is when $\mathbf{R}_i$ reside on the planes h_i perpendicular to $\mathbf{v}$.

Let us consider the first three measurements, made by three sensors residing on the plane $\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3$, let this plane be a horizontal one (the ground). In general, $\mathbf{v}$ is not coplanar with $\mathbf{R}_j$, $j = 1, 2, 3$. Assuming that $\mathbf{R}_j$ are on the ground, the bullet path, beginning at $\mathbf{S}$, has the azimuth α and the elevation β; $\mathbf{v}'$ is the velocity projection on the horizontal plane of sensors. The planes h_j cross the line $\mathbf{v}'$ at the points $\mathbf{A}_i$; it is evident that the horizontal lines $\mathbf{R}_j\mathbf{A}_j$ are perpendicular to $\mathbf{v}'$ which belongs to the vertical plane spanned by $(\mathbf{v}, \mathbf{v}')$.

At the measured times t_j, the bullet positions $\mathbf{P}_j$ are proportional to the bullet speed, Eq. (2). This determines certain proportion between $t_2 - t_1$ and $t_3 - t_1$, as well as between $|\mathbf{P}_2 - \mathbf{P}_1|$ and $|\mathbf{P}_3 - \mathbf{P}_1|$, and also $|\mathbf{A}_2 - \mathbf{A}_1|$ and $|\mathbf{A}_3 - \mathbf{A}_1|$, which are the projections of $|\mathbf{R}_2 - \mathbf{R}_1|$ and $|\mathbf{R}_3 - \mathbf{R}_1|$ on $\mathbf{v}'$:

$$\frac{(\mathbf{R}_3 - \mathbf{R}_1)\cdot\mathbf{v}'}{(\mathbf{R}_2 - \mathbf{R}_1)\cdot\mathbf{v}'} = \frac{t_3 - t_1}{t_2 - t_1}, \tag{4}$$

(dot means the scalar product). This proportion is satisfied only if the direction $\mathbf{v}'$ is correctly chosen. Thus, the bullet path azimuth (the angle α in Fig. 3) is determined by the correctly chosen direction $\mathbf{v}'$ satisfying Eq. 4. False choice of $\mathbf{v}'$, for example perpendicular to the line $\mathbf{R}_1\mathbf{R}_2$, would yield the same projection of $\mathbf{R}_1$ and $\mathbf{R}_2$ on it, clearly violating the above equation.

The bullet path elevation angle can be determined similarly by sensors residing on the vertical plane. Exploiting two earlier sensors, $\mathbf{R}_1$ and $\mathbf{R}_2$, for instance, the fourth sensor $\mathbf{R}_4$ must be placed somewhere above the ground (the plane $\mathbf{R}_1\mathbf{R}_2\mathbf{R}_4$ does not need to be vertical to make the evaluation, although it would make the evaluation harder). Hence, having arbitrary spatially positioned four Doppler sensors, Eq. (4) helps us to evaluate both the bullet velocity vector angles: azimuth and elevation.

Another measurement can be easily made by a Doppler sensor: the value of $\mathrm{d}f_D/\mathrm{d}t$ at t_i which, depending on both v and $|\mathbf{R}_i - \mathbf{P}_i|$, may help one to determine not only the bullet velocity direction but its value as well. Here, we neither discuss this possibility further, nor the performance of the system having only one active Doppler radar sensor and a number of passive sensors which measure the EM signal transmitted by the former and scattered by the bullet.

2. Calibration of radar systems

From Eq. (4), we easily notice the importance of using the correct coordinates $\mathbf{R}_i$ of the sensors. Obtaining them by triangulation may not be an easy task because they may reside on rough ground at distances of tens of meters from each other, particularly when the system is developed under enemy fire (in fact, the best system would exploit sensors sewn stochastically in front of the protected area, which area may not be easily accessed for ordinary triangulation [2]).

Here, we propose using a round of calibration shooting over the system of sensors which measured as described above, will provide information about the sensors' positions $\mathbf{R}_i$ instead of the bullet velocity direction. It is shown below that three shootings in different directions (different $\mathbf{v}$) are sufficient but larger round can improve the estimated values. Two Doppler radar systems are considered below: the first, having three active sensors, as presented schematically in Fig. 3, and the other having one active sensor and two passive ones, all three reporting the times t_i of observations of the zero Doppler frequency shift $f_D = 0$ of the EM signal radiated by the active sensor and scattered by the bullet.

2.1 Active Doppler radar system

Let the position $\mathbf{R}_1$ of the sensor is known only approximately and we need to correct it by observing the round of three calibration shootings from the different positions $\mathbf{S}_k$ and aimed at different directions (hence bullets have different velocities $\mathbf{v}_k$, $k = 1, 2, 3$). The zero Doppler shift time t_1 reported by the first sensor results from:

$$\begin{aligned} &\mathbf{P}_k = \mathbf{S}_k + \mathbf{v}_k t_1, \\ &(\mathbf{P}_k - \mathbf{R}_1) \cdot \mathbf{v}_k = 0, \text{ hence} \\ &\mathbf{v}_k \cdot \mathbf{R}_1 = \mathbf{P}_k \cdot \mathbf{v}_k, \end{aligned} \tag{5}$$

yielding three equations on three unknown coordinates of the vector $\mathbf{R}_1$. Knowing its approximate value may help us to choose $\mathbf{S}_k$, $\mathbf{v}_k$ in order to obtain the best system of equations.

The same round of shooting may be exploited for evaluation of the positions $\mathbf{R}_i$ of other sensors. Alternately, their relative positions $\mathbf{R}_{i1}$ with respect to exactly known $\mathbf{R}_1$ can be found from:

$$\mathbf{v}_k \cdot \mathbf{R}_{i1} = (t_{ik} - t_{1k})\,v^2, \quad \mathbf{R}_{i1} = \mathbf{R}_i - \mathbf{R}_1, \tag{6}$$

where t_{ik} are the times of zero Doppler frequency shift occurrences at $\mathbf{R}_i$ in case of kth shooting.

2.2. Passive Doppler system

In another version of antisniper Doppler radar system, there is one active Doppler radar placed at the position $\mathbf{R}_0$ (assumed known) that continuously lights the space sector of interest; the corresponding detection time of $f_D = 0$ is t_0. Another set of cheap, perhaps disposable, passive sensors (at $\mathbf{R}_i$, $\mathbf{R}$ in Fig. 3) receive both the direct and the scattered EM waves having zero Doppler shift occurrence at t_i, the times applied in computation of the object velocity direction in the way similar to that described above. Below, the problem of evaluation of $\mathbf{R}$ is discussed.

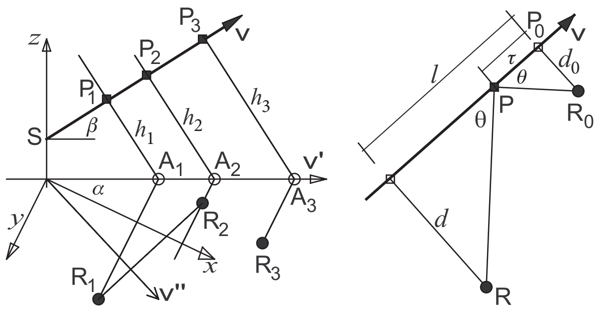

Fig. 3. Left: The rule for the calibrated active sensors: the projections $\mathbf{A}_i$ of the sensors' positions $\mathbf{R}_i$ on $\mathbf{v}'$ retain proportions between the bullet positions $\mathbf{P}_i$ at the observation times t_i. Wrong azimuth of $\mathbf{v}''$ makes projections of $\mathbf{R}_{1,2}$ identical, in clear disagreement with this rule. Right: Geometry of passive sensors

Let the series of bullets be shot with the known velocities $\mathbf{v}_k$. Their corresponding detections by active sensor placed at $\mathbf{R}_0$ start the time counting for the passive sensor which detects $f_D = 0$ at the corresponding times t_k, at which the bullet positions are $\mathbf{P}_k$. The time t_k results from:

$$\mathbf{P}_k = \mathbf{P}_0^{(k)} + \mathbf{v}_k t_k, \qquad \frac{(\mathbf{P}_k - \mathbf{R}_0)\cdot \mathbf{v}_k}{|\mathbf{P}_k - \mathbf{R}_0|} = \frac{(\mathbf{R} - \mathbf{P}_k)\cdot \mathbf{v}_k}{|\mathbf{R} - \mathbf{P}_k|}, \tag{7}$$

which is the condition of perfect balance of positive and negative Doppler frequency shift as the EM wave scatters from a moving object which distance from $\mathbf{R}$ grows and the distance to $\mathbf{R}_0$ shortens (Fig. 3). Here, the bullets' positions at the detection time by the active sensor $\mathbf{P}_0^{(k)}$ and their velocities $\mathbf{v}_k$, representing the parameters of calibration shootings, are known. Hence, a system of nonlinear equations results for the unknown $\mathbf{R}$ having three unknown coordinates (x, y, z); one may try the Newton method [3] of solution of these equations.

From geometrical point of view, the problem is even simpler. Let $t = 0$ be the time of zero Doppler shift observed by the active sensor in each case of shooting and

$\mathbf{P}_0^{(k)} = (X, Y, Z)^{(k)}$ be the corresponding bullet position on kth bullet path. We may evaluate the conical angle θ between the known $\mathbf{v}_k$ and $\mathbf{P}_k - \mathbf{R}_0$ (Fig. 3)

$$\cos\theta = v_k t_k \,/\left|\mathbf{P}_0^{(k)} + \mathbf{v}_k t_k - \mathbf{R}_0\right|. \tag{8}$$

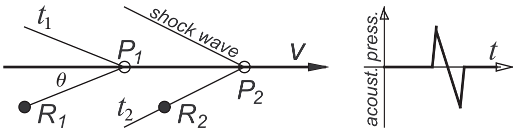

Fig. 4. Geometry of shock wave caused by supersonic bullet (left) and the distinctive N-shaped time dependence of acoustic pressure detected by microphones with its rising time much below 1 μs (right)

The passive sensor detecting $f_D = 0$ at the time t_k when the bullet position is $\mathbf{P}_k$ must reside on the same conical surface but on the other side of the cone tip $\mathbf{P}_k$, for the given kth shooting. Hence, $\mathbf{R} = (x, y, \mathrm{v})$ must be at the intersection point of three different cones associated with three calibration shootings.

For the given case k, the cone tip is determined by $\mathbf{P}_0 = (X, Y, Z)$ and $\mathbf{v}t = (\mathrm{u}, \mathrm{v}, \mathrm{w})$ (we neglect the subscripts k to simplify further notations): $\mathbf{P} = (X + \mathrm{u}, Y + \mathrm{v}, Z + \mathrm{w})$. The value of $\tau = |\mathbf{v}t\,|$ can be evaluated easily if we know the distances d_0 and d between the points $\mathbf{R}_0$ and $\mathbf{R}$ from the bullet straight path $\mathbf{P}_0 + \mathbf{v}t$, which last distance is (naturally, independent of t) [4]:

$$\begin{aligned} d^2 = (\mathrm{u}^2 + \mathrm{v}^2 + \mathrm{w}^2)^{-1}\{&[(x - X)\mathrm{v} - (y - Y)\mathrm{u}]^2 + [(y - Y)\mathrm{w} - (z - Z)\mathrm{v}]^2 + \\ &+ [(z - Z)\mathrm{u} - (x - X)\mathrm{w}]^2\} \end{aligned} \tag{9}$$

and similarly d_0 by replacing X, Y, Z by the coordinates of $\mathbf{R}_0$. Now, the distance $\tau = vt$ between $\mathbf{P}_0$ and $\mathbf{P}$ along the bullet path can be evaluated from the corresponding proportion:

$$\tau = vt = \frac{d_0 l}{d_0 + d}, \quad l = (\mathbf{R} - \mathbf{R}_0)\cdot\mathbf{v}/\,v, \tag{10}$$

from which we may obtain $t = \tau / v$ if $v = |\,\mathbf{v}\,|$ and $\mathbf{R}$ are known.

In the considered calibration problem, however, τ is known and the coordinates x, y, z of $\mathbf{R}$ are to be evaluated from the above equation repeated for all calibration shootings, which equation can be easily transformed to a four-order polynomial form for x, y, z:

$$[d^2 - (l^2/\tau^2 - 1)d_0^2]^2 - 4d_0^2 l^2/\tau^2 = 0, \tag{11}$$

(d_0, τ are known). There must be at least three such polynomials obtained from different calibration shootings to evaluate three unknown quantities x, y, z included in d and l. Similar triangulation problems have been already presented in literature [5] and there are also available Matlab subroutines [7, 8, 9] for their solutions.

3. Calibration of acoustic system

The shock wave is distinctive for supersonic bullet, thus its detection by microphones is widely used in antisniper systems [10] (the detection time inaccuracy is well below 1 μs, corresponding to a fraction of mm of the sound propagation distance). Here, we consider a system where several omnidirectional microphones (cheap and perhaps disposable) are scattered in the field (sewn randomly) and which only purpose is transmission of the raw detected waveform to the computation center through the radio link. Here, the characteristic shock wave is recognized and the corresponding time of its detection by the given microphone is evaluated. Further application of this observation for sniper detection requires exact coordinates of the microphones. Being sewn in the field spanning tens of meters, they may reside beyond optical access, preventing ones from using ordinary triangulation. The solution to this problem is proposed below, again by applying the round of calibration shooting.

Let the bullet velocity $\mathbf{v}$ (assumed constant here [11]) be greater than the sound velocity c (do not mistake it with light speed considered in previous sections). The cone tip of the shock wave resides at the moving bullet position $\mathbf{P}$, and its conical angle is 2θ, where

$$\sin\theta = c/v. \tag{12}$$

All observers residing on the same cone detect the shock wave at the same time; the different times t_i mean that observers (placed at $\mathbf{R}_i$) reside on different cones with tips at

$$\mathbf{P}_i = \mathbf{S} + \mathbf{v}t_i, \tag{13}$$

where $\mathbf{S}$ and $\mathbf{v}$ are the known parameters of the calibration shooting.

Summarizing, from the detection time of the shock wave by given microphone, we know the cone (characterized by the cone tip $\mathbf{P}$ and the angle θ) to which the microphone position $\mathbf{R}$ belongs. Having measurements of three or more calibration shootings, the microphone position can be evaluated as the intersection point of the corresponding cones. This is analogous triangulation problem to these considered in previous sections.

4. Conclusions

It is presented how typical measurements performed by antisniper sensors (the Doppler radar system is presented for the first time) can be exploited for evaluation of the sensor positions in the field that may be dangerous or difficult for direct access preventing ones of using ordinary triangulation. As the time measurements are typically quite accurate (corresponding to a fraction of millimeter in the acoustic case), the sensors' spatial positions accuracy is crucial for the system performance. In the analysis, we applied constant bullet velocity in order to simplify the considerations. Realistic model of bullet movement should include its deceleration $\mathbf{u}(\mathbf{v})$ due to friction in air (dependent on $\mathbf{v}$), gravity ($\mathbf{g}$), *etc.*, hence [12]

$$\mathbf{P} = \mathbf{S} + [\mathbf{v} - (\mathbf{u} + \mathbf{g})t/2]t, \tag{14}$$

is the bullet real path. This does not change much the equations presented above for the calibration purposes; it suffices to replace $\mathbf{v}$ by its actual value $\mathbf{v} = (\mathbf{u} + \mathbf{g})t$ at the observation time t. Also note that the shock wave conical angle, Eq. (12), changes respectively and its cone axis is tangential to the bullet path at given time of observation.

One may object, however, if both $\mathbf{v}$ and $\mathbf{u}$ are known with sufficient accuracy. This rises the question of self-calibration (deeply discussed in photogrammetrics, [13], for instance) of the system based on numerous sensors. The interesting problem of estimation [14] if not only the sensors' coordinates but also of the shooting parameters is far beyond the scope of this paper.

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E. DANICKI

Kalibracja radarowego I akustycznego systemu antysnajperskiego

Streszczenie. Antysnajperskie systemy wykorzystują pewną liczbę czujników (akustycznych lub mikrofalowych), współrzędne, których winny być znane z dużą dokładnością, w przeciwnym razie ich skuteczność będzie znacznie ograniczona. Triangulacja wielu czujników nie jest prostym zadaniem, w szczególności, jeśli miałaby być dokonana pod nieprzyjacielskim ogniem. Tu proponujemy pewną metodę wykorzystującą serię wystrzałów kalibracyjnych ponad rozsianymi w polu czujnikami, których pomiary pozwalają na określenie ich położenia. Metoda może być wykorzystana dla kalibracji systemu opartego na radarach Dopplerowskich prezentowanych w pewnej podstawowej konfiguracji, albo w systemach akustycznych, mierzących falę uderzeniową generowaną przez naddźwiękowe pociski.

Słowa kluczowe: systemy antysnajperskie, radar Dopplerowski, fale uderzeniowe
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