## The robust optimization method of functionally gradient materials under cyclic thermal and mechanical loading

G. Maciejewski<sup>†\*</sup> and Z. Mróz<sup>‡</sup>

Institute of Fundamental Technological Research Polish Academy of Sciences Pawińskiego 5b, 02-106 Warsaw, Poland

> <sup>†</sup>gmaciej@ippt.gov.pl <sup>‡</sup>zmroz@ippt.gov.pl

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## ABSTRACT

Functionally gradient materials (FGM) are currently a subject of intensive research. FGM provides opportunity to build a lightweight structure with unique properties that can not be attained from a homogeneous material. However, the design of composition distribution within FGM can be a challenging task for particular needs.

The composition distribution within FGM can be designed in a systematic way by the optimization procedure [1]. The objective function of optimization depends on the problem considered, but usually includes a measure of stress, fracture resistance, required temperature distribution, the strain energy density, or the peak of effective stresses [1, 2]. The optimization analysis of FGM is usually a complicated task. For a complex geometry, the topology optimization can be used. Material interfaces may be treated as zero level set functions and evolution of these functions will result in varying shape of the interfaces. When one material should be created inside another one, the topological derivative is used in the minimization procedure. Its numerical implementation is complicated within a finite element code. Additionally, when cyclic loading and temperature are present, as in exhaust valve analysis [3], the numerical optimization becomes computationally complex. Without simplification of the model, the optimization would be very time consuming.

In this presentation the robust optimization method of FGM is discussed for combined cyclic thermal and mechanical loading. The application to design of an exhaust valve of combustion engine is considered in detail, see Fig. 1. The density distribution of ceramic inclusions in a homogeneous metal matrix is specified by minimizing probability of plastic deformation and damage. The numerical analysis, cf. [4], exhibits very high stress levels at interface between ceramic coating and the metal substrate. The present analysis is simplified by considering static loading corresponding to extreme loading during one cycle and the equivalent heat flow condition. The heat loading is averaged over the one cycle of engine operation. The transient analysis of stress and temperature distribution field is used in the design study. The numerical example provided in the work illustrates the optimal density distribution of reinforcement for mechanical and thermal constraints.

The optimization problem of distribution of FGM is formulated as follows: find parameters  $x_i$  (corresponding to density of reinforcement phase within the matrix) to minimize the objective functional

$$J_{obj} = Max\{R_{pl}^{i}\} + Max\{R_{cr}^{i}\},$$
(1)



Figure 1: The schematic representation of the axisymmetric valve model used in the finite element simulations. The valve is subjected to mechanical loading at interfaces a and b, while thermal fluxes act at interfaces a, b, c, and d. The valve is in contact at interface b with the engine.

where yield resistivity and crack resistivity at point *i* are defined as following

$$R_{pl}^{i} = Max \left(\frac{\sigma_{ef}^{i}}{\sigma_{ef}^{l}}\right), \qquad (2)$$
$$R_{cr}^{i} = Max \left(\frac{\sigma_{cr}^{i}}{\sigma_{cr}^{l}}\right).$$

Here the  $\sigma_{ef}^i$  and  $\sigma_{cr}^i$  are the effective (Mises) stress and maximal tensile stress at point *i*, while  $\sigma_{ef}^l$  and  $\sigma_{cr}^l$  are their limit values. The objective functional should be minimized, while parameters  $R_{pl}^i$  and  $R_{cr}^i$  should be less than one in order to assure that the structure does not change its geometry and quality as an effect of plastic strains or crack generations.

The optimization procedure starts from the uniform ceramic material distribution and after transient thermal analysis the parameters  $R_{ef}^i$  and  $R_{cr}^i$  are determined at each point *i*. Next, the new distribution of material is designed by reducing concentration of the ceramic phase at places of high tensile stresses. After several steps, the optimal distribution of ceramic phase has been found. Because stress distribution is continuous, the obtained FGM has also continuous distribution of ceramic phase. Thus, the proposed method shares merits of standard optimization and topology optimization because it allows for creation of one phase of material inside the other. The numerical example shows practical usefulness of the method.

## References

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