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Comparison of two simple mathematical models for feed water heaters

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Abstract

The paper presents two mathematical models of feed water heaters. In the models the mass and energy balance equation and Peclet's law are used. In the first, simplified, model, there is no division into three zones: the heat exchanger is treated as a single zone and the mass and energy balance equation and Peclet's law are written for the heat exchanger. In the second model the heat exchanger is divided into three zones and the mass and energy balance equation and Peclet's law are written for each zone. The accuracy of the models was examined on the basis of real data. A comparison was performed for both models between water temperature at the outlet of the exchanger, measured and calculated.

Keywords: Feed water heater, heat transfer

1. Introduction

Feed water heaters are shell-tube heat exchangers whose task it is to heat boiler feed water in order to increase the efficiency of the system [1, 2]. The heat exchangers are supplied with steam from the turbine bleeds, mostly superheated steam. Heat from the condensed superheated steam is transmitted to heat the boiler feed water. The feed water heaters can be divided into three zones to effectively recover the heat from the superheated steam. In the first zone, which is called the desuperheating zone, the superheated steam cools down to the saturation state. In the second zone, a condensing zone, saturated steam condenses, whereas in the third zone, a subcooling zone, saturated water is cooled to below saturation temperature [3]. Feed water heaters can be horizontal or vertical. An example of a vertically oriented heat exchanger is presented with standard symbols in Fig. 1 [3, 4]. Depending on the steam pressure, high-pressure and low-pressure feed water heaters can be distinguished. An example of temperature distribution for the heat exchanger with standard symbols is presented in Fig. 2 [5, 6].

2. Mathematical models of feed water heaters

The high pressure feed water heater supplied by first bleed of a 200 MW turbine was analyzed. In reference conditions, the parameters have the following values: water pressure – 18.0 MPa, input temperature of water – 222.9°C, output temperature of water – 244.1°C, water mass flow – 180.5 kg/s, steam pressure – 4.11 MPa, steam temperature – 378°C, condensate temperature at the outlet – 238.3°C, condensate mass flux – 8.2 kg/s. The heat transfer surface area is 600 m^2. A block diagram of the heat ex-

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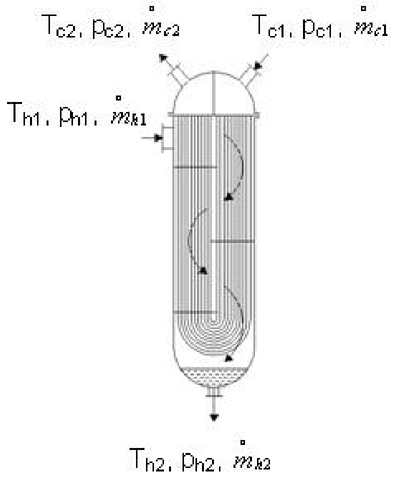

Figure 1: Diagram of a feed water heater

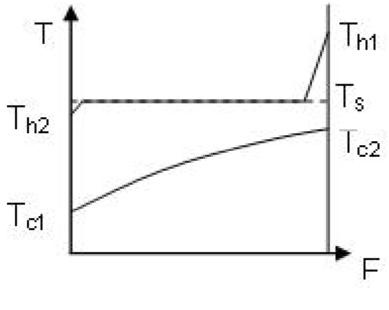

Figure 2: Example of temperature distribution for the feed water heater

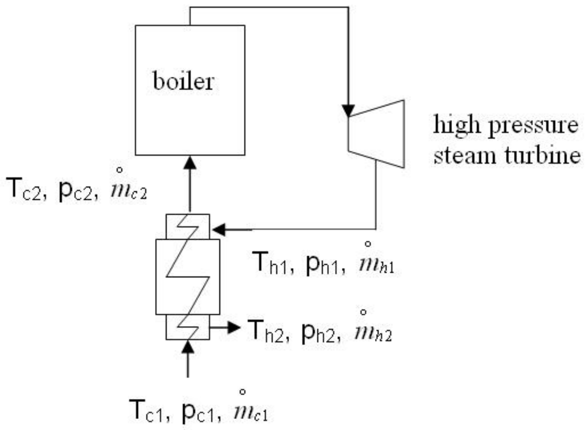

Figure 3: Diagram of a feed water heater with standard symbols

changer with standard symbols and location of the heat exchanger in the system is presented in Fig. 3 [2, 4].

Not all parameters within the feed water heater are measured. Therefore, it is reasonable to create mathematical models to determine the other unmeasured parameters. The paper presents two mathematical models of feed water heaters. In these models the mass and energy balance equation and Peclet's law are used. In order to simplify the models the following assumptions have been made: there is no pressure drop for both fluids and there is no heat loss to the environment. In the literature a mathematical model of a single-zone feed water heater can usually be found, which is a simpler model [1, 3, 7]. A three-zone model of a feed water heater is more complex, examples of which can be found in the literature [5, 7]. In some three-zone models overall heat transfer coefficients for each zone are functions of Nusselt, Reynold and Prandtl numbers [8–10] but for some approximate function for overall heat transfer coefficients the error is up $\pm 30\,\%$ [10]. In this article a three-zone model with constant overall heat transfer coefficients for each zone for the feed water heater is presented. This model was compared with the single-zone model as well as with real data.

2.1. The first mathematical model (single-zone model)

In this model the heat exchanger was not divided into three zones but is taken as a whole and a mass and energy balance equation and Peclet's law were written for the heat exchanger. With two equations (the mass balance equation is obvious) two unknown parameters can be determined. It was assumed that the searched values are water temperature at the outlet of the heat exchanger and mass flow of steam (it is also possible to seek a different combination of parameters). In this model the input parameters are: temperature, pressure and mass flow of water at the inlet to the heat exchanger, steam pressure, average heat transfer coefficient (k) and heat transfer surface (F). Computed values are: temperature, pressure and enthalpy of cooling water at the outlet of the heat exchanger and the mass flow of steam. The “average heat transfer coefficient” (k) was determined by means of the least squares method on the basis of the data for the first 20 points of work and the following value 7.656 kW/m^2/K was obtained.

Assuming there are no mass leaks in the feed water heater, the inlet mass flow is equal to the outlet mass flow by two sides

$$\dot{m}_{h1} - \dot{m}_{h2} = 0 \tag{1}$$

$$\dot{m}_{c1} - \dot{m}_{c2} = 0 \tag{2}$$

Assuming the change of kinetic and potential energy is negligible, the heat transfer can be written as

$$\dot{m}_{c1}(i_{c2} - i_{c1}) = \dot{Q} \tag{3}$$

Assuming there are no pressure drops for either of the fluids, outlet pressures can be written as

$$p_{h2} = p_{h1} \tag{4}$$

$$p_{c2} = p_{c1} \tag{5}$$

Saturation temperature can be determined from the dependence

$$T_s = f(p_{h1}) \tag{6}$$

The stream of heat transferred can be written as (Peclet’s law)

$$\dot{Q} = kF\triangle T_{ln} \tag{7}$$

Logarithmic mean temperature difference

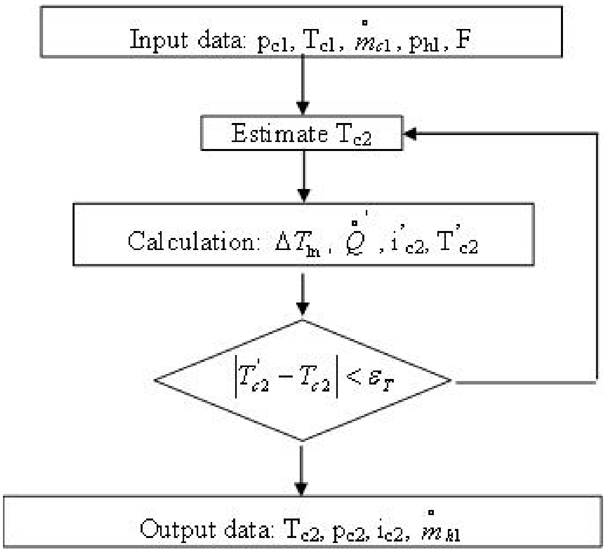

Figure 4: Block algorithm of computations for the single-zone model

$$\triangle T_{ln} = \frac{(T_s - T_{c2}) - (T_s - T_{c1})}{ln\frac{(T_s - T_{c2})}{(T_s - T_{c1})}} \tag{8}$$

Enthalpy of the outlet water from the heat exchanger can be determined as

$$i_{c2} = \frac{\dot{Q}}{\dot{m}_{c1}} + i_{c1} \tag{9}$$

Outlet temperature is a function of pressure and enthalpy

$$T_{c2} = f(p_{c2}, i_{c2}) \tag{10}$$

Steam mass is equal to

$$\dot{m}_{h1} = \frac{\dot{m}_{c1}(i_{c2} - i_{c1})}{i_{h1} - i_{h2}} \tag{11}$$

The block algorithm for a single-zone model with an indication of input and output data is presented in Fig. 4.

2.2. *The second mathematical model (three-zone model)*

In this model the heat exchanger was divided into three zones and for each zone the mass and energy balance equation and Peclet’s law are written. As with the single-zone model, the two unknown parameters can be determined for the three-zone model. It was assumed that the searched values are the water temperature at the outlet of the heat exchanger

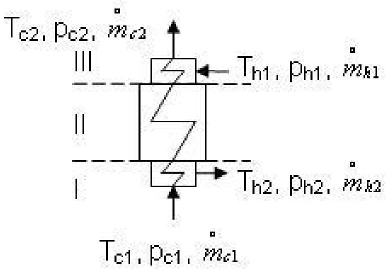

Figure 5: Diagram of the feed water heater with standard symbols and divided into three zones

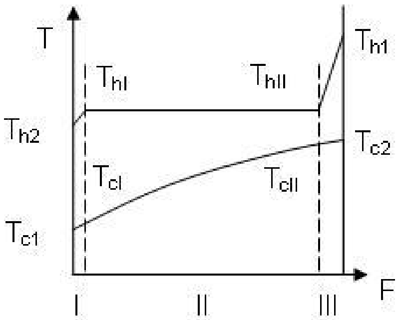

Figure 6: Example of temperature distribution for the feed water heater with standard symbols and divided into three zones

and the mass flow of steam (looking for a different combination of parameters is also possible). In this model input parameters are: temperature, pressure and mass flow of water at the inlet to the heat exchanger, pressure and steam temperature, condensate temperature, average heat transfer coefficients for each zone (k_{I}, k_{II}, k_{III}) and the heat transfer surface (F). Computed values are: temperature, pressure and enthalpy of cooling water at the outlet of the heat exchanger and the mass flow of steam. The average heat transfer coefficients for each zone (k_{I}, k_{II}, k_{III}) were determined by means of the least squares method on the basis of the data for the first 20 points of work and the following values 0.464, 3.439, 0.193 kW/m^2/K were obtained.

A diagram of a feed water heater with standard symbols and divided into three zones is presented in Fig. 5.

Example of temperature distribution for the feed water heater with standard symbols and divided into three zones is presented in Fig. 6.

After the division of the feed water heater into three zones an energy balance equation and a heat transfer equation are written for each zone. For the first zone of the feed water heater the following relations can be written

$$\dot{Q}_I = \dot{m}_{c1}(i_{cI} - i_{c1}) = \dot{m}_{h1}(i_{hI} - i_{h2}) \qquad (12)$$

Enthalpy at the inlet to the first zone is equal to

$$i_{hI} = i' = f(p_{h1}) \qquad (13)$$

The heat flux transferred

$$\dot{Q}_I = k_I F_I \triangle T_{lnI} \qquad (14)$$

Logarithmic mean temperature difference

$$\triangle T_{lnI} = \frac{(T_{hI} - T_{cI}) - (T_{h2} - T_{c1})}{ln\frac{(T_{hI}-T_{cI})}{(T_{h2}-T_{c1})}} \qquad (15)$$

Heat transfer surface area in the first zone can be written as

$$F_I = \frac{\dot{m}_{h1}(i_{hI} - i_{h2})}{k_I \triangle T_{lnI}} \qquad (16)$$

Enthalpy of the outlet water from the first zone can be determined as

$$i_{cI} = i_{c1} + \frac{\dot{m}_{h1}(i_{hI} - i_{h2})}{\dot{m}_{c1}} \qquad (17)$$

Outlet water temperature is a function of pressure and enthalpy

$$T_{cI} = f(p_{c1}, i_{cI}) \qquad (18)$$

For the second zone of the feed water heater the following equations can be written

$$\dot{Q}_{II} = \dot{m}_{c1}(i_{cII} - i_{cI}) = \dot{m}_{h1}(i_{hII} - i_{hI}) \qquad (19)$$

Enthalpy at the inlet to the second zone is equal to

$$i_{hII} = i' = f(p_{h1}) \qquad (20)$$

The heat flux transferred

$$\dot{Q}_{II} = k_{II} F_{II} \triangle T_{lnII} \qquad (21)$$

Logarithmic mean temperature difference

$$\begin{aligned} \triangle T_{lnII} &= \frac{(T_{hII} - T_{cII}) - (T_{hI} - T_{cI})}{ln\frac{(T_{hII}-T_{cII})}{(T_{hI}-T_{cI})}} \qquad (22) \\ &= \frac{T_{cI} - T_{cII}}{ln\frac{(T_{hII}-T_{cII})}{(T_{hI}-T_{cI})}} \end{aligned}$$

Heat transfer surface area in the second zone can be written as

$$F_{II} = \frac{\dot{m}_{h1}(i_{hII} - i_{hI})}{k_{II}\triangle T_{lnII}} \tag{23}$$

Enthalpy of the outlet water from the second zone can be determined as

$$i_{cII} = i_{cI} + \frac{\dot{m}_{h1}(i_{hII} - i_{hI})}{\dot{m}_{c1}} \tag{24}$$

Outlet water temperature is a function of pressure and enthalpy

$$T_{cII} = f(p_{c1}, i_{cII}) \tag{25}$$

For the third zone of the feed water heater the following equations can be written

$$\dot{Q}_{III} = \dot{m}_{c1}(i_{c2} - i_{cII}) = \dot{m}_{h1}(i_{h1} - i_{hII}) \tag{26}$$

Enthalpy of steam is equal to

$$i_{h1} = f(p_{h1}, T_{h1}) \tag{27}$$

The heat flux transferred

$$\dot{Q}_{III} = k_{III}F_{III}\triangle T_{lnIII} \tag{28}$$

Logarithmic mean temperature difference

$$\triangle T_{lnIII} = \frac{(T_{h1} - T_{c2}) - (T_{hII} - T_{cII})}{ln\frac{(T_{h1}-T_{c2})}{(T_{hII}-T_{cII})}} \tag{29}$$

Heat transfer surface area in the third zone can be written as

$$F_{III} = \frac{\dot{m}_{h1}(i_{h1} - i_{hII})}{k_{III}\triangle T_{lnIII}} \tag{30}$$

Enthalpy of the outlet water from the heat exchanger can be determined as

$$i_{c2} = i_{cII} + \frac{\dot{m}_{h1}(i_{h1} - i_{hII})}{\dot{m}_{c1}} \tag{31}$$

Outlet water temperature is a function of pressure and enthalpy

$$T_{c2} = f(p_{c1}, i_{c2}) \tag{32}$$

The total heat transfer surface area of the feed water heater is equal to

$$F = F_I + F_{II} + F_{III} \tag{33}$$

The block algorithm for a three-zone model with an indication of input and output data is presented in Fig. 7.

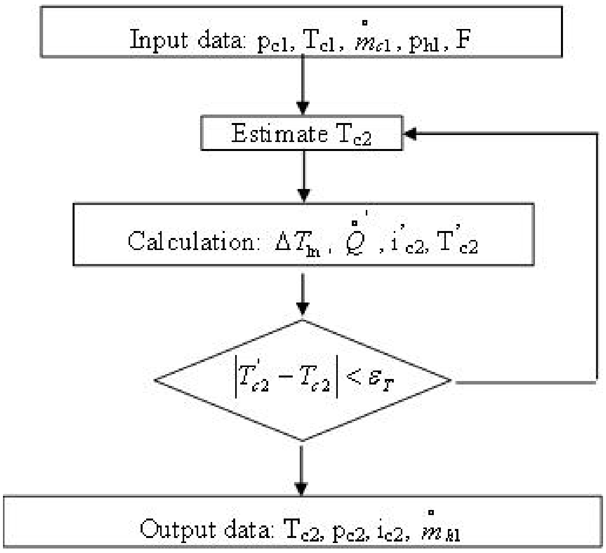

Figure 7: Block algorithm of computations for the three-zone model

3. Results

The high-pressure feed water heater supplied by the first 200 MW turbine bleed was analyzed. The verification of the models was based on measured data. The data in the system were recorded every 1 hour during normal operation of the block (average value for 1 hour). The analysis of the models was performed for 200 points. The distribution of temperatures in the feed water heater for the first point of work is presented in Fig. 8. The changes for the 200 points of work for the following parameters: water temperature at the inlet to the heat exchanger, steam temperature, steam pressure, mass flow of cooling water and the water pressure at the outlet of the heat exchanger are presented in Figures 9, 10, 11, 12 and 13.

The validation for the single-zone model was based on a comparison between the outlet water temperature from the exchanger, measured and calculated. The comparison between the water temperature at the outlet of the heat exchanger, calculated and measured, is presented in Fig. 14. The points on

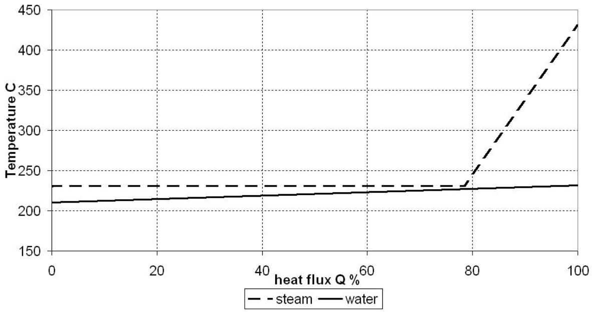

Figure 8: Temperature distribution in the feed water heater

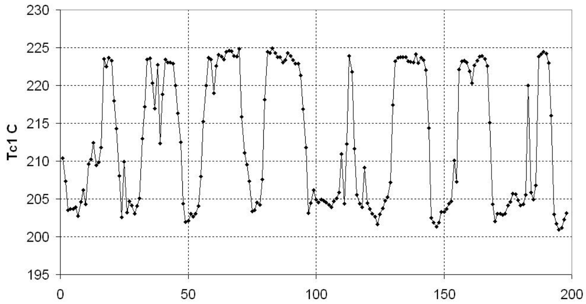

Figure 9: Water temperature at the inlet to the heat exchanger

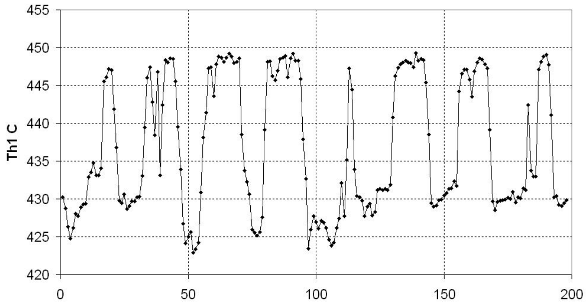

Figure 10: Steam temperature at the inlet to the heat exchanger

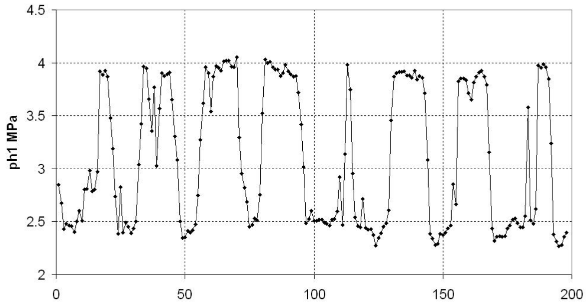

Figure 11: Steam pressure at the inlet to the heat exchanger

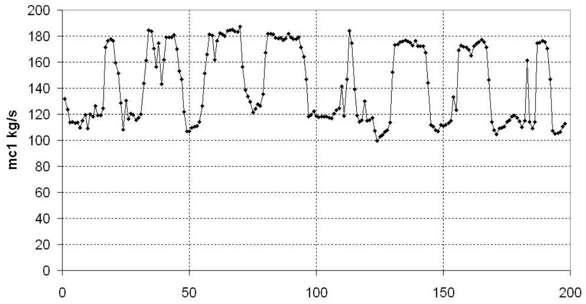

Figure 12: Mass flow of cooling water at the inlet to the heat exchanger

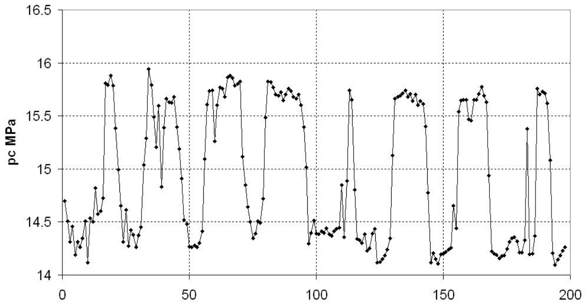

Figure 13: Water pressure at the outlet of the heat exchanger

the graph are arranged in a fairly good approximation of the equation along the y = x line. The change in water temperature at the outlet of the exchanger, measured and calculated for 200 points, is presented in Fig. 15. Significant differences occur between the measured and calculated temperatures at higher temperatures and are in the order of about 4 - 5 degrees Celsius.

The same comparison was performed for the three-zone model. The comparison between the wa-

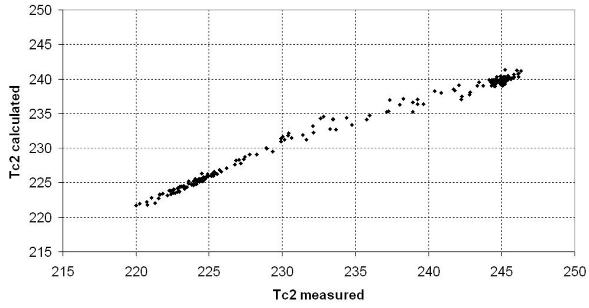

Figure 14: The comparison between water temperature at the outlet of the heat exchanger, measured and calculated (single-zone model)

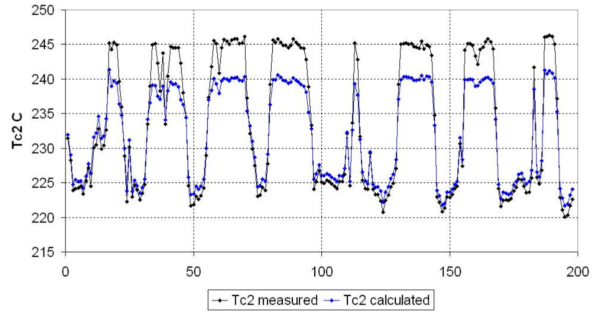

Figure 15: The change of water temperature at the outlet of the exchanger, measured and calculated (single-zone model)

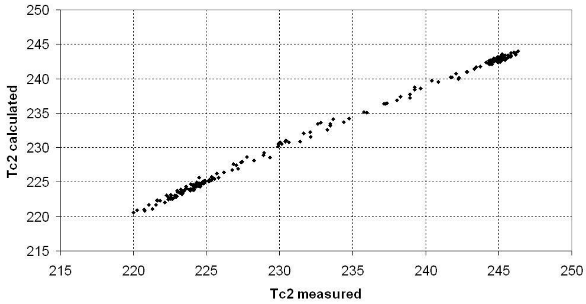

Figure 16: The comparison between water temperature at the outlet of the heat exchanger, measured and calculated (three-zone model)

ter temperature at the outlet of the heat exchanger, calculated and measured, is presented in Fig. 16. The points on the graph, as with the single-zone model, are arranged in a fairly good approximation of the equation along the y = x line. The change in water temperature at the outlet of the exchanger, measured and calculated for 200 points, is presented in Fig. 17. A more accurate water temperature at the outlet of the heat exchanger can be seen in Fig. 17.

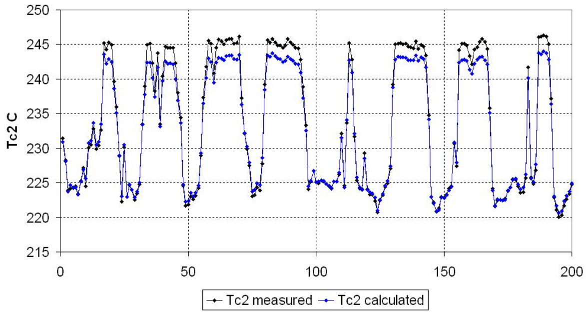

Figure 17: The change of water temperature at the outlet of the exchanger, measured and calculated (three-zone model)

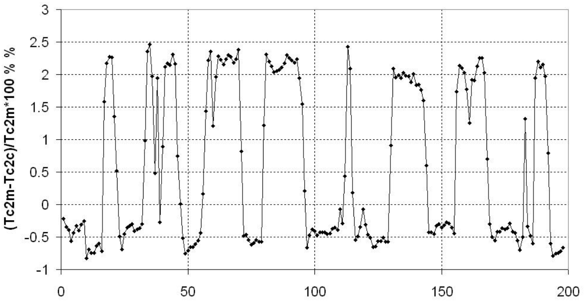

Figure 18: A relative difference between water temperature at the outlet of the heat exchanger measured (m) and calculated (c) in % (single-zone model)

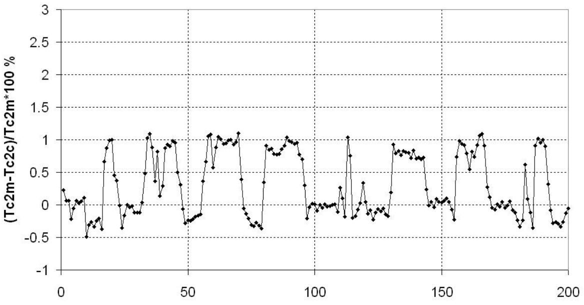

Figure 19: A relative difference between water temperature at the outlet of the heat exchanger measured (m) and calculated (c) in % (three-zone model)

The relative difference between water temperature at the outlet of the heat exchanger, measured and calculated in % for the single-zone model and the three-zone model, is presented in Fig. 18 and 19.

The calculated mass flow of steam for single-zone model and three-zone model is presented in Fig. 20 and 21.

Due to the less accurate single-model, the calculated mass flow of steam for the single-zone model is about three times higher than the calculated mass flow of steam for the three-zone model (Fig. 20, Fig. 21).

Calculated heat transfer surfaces for each zone ($F_{\mathrm{I}}, F_{\mathrm{II}}, F_{\mathrm{III}}$) for 200 points of work are presented in Fig. 22. Calculated heat transfer for each zone ($\dot{Q}_I, \dot{Q}_{II}, \dot{Q}_{III}$) are presented in Fig. 23. For the reference parameters calculated heat transfer surfaces are F_{I}=77.34 m^2, F_{II}= 218.72 m^2, F_{III}=303.94 m^2 (F_{I}=12.89 %, F_{II}=36.45 %, F_{III}=50.66 %) and calculated heat transfers are equal to $\dot{Q}_I$=758.50 kW, $\dot{Q}_{II}$=13 366.25 kW, $\dot{Q}_{III}$=2 848.44 kW ($\dot{Q}_I$=4.47 %,

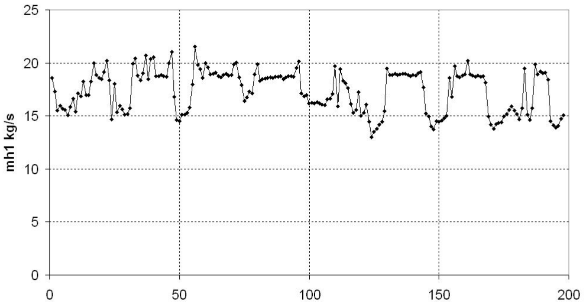

Figure 20: A calculated mass flow of steam for single-zone model

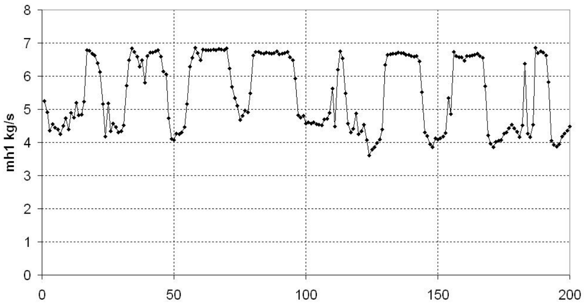

Figure 21: A calculated mass flow of steam for three-zone model

$\dot{Q}_{II}$=78.75 %, $\dot{Q}_{III}$=16.78 %).

About 5 % of heat is transferred in the subcooling zone (I), 75 % in the condensing zone (II) and 20 % in the desuperheating zone (III).

4. Conclusions

The paper presents two mathematical models describing the work of a high-pressure feed water heater supplied by the first 200 MW turbine bleed. In both models the overall heat transfer coefficients are constant. The single-zone model is simpler but less accurate (Fig. 18). The calculated mass flow of steam for the single-zone model is on average three times higher than the calculated mass flow of steam for the three-zone model. The three-zone model is more complex and more accurate (Fig. 19). Both models allow for the determination of two unknown parameters in the heat exchanger on the basis of measured parameters. In both models the water temperature at the outlet of the heat exchanger and the mass flow of steam were calculated. Any combination of two unknown parameters can be determined. In the second model the estimated value of the heat transfer surface (Fig. 22) and heat transfer (Fig. 23) in each zone can also be determined. The accuracy of models can be increased by expressing the overall heat transfer coefficients of Nusselt, Reynold and Prandtl numbers. Both models are simple and, importantly, user-friendly. Use of the three-zone model is recommended due to its greater accuracy.

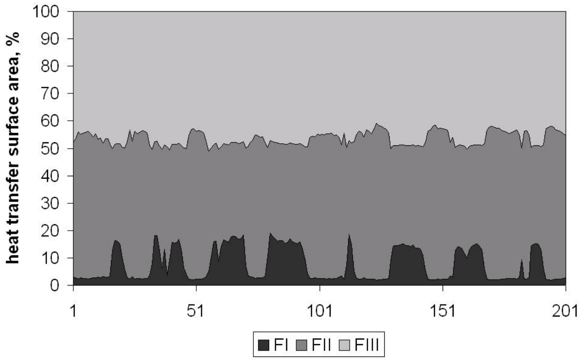

Figure 22: Calculated heat transfer surface area for each zone in %

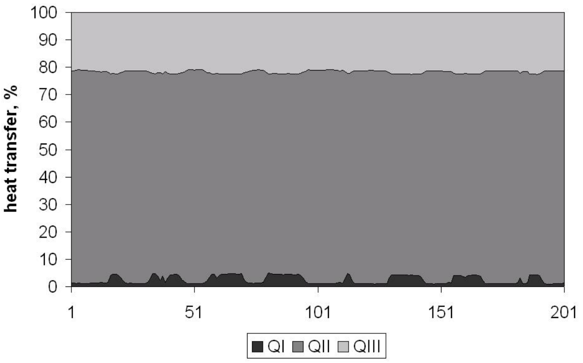

Figure 23: Heat transfer for each zone in %

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