# AN APPLICATION OF THE MAGNETO-RHEOLOGICAL ACTUATORS TO TORSIONAL VIBRATION CONTROL OF THE ROTATING ELECTRO-MECHANICAL SYSTEMS

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### **KEYWORDS**

Semi-active control, torsional vibrations, electro-mechanical drive system, control dampers, magneto-rheological fluid

#### ABSTRACT

In the paper control of transient and steady-state torsional vibrations of the driven by the asynchronous motor laboratory drive system of the imitated coal pulverizer is performed by means of actuators with the magneto-rheological fluid. The main purpose of these studies is a minimisation of vibration amplitudes in order to increase the fatigue durability of the most responsible elements. The theoretical investigations are based on a hybrid and finite element structural model of the vibrating mechanical system as well as on sensitivity analysis of the response with respect to the actuator damping characteristics. For suppression of transient torsional vibrations excited by electro-magnetic torques generated by the motor and by the coal pulverizer tool there is proposed a control strategy based on actuators in the form of rotary control dampers.

#### INTRODUCTION

Active vibration control of drive systems of machines, mechanisms and vehicles creates new possibilities of improvement of their effective operation. From among various kinds of vibrations occurring in the drive systems the torsional ones are very important as naturally associated with their fundamental rotational motion. Torsional vibrations of drive trains driven by electric motors are very dangerous for material fatigue of the most heavily affected and responsible elements of these mechanical systems. Thus, this problem has been considered for many years by many authors, e.g. in [1-3]. Nevertheless, till present majority of these studies are reduced to possibly accurate standard transient vibration analyses taking into consideration additional dynamic effects caused by elastic couplings, dry friction in clutches, electromechanical coupling effects, properties of the gear stage meshings, e.g. backlash, and others. Control of torsional vibrations occurring in the drive systems could effectively

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minimize material fatigue and in this way it would enable us to increase their operational reliability and durability. Unfortunately, one can find not so many published results of research in this field, beyond some attempts performed by active control of shaft torsional vibrations using piezo-electric actuators, [4]. But in such cases relatively small values of control torques can be generated and thus the piezo-electric actuators can be usually applied to low-power drive systems. Moreover, even if a relatively big number of the piezo-electric actuators are attached to the rotor-shafts of the entire drive system, as it follows e.g. from [4], only higher eigenmodes can be controlled, whereas control of the most important fundamental eigenmodes is often not sufficiently effective.

Torsional vibrations are in general rather difficult to control not only from the viewpoint of proper control torque generation, but also from the point of view of a convenient technique of imposing the control torques on quickly rotating parts of the drive-systems. In this paper there is proposed the semi-active control technique based on the actuators in the form of rotary dampers with the magneto-rheological fluid (MRF). In these actuators between the shaft and the inertial ring, which is freely rotating with a velocity close or equal to the system average rotational speed, the magneto-rheological fluid of adjustable viscosity is used. Such actuators generate control torques that are functions of the shaft actual rotational speed, which consist of the average component corresponding to the rigid body motion and of the fluctuating component caused by torsional vibrations.

The general purpose of the considerations is to control torsional vibrations of the power-station coal-pulverizer drive system driven by means of the asynchronous motor. In order to develop the most effective control algorithm the proper test-rig of analogous function and structure as these of the coal-pulverizer drive system is under construction. Then, using this test-rig the theoretical findings are going to be experimentally verified in the nearest future. This test-rig drive system is also driven by the asynchronous motor and loaded by constant and variable retarding torques generated by the programmable electric brake. In order to increase a reliability of the obtained theoretical results, the investigations are performed by means of the hybrid and the classical finite element (FEM) structural electro-mechanical

models of the considered drive system as well as using sensitivity analysis of the responses with respect to the actuator damping characteristics. These two models consisting of rigid bodies and continuous or discretized deformable finite elements, respectively, are employed here for eigenvalue analyses as well as for numerical simulations of torsional vibrations of the drive system coupled with electrical vibrations in the motor windings.

In the computational examples transient torsional vibrations induced by the electromagnetic motor torque during start-ups as well as by the variable dynamic retarding torque during steady-state operation of the drive system have been efficiently attenuated to the quasi-static level. Here, the dynamic response and its control were investigated in the space of modal functions in the case of the hybrid model and in the domain of generalized co-ordinates in the case of the finite element model application.

#### ASSUMPTIONS FOR THE MECHANICAL MODELS AND FORMULATION OF THE PROBLEM

In the considered laboratory drive system imitating operation of the coal pulverizer power is transmitted from the servo-asynchronous motor to the driven machine tool in the form of electric brake by means of the two multi-disk elastic couplings with built-in torque-meters, electro-magnetic overload coupling and by the shaft segments. Moreover, this system is equipped by two rotary magneto-rheological dampers and two inertial disks of adjustable mass moments of inertia and axial positions, which enable us to tune-up the drive train to the proper natural frequency values. The considered real laboratory drive system is presented in Fig. 1. Its corresponding hybrid and finite element mechanical models are shown in Fig. 2a and 2b, respectively.



M-R damper(1 or 2) overload coupling

torque - meter



Fig.2 Mechanical models of the drive systems: the hybrid model (a), the finite element model (b).

In order to perform a theoretical investigation of the semi-active control applied for this mechanical system, a reliable and computationally efficient simulation models are required. In this paper dynamic investigations of the entire drive system are performed by means of two structural models consisting of torsionally deformable one-dimensional beam-type finite elements and rigid bodies. These are the discrete-continuous (hybrid) model presented in Fig. 2a and the classical finite element model shown in Fig. 2b. Both models are employed here for eigenvalue analyses as well as for numerical simulations of torsional vibrations of the drive train. In the hybrid model successive cylindrical segments of the stepped rotor-shaft are substituted by the cylindrical macro-elements of continuously distributed inertial-viscoelastic properties, as presented in Fig. 2a. However, in the finite element model these continuous macro-elements have been discretized with a proper mesh density assuring a sufficient accuracy of results. Since in the real drive system the electric motor coils and coupling disks are attached along some rotor-shaft segments by means of shrink-fit connections, the entire inertia of such components is increased, whereas usually the shaft cross-sections only are affected by elastic deformations due to transmitted loadings. Thus, the corresponding visco-elastic macro-elements in the hybrid model and the discretized finite elements in the FEM model must be characterized by the geometric cross-sectional polar moments of inertia  $J_{Ei}$  responsible for their elastic and inertial properties as well as by the separate layers of the polar moments of inertia  $J_{Ii}$  responsible for their inertial properties only, i=1,2,...,n, where *n* is the total number of macroelements in the considered hybrid model, Fig. 2a. Moreover, on the actual operation temperature  $T_i$  can depend values of Kirchhhoff's moduli  $G_i$  of the rotor-shaft material of density  $\rho$ for each *i*-th macro-element representing given rotor-shaft segment. In the proposed hybrid and FEM model of the coal pulverizer drive system inertias of the inertial disks are represented by rigid bodies attached to the appropriate macroelement extreme cross-sections, which should assure a reasonable accuracy for practical purposes.

Torsional motion of cross-sections of each visco-elastic macro-element in the hybrid model is governed by the hyperbolic partial differential equations of the wave type

$$G_{i}(T_{i})J_{\mathrm{E}i}\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial^{2}\theta_{i}(x,t)}{\partial x^{2}}-c_{i}\frac{\partial\theta_{i}(x,t)}{\partial t}-$$
$$-\rho\left(J_{\mathrm{E}i}+J_{\mathrm{I}i}\right)\frac{\partial^{2}\theta_{i}(x,t)}{\partial t^{2}}=q_{i}\left(x,t,\theta_{i}(x,t)\right),\quad(1)$$

where  $\theta_i(x,t)$  is the angular displacement with respect to the shaft rotation with the average angular velocity  $\Omega$ ,  $\tau$  denotes the retardation time in the Voigt model of material damping and  $c_i$  is the coefficient of external (absolute) damping. The response dependent external active and passive torques are continuously distributed along the respective macro-elements of the lengths  $l_i$ . These torques are described by the twoargument function  $q_i(x,t,\theta_i(x,t))$ , where x is the spatial coordinate and t denotes time.

Mutual connections of the successive macro-elements creating the stepped shaft as well as their interactions with the rigid bodies are described by equations of boundary conditions. These equations contain geometrical conditions of conformity for rotational displacements of the extreme cross sections for  $x=L_i=l_1+l_2+...+l_{i-1}$  of the adjacent (*i*-1)-th and the *i*-th elastic macro-elements:

$$\theta_{i-1}(x,t) = \theta_i(x,t) \quad \text{for } x = L_i.$$
(2a)

The second group of boundary conditions are dynamic ones, which contain equations of equilibrium for external and control torques as well as for inertial, elastic and external damping moments. For example, the dynamic boundary condition describing a simple connection of the mentioned adjacent (i-1)-th and the i-th elastic macro-elements has the following form:

$$M_{i}(t) - I_{0i} \frac{\partial^{2} \theta_{i}}{\partial t^{2}} - M_{i}^{D}(t) + G_{i}(T_{i})J_{Ei} \left(1 + \tau \frac{\partial}{\partial t}\right) \frac{\partial \theta_{i}}{\partial x} - G_{i-1}(T_{i-1})J_{E,i-1} \left(1 + \tau \frac{\partial}{\partial t}\right) \frac{\partial \theta_{i-1}}{\partial x} = 0$$
(2b)

for 
$$x = L_i$$
,  $i = 2, 3, ..., n$ ,

where  $M_i(t)$  denotes the external concentrated torque,  $I_{0i}$  is the mass polar moment of inertia of the rigid body and  $M_i^{\rm D}(t)$  denotes the control damping torque.

In order to perform an analysis of natural elastic vibrations, all the forcing and viscous terms in the motion equations (1) and boundary conditions (2b) have been omitted. An application of the solution of variable separation for Eqs. (1) leads to the following characteristic equation for the considered eigenvalue problem:

$$\mathbf{C}(\boldsymbol{\omega}) \cdot \mathbf{D} = \mathbf{0}, \qquad (3)$$

where  $C(\omega)$  is the real characteristic matrix and **D** denotes the vector of unknown constant coefficients in the analytical local eigenfuctions of each *i*-th macro-element. Thus, the determination of natural frequencies reduces to the search for values of  $\omega$ , for which the characteristic determinant of matrix **C** is equal to zero. Then, the torsional eigenmode functions are obtained by solving equation (3).

The solution for forced vibration analysis has been obtained using the analytical - computational approach. Solving the differential eigenvalue problem  $(1)\div(3)$  and an application of the Fourier solution in the form of series in the orthogonal eigenfunctions lead to the set of uncoupled modal equations for time coordinates  $\xi_m(t)$ :

$$\ddot{\xi}_{m}(t) + (\beta + \tau \omega_{m}^{2}) \dot{\xi}_{m}(t) + \omega_{m}^{2} \xi_{m}(t) = \frac{1}{\gamma_{m}^{2}} Q_{m}(t),$$
$$m = 1, 2, \dots, \quad (4)$$

where  $\omega_m$  are the successive natural frequencies of the drive system,  $\beta$  denotes the coefficient of external damping assumed here as proportional damping to the modal masses  $\gamma_m^2$  and  $Q_m(t)$  are the modal external excitations.

For the assumed analogous linear finite element model the mathematical description of its motion has the classical form of a set of coupled ordinary differential equations

$$\mathbf{M}\,\ddot{\mathbf{s}}(t) + \mathbf{C}\,\dot{\mathbf{s}}(t) + \mathbf{K}\,\mathbf{s}(t) = \mathbf{F}\big(t, s(t), \dot{s}(t)\big),\tag{5}$$

where  $\mathbf{s}(t)$  denotes the vector of generalized co-ordinates  $s_j(t)$ , **M**, **C** and **K** are respectively the mass, damping and stiffness matrices and **F** denotes the time- and system response – dependent external excitation vector. By means of equations (5) numerical simulations of the forced torsional vibrations for the passive and controlled system can be carried out. In order to determine natural frequencies and eigenvectors for the FEM model of this drive system it is necessary to reduce (5) into the form of so called standard eigenvalue problem.

## CONTROL CONCEPTS OF THE TRANSIENT AND STEADY-STATE TORSIONAL VIBRATIONS

As it was mentioned above, since torsional vibrations are rather difficult to control, apart from some attempts with the piezo-electic actuators other control strategies can be hardly found in the literature. Thus, an application of actuators with the magneto-rheological fluid (MRF) for the considered purpose seems to be very promising. The magnetorheological fluids are functional fluids whose effective viscosity depends on externally provided magnetic field. This feature makes them perfectly suitable for large brakes and clutches with controllable damping characteristic. Besides the ability to generate large damping torques, an important advantage of the MRF-based devices is a low power consumption. External power is needed to supply the electromagnetic windings only, i.e. to modify the dynamic characteristics of the mechanical system, which is the distinguishing feature of the semi-active control. Moreover, the semi-active damping-based approach eliminates the risk of instability, which is intrinsically related to the active control paradigm and which could potentially occur in the case of an electrical failure, unexpected control time delays or in the case of an inaccurate modeling. In addition, it should be remarked that, as demonstrated in [6], there exist effective control methodologies based on actuators with the MRF, which are characterized by actual control delays reaching few milliseconds only. Thus, this technique enables us to control effectively vibrations of frequency exceeding 100 Hz or more.

According to the above, in [7] in order to attenuate transient torsional vibrations occurring during start-ups of the radial compressor geared drive system driven by electric motors, two actuating techniques were proposed: The first one applied actuators in the form of control brakes similar to the journal bearings, but instead of the oil film between the shaft and the motionless bushing, the MRF of variable viscosity was used. As it follows from [7], when maximal fluctuation of the vibratory component of the shaft rotational speed during run-ups occurs at the beginning of the process for relatively small system average rotational speeds, such actuators can suppress the predominant vibratory component of the shaft rotational speed. But for greater average rotational speed values the control brakes could even increase the system oscillations by retarding the rigid body motion and by keeping longer in this way the rotor-to-stator slip together with the external excitation frequency almost constant, which would lead to a greater fatigue effort of the shaft material and to bigger energy consumption. Moreover, such actuators would be prone to excessive wearing due to large abrasive forces.

Thus, the second actuating technique proposed in [7] was based on rotary dampers and realized in an analogous way to the known torsional vibration viscous dampers installed on the reciprocating engine crankshafts. But here, instead of the silicon oil between the shaft and the inertial ring, which can freely rotate with a velocity close or equal to the system average rotational speed, the magneto-rheological fluid of an adjustable viscosity is used. Such actuators generate control torques that are functions of the difference between the rotational speeds of the rings and of the shaft, where the latter consists of the average component corresponding to the rigid body motion and of the fluctuating component caused by the torsional vibrations. Since the average rotational speeds of the rings and of the shaft are similar, only small wearing effects can be expected and vibrations can be suppressed without influencing much the rigid body motion of the drive system.

Because of the abovementioned reasons, in this paper, the control is going to be realized by the rotary dampers with the magneto-rheological fluid (MRF) applied to attenuate transient and steady-state torsional vibrations excited during start-ups and nominal operation of the laboratory drive system driven by the asynchronous motor. The control dampers suppress the vibrations by attenuating the differences between the rotational velocities of the vibratory and the average drive system motion. Thus, in the considered laboratory drive system there are installed two rotary dampers with freely rotating inertial rings, each with the independently controllable damping coefficient  $k_j(t)$ , j=1,2. Each damper generates the following damping torque:

$$M_{j}^{D}(t) = -k_{j}(t)\Delta\Omega_{j}(t) =$$
$$= -k_{j}(t) \left[ \Omega(t) + \sum_{m=1}^{\infty} \dot{\theta}_{m}(x_{j}, t) - \Omega_{j}(t) \right], \quad j = 1, 2. \quad (6)$$

where  $x_j$  is the location of the *j*-th damper and  $\Omega_j(t)$  is the rotational speed of the *j*-th inertial ring, which obeys

$$J_{j}\dot{\Omega}_{j}(t) = -k_{j}(t) \left[\Omega_{j}(t) - \Omega(t) - \sum_{m=1}^{\infty} \dot{\theta}_{m}\left(x_{j}, t\right)\right], \quad (7)$$

where  $J_j$  denotes the *j*-th inertial ring polar mass moment of inertia.

The damping torques  $M_j^D(t)$  can be regarded as the response-dependent control external excitations. Then, by a transformation of them into the space of modal coordinates  $\xi_m(t)$  and upon a substitution of (6) into independent equations (4), the following set of coupled modal equations is yielded:

$$\mathbf{M}_{0}\ddot{\mathbf{r}}(t) + \mathbf{D}(k_{i}(t),\dot{\mathbf{r}}(t))\dot{\mathbf{r}}(t) + \mathbf{K}_{0}\mathbf{r}(t) = \mathbf{F}(t,\dot{\mathbf{r}}(t)), \quad (8)$$

where 
$$\mathbf{D}(\dot{\mathbf{r}}(t)) = \mathbf{D}_0 + \mathbf{D}_C(k_j(t), \dot{\mathbf{r}}(t)), \ j = 1,2.$$

The symbols  $\mathbf{M}_0$ ,  $\mathbf{K}_0$  and  $\mathbf{D}_0$  denote, respectively, the constant diagonal modal mass, stiffness and damping matrices. The full matrix  $\mathbf{D}_{C}(k_{j}(t), \dot{\mathbf{r}}(t))$  plays here a role of the semi-active control matrix and the symbol  $\mathbf{F}(t, \dot{\mathbf{r}}(t))$  denotes the response dependent external excitation vector due to the electromagnetic torque generated by the electric motor and due to the retarding torque produced by the driven imitated coal pulverizer. The Lagrange coordinate vector  $\mathbf{r}(t)$  consists of the unknown time functions  $\xi_m(t)$  in the Fourier solutions,  $m=1,2,\ldots$ . The number of equations (8) corresponds to the number of torsional eigenmodes taken into consideration in the range of frequency of interest. These equations are mutually coupled by the out-of-diagonal terms in matrix **D** regarded as external excitations expanded in series in the base of orthogonal analytical eigenfunctions. A fast convergence of the applied Fourier solution enables us to reduce the appropriate number of the modal equations to solve in order to obtain a sufficient accuracy of results in the given range of frequency. Here, it is necessary to solve only 6÷10 coupled modal equations (8), contrary to the classical one-dimensional rod finite element formulation leading in general to a relatively large number of motion equations (5) in the generalized coordinates.

Since in the investigated case, during start-ups and steady-state operation the drive system can be affected by external excitations that are unknown in advance, any effective and practically usable control strategy has to be passive or closed-loop. According to the above, it is assumed here that the feed-backs are the current frequency spectra  $\omega_e(f,t)$  and  $\omega_r(f,t)$  of the electric and the retarding torques, which can be estimated on-line from the current slip values determined form measurements of the rotational velocities of the appropriate drive system shaft cross-sections. The locally optimum control functions  $k_j(t)$  are determined with respect to the frequency response functions  $\text{FRF}_e(f,\mathbf{k})$  and  $\text{FRF}_r(f,\mathbf{k})$  of the damped drive system excited respectively by the electric motor and by the receiver. In this study two basic control modes are proposed:

1. Semi-active control, for which the optimum value  $\mathbf{k}_0(t)$  of the vector of the damping coefficients  $k_j(t)$  is determined as

$$\mathbf{k}_{\mathbf{0}}(t) = \arg\min_{\mathbf{k}} \int [\omega_{e}(f,t) \operatorname{FRF}_{e}(f,\mathbf{k}) + \omega_{r}(f,t) \operatorname{FRF}_{r}(f,\mathbf{k})] df.$$
(9)

2. Passive control, for which the damping coefficients remain constant during the whole run-up process and the steadystate operation. However, their values are optimum with respect to the frequency response function of the damped drive system excited by the receiver, as defined by

$$\mathbf{k}_{0} = \arg\min_{\mathbf{k}} \max_{f} \operatorname{FRF}_{\mu}(f, \mathbf{k}).$$
(10)

Both control modes are going to be tested numerically in the computational examples described below.

#### MODELLING OF THE ELECTRICAL EXTERNAL EXCITATION GENERATED BY THE SERVO-ASYNCHRONOUS MOTOR

The torsional vibrations of the drive system usually result in significant fluctuation of rotational speed of the rotor of the driving electric motor. Such oscillation of the angular velocity superimposed on the average rotor rotational speed cause more or less severe perturbation of the electro-magnetic flux and thus additional oscillation of the electric currents in the motor windings. Then, the generated electromagnetic torque is also characterized by additional variable in time components which induce torsional vibrations of the drive system. According to the above, the mechanical vibrations of the drive system become coupled with the electrical vibrations of the currents in the motor windings. Such coupling is often complicated in character and thus computationally troublesome. Because of this reason, till present majority of authors used to simplify the matter regarding mechanical vibrations of the drive systems and electric current vibrations in the motor windings as mutually uncoupled. Then, the mechanical engineers apply the electromagnetic torques generated by the electric motors as 'a priori' assumed excitation functions of time or the rotor-tostator slip, e.g. in [1,5], usually determined by means of experimental measurements for numerous dynamic behaviours of the given electric motor. However, the electricians thoroughly model generated electromagnetic torques basing on electric current flows in the electric motor windings, but they usually reduce the mechanical drive system to one or seldom to few rotating rigid bodies. In many cases such simplifications yield insufficiently exact results, both for the mechanical and the electrical part of the investigated object.

In order to develop a proper control algorithm for the given vibrating drive system the electromagnetic external excitation produced by the motor should be described possibly accurately and thus the electromechanical coupling between the electric motor and the torsional train ought to be taken into consideration. According to the above, apart of the sufficiently realistic mechanical models of the vibrating object, it is also necessary to introduce a proper mathematical model of the electric motor. In the considered case of the symmetrical three-phase asynchronous motor electric current oscillations in its windings are described by six voltage equations, which can be found e.g. in [8]. Then, the electromagnetic torque generated by such motor can expressed by the following formula

$$T_e = \frac{3}{2} pM \left[ (i_{\beta}^{s} i_{\alpha}^{r} - i_{\alpha}^{s} i_{\beta}^{r}) \cos p \mathcal{I} - (i_{\alpha}^{s} i_{\alpha}^{r} + i_{\beta}^{s} i_{\beta}^{r}) \sin p \mathcal{I} \right], (11)$$

where *M* denotes the relative rotor-to-stator coil inductance, *p* is the number of pairs of the motor magnetic poles,  $\vartheta$  denotes the rotation angle between the rotor and the stator and  $i_{\gamma}^{q}$ ,  $\gamma=\alpha,\beta$ , are the electric currents in the rotor for q=r and the stator for q=s reduced to the electric field equivalent axes  $\alpha$  and  $\beta$ , [8]. From the abovementioned system of voltage equations as well as from formula (11) it follows that the coupling between the electric and the mechanical system is

non-linear in character, which leads to very complicated analytical description resulting in rather harmful computer implementation. Thus, this electromechanical coupling has been realized here by means of the step-by-step numerical extrapolation technique, which for relatively small direct integration steps for equations (8) results in very effective, stable and reliable results of computer simulation.

#### **COMPUTATIONAL EXAMPLES**

In the computational examples there are investigated start-ups and following after them steady-state operation of the considered laboratory drive system of the imitated coal pulverizer. This system presented in Fig. 1 is accelerated from a standstill to the nominal operating conditions characterized by the rated retarding torque  $M_n$ =21.5 Nm at the constant rotational speed 1680 rpm. In order to imitate operation of the coal pulverizer in a possibly realistic way, the retarding torque produced by the machine working tool has been assumed as linearly proportional to the current shaft rotational speed with a superimposed step-wise fluctuation component of also velocity dependent amplitude

$$M_r(\Omega_r(t)) = \frac{M_n}{\Omega_n} \Omega_r(t) \cdot \left(1 + z \cdot \operatorname{sgn}\left(\sin(k\vartheta_r(t))\right)\right), \quad (12)$$

where  $\Omega_n$  denotes the nominal angular velocity,  $\Omega_r$  (t) is the current angular velocity of the working tool, h denotes the step fluctuation ratio,  $\vartheta_r(t)$  is the working tool current phase angle and z denotes the frequency parameter of the retarding torque oscillation. It is to remark that function (12) describing the assumed retarding torque consists of the average component expressing a mean resistance of the comminuted coal of a given average density and of the fluctuating one, which represents rapid changes of the braking moment caused by a non-homogeneous structure of the pulverized material. Here, for temporary negative values of the retarding torque  $M_r(t)$  is assumed to be equal to zero. The retarding torque (12) as well as the electromagnetic torque (11) generated by the servo-asynchronous motor are assumed to be uniformly distributed along the mechanical model elements representing the machine working tool and the motor rotor, respectively.

The qualitative dynamic properties of the considered drive system have been determined first in the form of an eigenvalue analysis by solving equation (3). In Fig. 3 there are depicted the lowest for this system first three eigenfunctions together with the corresponding natural frequency values. It is to emphasize that all eigenfunctions presented in Fig. 3 respectively almost overlay with the corresponding plots of eigenvectors obtained using the FEM model of this drive system consisting of 74 two-node rod elements shown in Fig. 2b, where the respective differences of successive natural frequencies did not exceed 1 %.

From the viewpoint of the studied dynamic process and the vibration control in the range of the most fundamental frequency range  $0\div300$  Hz these first three eigenforms seem to be significant. First of all, their natural frequency values are relatively close to frequencies of excitations generated by the considered asynchronous motor as well as generated by the retarding torque. The first two eigenfuctions are characterized by the greatest modal displacements



Fig.3 Eigenfuctions with the corresponding natural frequencies of the drive system

corresponding to the positions of the dynamic excitation source, i.e. the driving motor. The first eigenform of the natural frequency 42.6 Hz is also characterized by the greatest modal displacements corresponding to the position of the imitated coal pulverizer working tool, which is very essential for imposing the control torques on the vibrating system, Fig. 3. However, the third eigenform of the natural frequency 296.4 Hz is characterized by the greatest modal displacements corresponding to the position of the imitated coal pulverizer working tool, but as it follows from Fig. 3, its eigenfunction is almost insensitive to the excitation by the motor and the control torques, since the modal displacements corresponding to the motor and actuator positions are close to zero. According to the above, because the external excitation produced by the asynchronous motor is expected as particularly severe, in the considered system for the possibly effective semi-active control, two actuators with the magnetorheological fluid have been applied, both located between the driving motor and the left-hand side heavy inertial disk, Fig.1.

The electromagnetic torque generated by the servoasynchronous motor is determined using formula (11) during numerical simulations of the start-up and steady-state operation process regarding torsional vibrations of the mechanical system mutually coupled with oscillations of electrical currents in the motor windings. The retarding torque described by (12) was assumed for h=2 and z=2, which denotes its four step-wise changes per one working tool revolution with an amplitude two-times greater than the average retarding torque value. The time history plots of the driving and retarding torques normalized by the rated torque  $M_n$  during the start-up and the beginning of nominal operation are illustrated in Fig. 4 by the black and grey lines, respectively. From these plots it follows that such relatively small asynchronous motor is able to accelerate the drive system of the entire mass moment of inertia equal to 0.215 kgm<sup>2</sup> in ca. 0.51 s, where the initial value of the electromagnetic torque generated by this motor is 3.9 times greater than the nominal torque  $M_n$ =21.5 Nm, the maximal quasi-static value of the decaying with time dynamic component oscillating with frequency 60 Hz is 5.8 times greater than  $M_n$ .



Fig.4 Time histories of the driving and retarding torques.

The passive system transient dynamic response caused by this start-up and the steady-state operation is presented by means of black lines in Fig. 5 in the form of the time histories of the normalized elastic torques transmitted by the shafts between the magneto-rheological dampers, Fig. 5a, in the vicinity of the overload coupling, Fig. 5b, and at the input to the electric brake imitating the working tool, Fig. 5c. From these plots it follows that the dynamic component of the asynchronous motor torque in the form of the attenuated sinusoid of frequency 60 Hz significantly induces severe transient torsional vibrations during start-up of the drive system. By means of the FFT analysis performed for the above time histories one can testify that the shaft segment between the magneto-rheological dampers is affected by vibrations excited according to the second eigenform characterized by the relatively great modal displacement gradient at this part of the entire drive train, which follows from Fig. 3. Since the greatest modal displacement gradient of the first eigenform occurs in the vicinity of the overload coupling, Fig. 3, an excitation of this torsional vibration mode is observed in the shaft segment adjacent to this device, as shown in Fig. 5b. However, the presented in Fig. 5c dynamic response of the shaft segment at the working tool input is characterized by transient vibrations exciting the third eigenvibration mode of natural frequency 296.4 Hz being systematically repeated due to successive jumps of the retarding torque. The maximum amplitudes of the dynamic torques transmitted by these most heavily affected shafts are almost 15-20 times greater than the rated torque, Fig. 5, which is very dangerous for their fatigue durability. Thus, control of transient torsional vibrations occurring in this drive system during start-ups and steady-state operation is very required.



Fig.5 Time histories of the dynamic elastic torques transmitted by the drive system: between the MR dampers (a), at the overload coupling (b) and in the vicinity of the working tool input (c).

In Fig. 5 by means of the grey lines there are presented analogous plots of time histories of the elastic torques transmitted by the above mentioned shafts and excited in the system controlled by the magneto-rheological dampers with an adjustable viscosity coefficients determined according to the semi-active control strategy defined in (9). From these plots it follows that the corresponding extreme values of the elastic torques in Figs. 5a and 5b have been essentially minimized in comparison with these obtained for the uncontrolled system (black lines). However, by means of the M-R dampers located "far away" from the power receiver it was not possible to suppress the dynamic torque transmitted by the shaft at the input to the electric brake, which is substantiated by the shape of the system third eigenform shown in Fig. 3. For this case the time history of the optimum control damping coefficients  $k_i(t)$  is presented by the black line in Fig. 6.

Figures 7 and 8 depict the system frequency response functions  $FRF_e(f,\mathbf{k})$  and  $FRF_r(f,\mathbf{k})$  obtained respectively for the drive system excited by the electric motor and the power receiver. Besides the semi-active, the passive control scheme defined by (10) has been also tested. In the RMS terms, the dynamic torques transmitted by three chosen crucial shaft segments have been reduced in the investigated control schemes to 51.4% and 49.7% of the level attained in the undamped system.

In order to attenuate the dynamic torque amplitudes transmitted by the shafts in the vicinity of the driven machine working tool it would be necessary to install the M-R



#### Fig. 6 Time histories of the control damping coefficient k(t) for the nominal (black line) and the "reverse" drive system (grey line).

dampers close to the power receiver. The real laboratory drive system presented in Fig. 1 enables us a mutual change of the driving motor and the electric brake locations creating in this way a "reverse" drive system, where all its parameters and excitations remain unchanged. In this case the time histories of the driving and retarding torques generated by the electric motor and the working tool installed respectively on the opposite sides of the drive train are almost identical as these shown in Fig. 4. In Fig. 9 in the same way as in Fig. 5 there are presented plots of time histories of the dynamic torques transmitted by the same shaft segments in the uncontrolled system and in the semi-actively controlled system. The corresponding time history of the optimum control damping coefficient  $k_j(t)$  is presented by the grey line in Fig. 6.



Fig.7 Frequency response function of the damped drive system excited by the electric motor

From Fig. 9 it follows that the MR dampers installed close to the power receiver suppress very effectively the steady state vibrations induced by the driven machine working tool, as shown in Fig. 9c. In this case the transient and steady state vibrations in the vicinity of the overload coupling are also significantly attenuated. However, the control effect of the vibrations at the driving motor input became now almost negligible. In the "reverse" system, the dynamic torques transmitted by the three investigated shaft segments have been reduced to 42.9% and 41.7% of the level attained in the undamped system using respectively the semi-active and the passive control strategy.



Fig. 8 Frequency response function of the damped drive system excited by the power receiver





In order to verify the results of computations obtained using the hybrid model of the drive system, the proposed control algorithm has been tested by means of the FEM model shown in Fig. 2b. Here, numerical integration of the 74+2 coupled motion equations (5) in the generalized coordinates, where the damping matrix C played a role of control matrix, yielded almost the same results as these obtained by the use of the hybrid model in the modal coordinates, but it was much more time consuming.

#### CONCLUSIONS

In the paper a control of transient and steady-state torsional vibrations of the laboratory drive system driven by the asynchronous motor has been computationally performed by means of rotary dampers with the magneto-rheological fluid (MRF). Two control strategies have been proposed: a passive and a closed-loop semi-active, which depends on the driving torque generated by the electric motor and the retarding torque produced by the working tool imitating a coal pulverizer as well as on dynamic behavior of the drive system during run-up and nominal operation. As it follows from the numerical examples, in both considered cases the optimum control carried out by means of the applied actuators with the MRF can effectively reduce the transient torsional vibrations of the selected shaft segments to the quasi-static level of the loading transmitted by the drive system, where dynamic amplifications of the responses due to resonance effects have been almost completely suppressed. Nevertheless, it should be remarked that the additional inertia of the dampers results in some braking of the rigid body motion of the drive system, which leads to a slight run-up retardation in time.

In the next step of research in this field, by means of the already constructed real laboratory drive system depicted in Fig. 1, the presented above control strategies are going to be experimentally verified.

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