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DSA FOR ELASTIC-PLASTIC FINITE ROTATION SHELLS UNDER DYNAMIC LOADS

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Summary The paper describes a constitutive DSA algorithm for elastic-plastic finite rotation shells and explicit dynamics, by which design derivatives are calculated w.r.t. material parameters. The paper shows that despite a great complexity of the solution algorithm for the finite-rotation elastic-plastic shells, it is feasible to compute analytical design derivative of this algorithm, and the yielded sensitivities are of very good accuracy.

CONSTITUTIVE DSA FOR HUBER-MISES PLASTICITY

Derivative w.r.t. material parameters.

In the paper we consider the constitutive model consisting of Hooke's elasticity and Huber-Mises plasticity with nonlinear isotropic hardening of the saturation type, $\kappa(\bar{e}^p) = \sigma_{y0} + \kappa_1' \, \bar{e}^p + (\sigma_{y1} - \sigma_{y0})(1 - e^{-a\bar{e}^p})$, and a linear kinematic hardening function $H_\alpha(\bar{e}^p) = H_0 + H' \, \bar{e}^p$. Hence, the set of material parameters is $\mathbf{m} \doteq \{E, \, \nu, \, \sigma_{y0}, \, \kappa_1', \, \sigma_{y1}, a\}$. For a design parameter h and material parameters $\mathbf{m}$ depending on h, i.e. $\mathbf{m} = \mathbf{m}(h)$, we have $\mathrm{D}_h(\,\cdot\,) = \mathrm{D}_{\mathbf{m}}(\,\cdot\,) \, \mathrm{D}_h\mathbf{m}$, where $\mathrm{D}_h\mathbf{m}$ is specified additionally, and it suffices to consider derivatives w.r.t. $\mathbf{m}$, only. For some $A(\mathbf{m}, B(\mathbf{m}))$, the total design derivative of A with respect to $\mathbf{m}$ is written as $\mathrm{D}_{\mathbf{m}}A = \partial_{\mathbf{m}}A + A_{,B} \, \mathrm{D}_{\mathbf{m}}B$, where $\partial_{\mathbf{m}}(\cdot)$ is the explicit derivative.

Update of design derivatives of state variables.

Let us denote by $\mathbf{s}$ the state variables which are updated incrementally, stored and retrieved as constitutive history data, $\mathbf{s} = \{\boldsymbol{\Sigma}, \, \bar{e}^{\mathrm{p}}, \, \boldsymbol{\alpha}_R^p, \, \varepsilon^p, \, \varepsilon_{33}, \, \boldsymbol{\psi}_{\mathbf{R}}\}$, where $\boldsymbol{\psi}_{\mathbf{R}}$ are rotational parameters of the constitutive algorithm. The algorithm for $\mathbf{s}_{n+1}$ can be written as $\mathbf{s}_{n+1} = \mathbf{s}_{n+1}(\mathbf{m}, \, \mathbf{s}_n(\mathbf{m}), \, \mathbf{q}_{n+1}(\mathbf{m}), \, \mathbf{q}_n(\mathbf{m}))$, and upon differentiation w.r.t. $\mathbf{m}$ we obtain the update formula

$$\mathrm{D}_{\mathbf{m}}\mathbf{s}_{n+1} = \frac{\partial \mathbf{s}_{n+1}}{\partial \mathbf{m}} + \frac{\partial \mathbf{s}_{n+1}}{\partial \mathbf{s}_n} \, \mathrm{D}_{\mathbf{m}}\mathbf{s}_n + \frac{\partial \mathbf{s}_{n+1}}{\partial \mathbf{q}_{n+1}} \, \mathrm{D}_{\mathbf{m}}\mathbf{q}_{n+1} + \frac{\partial \mathbf{s}_{n+1}}{\partial \mathbf{q}_n} \, \mathrm{D}_{\mathbf{m}}\mathbf{q}_n. \tag{1}$$

The derivatives, $\mathrm{D}_{\mathbf{m}}\mathbf{s}_n$ and $\mathrm{D}_{\mathbf{m}}\mathbf{q}_n$ are known as they are history variables, and $\mathrm{D}_{\mathbf{m}}\mathbf{q}_{n+1}$ in explicit dynamics is predicted. The derivatives, $\partial\mathbf{s}_{n+1}/\partial\mathbf{m}$, $\partial\mathbf{s}_{n+1}/\partial\mathbf{z}_n$, $\partial\mathbf{s}_{n+1}/\partial\mathbf{q}_{n+1}$, $\partial\mathbf{s}_{n+1}/\partial\mathbf{q}_n$, must be explicitly calculated. This update includes the design derivative of back-rotated Kirchhoff stress $\boldsymbol{\Sigma}$ which belongs to $\mathbf{s}$.

Design derivative of stress update algorithm for finite rotation shells.

The algorithmic approach to plasticity for finite-rotation shells follows this of [1], i.e. the constitutive equations are written for the back-rotated Kirchhoff stress $\boldsymbol{\Sigma}$, and equations analogous to those for the small deformation case are used. The constitutive model is the 3D Huber-Mises material with nonlinear isotropic/linear kinematic hardening. The plane stress constraints are directly incorporated into constitutive equations, as in [3] or [2], i.e. the deviatoric stress is parameterized by the plane stress components. The resulting yield surface is ellipsoidal, and not a radial return algorithm but a return map algorithm, with a constitutive Newton loop, must be used. This model is applied to shell laminas.

The algorithm to calculate the 2nd Piola-Kirchhoff stress $\mathbf{S}$ can be written as $\mathbf{S}_{n+1} = \mathbf{S}_{n+1}(\mathbf{m}, \, \mathbf{s}_n(\mathbf{m}), \, \mathbf{q}_{n+1}(\mathbf{m}), \, \mathbf{q}_n(\mathbf{m}))$ and upon differentiation w.r.t. $\mathbf{m}$ we obtain

$$\mathrm{D}_{\mathbf{m}}\mathbf{S}_{n+1} = \frac{\partial \mathbf{S}_{n+1}}{\partial \mathbf{m}} + \frac{\partial \mathbf{S}_{n+1}}{\partial \mathbf{s}_n} \, \mathrm{D}_{\mathbf{m}}\mathbf{s}_n + \frac{\partial \mathbf{S}_{n+1}}{\partial \mathbf{q}_{n+1}} \, \mathrm{D}_{\mathbf{m}}\mathbf{q}_{n+1} + \frac{\partial \mathbf{S}_{n+1}}{\partial \mathbf{q}_n} \, \mathrm{D}_{\mathbf{m}}\mathbf{q}_n, \tag{2}$$

where $\left\{\mathrm{D}_{\mathbf{m}}\mathbf{s}_n, \, \mathrm{D}_{\mathbf{m}}\mathbf{q}_{n+1}, \, \mathrm{D}_{\mathbf{m}}\mathbf{q}_n\right\}$ are known, and the derivatives, $\left\{\partial\mathbf{S}_{n+1}/\partial\mathbf{m}, \partial\mathbf{S}_{n+1}/\partial\mathbf{s}_n, \partial\mathbf{S}_{n+1}/\partial\mathbf{q}_{n+1}, \partial\mathbf{S}_{n+1}/\partial\mathbf{q}_n\right\}$ can be explicitly calculated.

DESIGN DERIVATIVE OF EXPLICIT DYNAMICS ALGORITHM

The equations of dynamics for shells are complicated due to rotational inertia terms and parametrization of rotations (we use the canonical rotation vector). The design differentiation of the explicit dynamics algorithm yields, among others, the following equation

$$\mathrm{D}_{\mathbf{m}}\ddot{\mathbf{q}}_{n+1} = \mathbf{M}^{-1}[-\mathrm{D}_{\mathbf{m}}\mathbf{f}_{n+1} - \mathrm{D}_{\mathbf{m}}\mathbf{c}(\boldsymbol{\omega}) - \mathbf{C} \, \mathrm{D}_{\mathbf{m}}\dot{\mathbf{q}}_{n+1/2}], \tag{3}$$

where $\mathbf{q} = \{\mathbf{x}_{0I}, \, \boldsymbol{\psi}_I\}$ and $\boldsymbol{\psi}_I$ is a canonical rotation vector. The design derivative of the internal force $\mathbf{f}$ is as follows

$$\mathrm{D}_{\mathbf{m}}\mathbf{f} \doteq \frac{\partial}{\partial \mathbf{m}} \int_V \mathbf{B}^T\mathbf{S} \, dV = \int_V \left(\mathrm{D}_{\mathbf{m}}\mathbf{B}^T \, \mathbf{S} \; + \; \mathbf{B}^T \, \mathrm{D}_{\mathbf{m}}\mathbf{S}\right) \, dV, \tag{4}$$

where $\mathbf{S}$ is the 2nd Piola-Kirchhoff stress, $\mathbf{E}$ is the Green strain. The kinematical operator $\mathbf{B} \doteq \partial \mathbf{E}/\partial \mathbf{q}$ depends on $\mathbf{q}$ for finite rotation shells, hence its design derivative is

$$\mathrm{D}_{\mathbf{m}}\mathbf{B} = \frac{d\mathbf{B}}{d\mathbf{q}}\frac{d\mathbf{q}}{d\mathbf{m}} = \mathbf{B}_{,\mathbf{q}}\,\mathrm{D}_{\mathbf{m}}\mathbf{q}, \tag{5}$$

where $\mathbf{B}_{,\mathbf{q}} = \partial^2\mathbf{E}/\partial\mathbf{q}^2$ is a $(n_s \times n_{dof} \times n_{dof})$ matrix, and n_s is a number of strain components.

Design derivative of inertial term.

The inertial term $\mathbf{c}(\boldsymbol{\omega}) = I_\rho \boldsymbol{\omega} \times (\boldsymbol{\Pi}\boldsymbol{\omega})$ depends on $\boldsymbol{\psi}_{n+1}$ and $\dot{\boldsymbol{\psi}}_{n+1/2}$, because $\boldsymbol{\omega} \approx \mathbf{T}(\boldsymbol{\psi}_{n+1})\,\dot{\boldsymbol{\psi}}_{n+1/2}$. Hence, the design derivative of it is computed as follows

$$\mathrm{D}_h\mathbf{c}(\boldsymbol{\omega}) = \frac{\partial \mathbf{c}}{\partial \boldsymbol{\psi}_{n+1}}\,\mathrm{D}_h\boldsymbol{\psi}_{n+1} + \frac{\partial \mathbf{c}}{\partial \dot{\boldsymbol{\psi}}_{n+1/2}}\,\mathrm{D}_h\dot{\boldsymbol{\psi}}_{n+1/2}. \tag{6}$$

NUMERICAL EXAMPLE: ELASTIC-PLASTIC COMPRESSION OF RECTANGULAR RAIL

This example tests the DSA procedure in a realistic example of compression of a steel thin-walled rail, a quarter of which is shown in Fig. 1. To avoid contact modelling and enable clear conclusions regarding accuracy of the DSA, computations are performed for first few milliseconds only.

The load is a 270 kg mass with the initial velocity $v_3 = -7.7778$ m/s, applied at the rail end $x_3 = 370$ mm. The material data $E = 210000$, $\nu = 0.3$, $\sigma_{y1} = 250$, $\sigma_{y0} = 250$, $H' = 2000$, $a = 1$, $H'_{kin} = 0$, thickness $h = 1.47$.

The shell element is a 4-node bilinear with 6 dofs/node, based on Reissner's kinematics. The drilling rotation is included on use of the drilling RC-equation by the penalty method. The rotations are parameterized by a canonical rotation vector. The Green strain and 2nd Piola-Kirchhoff stress are used as a work-conjugate pair. For the transverse shear the ANS (Bathe, Dvorkin) approximation is used. A numerical integration with 2x2 Gauss points in lamina, and 5-point Simpson rule over thickness is used.

Design derivatives are computed analytically and by FD method; the derivatives of rotation at point P w.r.t. yield stress σ_{y0} is given in Fig. 1b. As it can be seen, the finite-difference DSA results coincide with the analytical DSA results, which proves that, despite a substantial complexity of the algorithms, the analytical DSA equations are correct.

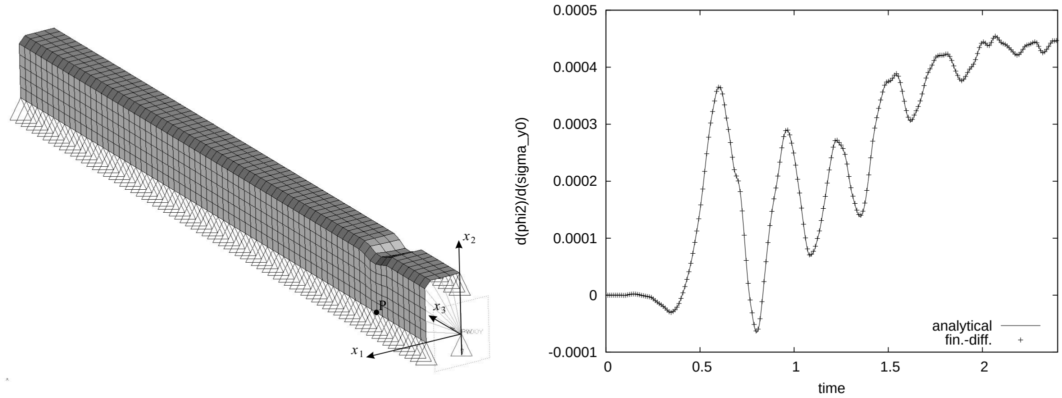

Figure 1. Elastic-plastic compression of rail: (a) geometry, (b) design derivatives of rotation at point P w.r.t. yield stress σ_{y0}.

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References

[1] Hughes T.J.R.: Numerical implementation of constitutive models: rate independent deviatoric plasticity. In S. Nemat-Nasser *et al.*, ed., *Theoretical Foundations for Large-Scale Computing for Nonlinear Material Behaviour*, 29–57. Nijhoff 1984.

[2] Jetteur, P.: Implicit integration algorithm for plane stress elastoplasticity in plane stress analysis. *EC*, **3**:251–253, 1986.

[3] Simo J.C., Taylor R.L.: A return mapping algorithm for plane stress elstoplasticity. *IJNME*, **22**:649–670, 1986

[4] Wisniewski K., Kowalczyk P., Turska E.: On computation of design derivatives for Huber-Mises plasticity with non-linear hardening, *IJNME*, **57**: 271–300, 2003.

[5] Wisniewski K., Kowalczyk P. Turska E.: Design Sensitivity Analysis for elasic-plastic shells and explicit dynamics. *in preparation*, 2004.