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RECENT RESULTS ON NINE-NODE SHELL ELEMENTS USING TWO-LEVEL APPROXIMATION OF STRAINS

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1. Introduction

The paper concerns 9-node quadrilateral shell elements derived for the Reissner-Mindlin kinematics and based on the potential energy functional and the Green strain. The standard (unmodified) element of this class suffers from locking in unidirectional out-of-plane bending and also is too stiff in bending by in-plane forces, so several special methods were developed to improve its performance.

In the current paper we focus on the method using *two-level approximations of strains* and the presentation will discuss our recent results related to this technique described in detail in [3, 4, 5] .

2. Basic shell equations

Two-field functional. In the present work, we use a two-field 3D functional depending on displacements and rotations,

$$F_2(\boldsymbol{\chi}, \mathbf{Q}) \doteq \int_B \mathcal{W}(\mathbf{C})\, \mathrm{d}V + F_{\text{drill}} + F_{\text{ext}}, \tag{1}$$

where $\boldsymbol{\chi}$ is the deformation function and $\mathbf{Q} \in SO(3)$ is the rotation tensor. The strain energy density $\mathcal{W}$ depends on the right Cauchy-Green deformation tensor $\mathbf{C} \doteq \mathbf{F}^T\mathbf{F}$, where $\mathbf{F} \doteq \nabla\boldsymbol{\chi}$ is the deformation gradient. F_{ext} is the potential of external loads.

The drilling rotation is introduced by the equation $c = 0$ derived from the Rotation Constraint (RC) equation $\text{skew}(\mathbf{Q}^T\mathbf{F}) = \mathbf{0}$, and is imposed using in the penalty method,

$$F_{\text{drill}} \doteq \frac{1}{2}\int_M \gamma\, c^2\, \mathrm{d}A, \qquad c \doteq \frac{1}{2}\left[(\mathbf{F}_0\mathbf{t}_2)\cdot(\mathbf{Q}_0\mathbf{t}_1) - (\mathbf{F}_0\mathbf{t}_1)\cdot(\mathbf{Q}_0\mathbf{t}_2)\right], \tag{2}$$

where $\gamma \in (0, \infty)$ is the regularization parameter. Besides, $\mathbf{F}_0$ and $\mathbf{Q}_0$ are associated with the reference surface and $\mathbf{t}_\alpha$ $(\alpha = 1, 2)$ are vectors of the local tangent Cartesian basis $\{\mathbf{t}_k\}$ $(k = 1, 2, 3)$.

Reissner-Mindlin hypothesis. The initial configuration of the shell is parameterized by the natural coordinates on the reference (middle) surface, $\xi^\alpha \in [-1, +1]$, and the normal coordinate $\zeta \in [-h/2, +h/2]$, where h is the initial shell thickness. For the deformed configuration, we use the Reissner-Mindlin kinematical hypothesis,

$$\mathbf{x}(\xi^\alpha, \zeta) = \mathbf{x}_0(\xi^\alpha) + \zeta\, \mathbf{Q}_0(\xi^\alpha)\, \mathbf{t}_3(\xi^\alpha), \tag{3}$$

where $\mathbf{x}$ is a position vector at arbitrary ζ and $\mathbf{x}_0$ at $\zeta = 0$. Besides, $\mathbf{t}_3$ is the unit normal vector (director). The rotation tensor $\mathbf{Q}_0$ is parameterized by the canonical rotation vector $\boldsymbol{\psi}$,

$$\mathbf{Q}_0(\boldsymbol{\psi}) \doteq \mathbf{I} + \frac{\sin\omega}{\omega}\tilde{\boldsymbol{\psi}} + \frac{1-\cos\omega}{\omega^2}\tilde{\boldsymbol{\psi}}^2, \tag{4}$$

where $\omega = \|\boldsymbol{\psi}\| = \sqrt{\boldsymbol{\psi}\cdot\boldsymbol{\psi}} \geq 0$ and $\tilde{\boldsymbol{\psi}} \doteq \boldsymbol{\psi} \times \mathbf{I}$. This parametrization is used within the load step and combined with the quaternion update scheme to enable unrestricted (finite) rotations.

3. Two-level approximation of strains for nine-node elements

Below are presented two methods improving performance of the nine-node element but other methods also exist, e.g. the *Uniform Reduced Integration* plus stabilization and the *EAS* method.

Selective Reduced Integration (SRI). This method applies different integration rules to particular parts of the strain energy. For the considered class of shells, the Green strain is linear over the thickness, i.e. $\mathbf{E}(\zeta) \approx \boldsymbol{\varepsilon} + \zeta\boldsymbol{\kappa}$. If material properties are symmetric w.r.t. the mid-surface then the integration over the thickness decouples particular terms of the shell strain energy so they can be integrated over the middle surface using different rules.

For instance, the 9-SRI shell element with drilling rotation of [2] is integrated using the 3×3, 2×3, 3×2 and 2×2 Gauss rules, see Table 2 therein. This element has no spurious zero eigenvalues, is free of locking, has good mesh convergence and is very fast. However, its range of application is restricted by the assumption about material symmetry, so a more general formulation is needed and the most reliable one is described below.

Two-level approximation of strains. In this method, the strain components are sampled at certain points and extrapolated over the element. In effect, we can use only one set of integration points and all strain components are available at each of these points. Different variants of this method exist in the literature and are called the *Assumed Strain* (AS) method and the *Mixed Interpolation of Tensorial Components* (MITC) method, see e.g. books: [Huang, 1989] and [Chapelle, Bathe, 2003]. In our recent papers [3, 4, 5] we managed to improve the performance of both variants as follows.

In [3], we proposed to modify the transformations of the well-established MITC9 element which does not pass the patch test. Our improved 2D element (designated MITC9i) passes the membrane patch test for the mesh with straight elements sides for : (i) arbitrary positions of side nodes along the boundary, and (ii) an arbitrary position of the interior node, which is a clear improvement.

In [4], we tested the corrected shape functions of [Celia, Gray, 1984] instead of the standard isoparametric ones applied to four 2D nine-node elements. Three elements, MITC9i, 9-AS and QUAD9** passed the patch test for the meshes with straight elements' sides (identically to this described above for [3]); no such improvement is obtained for MITC9 although the errors are reduced.

In [5], we describe the modified transformations of the MITC9 plate and shell elements, which enable passing the bending patch test, and a modification of the corrected shape functions, which enables their application to curved shells.

Numerical tests quantifying effects of the above modifications will be presented.

4. References

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