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RESEARCH ARTICLE

Reliability based design of frames with limited residual strain energy capacity

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Abstract

The aim of this paper is to create new type of plastic limit design procedures where the influence of the limited load carrying capacity of the beam-to-column connections of elasto-plastic steel (or composite) frames under multi-parameter static loading and probabilistically given conditions are taken into consideration. In addition to the plastic limit design to control the plastic behaviour of the structure, bound on the complementary strain energy of the residual forces is also applied. If the design uncertainties (manufacturing, strength, geometrical) are taken into consideration at the computation of the complementary strain energy of the residual forces the reliability based extended plastic limit design problems can be formed. Two numerical procedures are elaborated. The formulations of the problems yield to nonlinear mathematical programming which are solved by the use of sequential quadratic algorithm.

Keywords

reliability analysis · limit analysis · residual strain energy · Monte Carlo simulation · optimal design

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1 Introduction

At the Twelfth International Conference on Civil, Structural and Environmental Engineering Computing "CC2009" a special session was organized to problems dedicated to robust optimal design by stochastic optimization procedures. This paper is a revised and extended version of the CC2009 Conference presentation of Logo et al. [1]. Knabel et al. [10] gave a rather effective reliability based limit design method for skeletal structures what is based on a response surface technique. A complex and "real life" application was introduced by Kirchner and Vietor [9] which can be a promising development in the field of vehicle body development. The robust design of plane frames in the case of uncertainty was discussed by Zier [17]. Here comparison of four approaches to the linear approximation of the yield condition was presented. The numerical approach presented by Beer and Liebscher [2] could be applied in combination with a nonlinear structural analysis and any initial uncertainty analysis, such as Monte Carlo simulation, interval analysis or fuzzy analysis.

In engineering practice the uncertainties play a very important role [13–15] and need intensive calculations. There are several engineering problem where the designer should face to the problem of limited load carrying capacity of the connected elements of the structures [8, 12]. Such problem can be found during the rehabilitation of the old buildings with composite plates (floors) or in the case of steel frame structures. The main structural elements of steel frame multi-storey structures are the columns, the beams and their connections. The assumption that the connections are either rigid or pinned has been widely applied in the past. The actual behaviour of the connections is however somewhere between these limits and they are semi-rigid [6, 11]. This circumstance can influence significantly the behaviour of the structure therefore has to be taken into account in the analysis and design. At the application of the plastic analysis and design methods the control of the plastic behaviour of the structures is an important requirement. In structural plasticity the static and kinematic limit theorems provide appropriate tools to solve these complex problems [4, 7, 8].

Comprehensive reliability limit analysis of frames (structure with rigid connections) was considered by Corotis and Nafday [3]. They proposed simulation method for failure probability estimation with assumed random nature of load and scatter of variables relevant to the resistance of the structure. Load variability description comes from observations and appropriate probabilistic distribution can easily be adjusted. However, resistance distribution in their work is defined by limit load multiplier which is determined for each dominant failure mode associated with variability ranges of all variables.

In classical plasticity the limit and the shakedown analysis are among the most important basic problems. Since the shakedown and limit analysis provide no information about the magnitude of the plastic deformations and residual displacements accumulated before the adaptation of the structure, therefore for their determination several bounding theorems and approximate methods have been proposed. Among others Kaliszky and Lógó [8, 11] suggested that the complementary strain energy of the residual forces could be considered an overall measure of the plastic performance of structures and the plastic deformations should be controlled by introducing a limit for magnitude of this energy. In engineering the problem parameters (geometrical, material, strength, manufacturing) are given or considered with uncertainties. The obtained analysis and/or design task is more complex and can lead to reliability analysis and design.

Instead of variables influencing performance of the structure (manufacturing, strength, geometrical) only one bound modelling resistance scatter can be applied. The bound on the complementary strain energy of the residual forces controlling the plastic behaviour of the structure can be utilized. This bound has significant effect for the limit load multipliers [11]. Moreover, linear programming with complementary strain energy boundary yields at once limit load multiplier conditioned by assumed value of resistant failure probability. The reliability based extended plastic limit design problem can be formed and whole limit load envelope for different directions of load can be determined. In the case of semi-rigid connections the boundaries between dominant failure modes are not clear. Therefore, suitable approximation of limit load envelope, so called response surface method can be applied. It gives the best numerical efficiency required in engineering applications, e.g. see [4, 10]. Surrogate but analytical model of limit load envelope enables application of any reliability analysis method, even the most expensive simulation, Crude Monte Carlo method.

The aim of this paper is to create new type of plastic limit design procedures where the influence of the limited load carrying capacity of the beam-to-column connections of elasto-plastic steel (or composite) frames under multi-parameter static loading and probabilistically given conditions are taken into consideration. In addition to the plastic limit design to control the plastic behaviour of the structure, bound on the complementary strain energy of the residual forces is also applied. This bound has significant effect for the load parameter [12]. If the design uncertainties (manufacturing, strength, geometrical) are expressed by the calculation of the complementary strain energy of the residual forces the reliability based extended plastic limit design problems can be formed. Two numerical procedures are elaborated: the first one is based on the extended plastic limit design method with a direct integration technique and the uncertainties are assumed to follow Gaussian distribution. The formulations of the problems yield to nonlinear mathematical programming which are solved by the use of sequential quadratic algorithm. The nested optimization procedure is governed by the reliability index calculation. The second procedure is based on the Crude Monte Carlo simulation where the extended limit design procedure is applied [10]. The multi-parameter static loads follow Gumbel distribution and the "design" uncertainties are assumed Gaussian distributed data. Because of the demand of the high efficiency, the response surface method is applied.

The parametric study is illustrated by the solution of examples.

2 Elements of the mechanical modelling and the analysis

2.1 Notations and loadings

In the paper the following notations are used:

$\mathbf{P}_d$: dead load;

$\mathbf{P}_1, \mathbf{P}_2$: Static working loads;

$\mathbf{M}_h^e, \mathbf{M}_d^e$: Fictitious elastic moments calculated from the live and dead loads assuming that the structure is purely elastic;

$\mathbf{Q}^r, \mathbf{M}^r$: residual internal forces and moments;

$\mathbf{M}_d^p, \mathbf{M}_h^p$: plastic moments;

$\overline{\mathbf{M}}^p$: limit moments of the bounded beam to column joints;

W_{p0} : allowable complementary strain energy of the residual forces;

$\boldsymbol{\sigma}_y, E$: yield stress and Young's modulus;

A_i, I_i, S_{0i} and ℓ_i : areas, moment of inertias of the cross-sections and length of the finite elements ($i = 1, 2, \ldots, n$), respectively;

$\overline{S}_j$: stiffness of the semi-rigid connection.

$\mathbf{F}, \mathbf{K}, \mathbf{G}, \mathbf{G}^*$: flexibility, stiffness, geometrical and equilibrium matrices, respectively; ($j = 1, 2, \ldots, k$) is the number of semi-rigid connections. They are subsets of ($i = 1, 2, \ldots, n$).

β : reliability index; Φ^{-1}: inverse cumulative distribution function (so called probit function) of the Gaussian distribution,

$f\left(W_{p0}\right)$: the Gaussian probability density function of the complementary strain energy of the residual forces.

V_0 : represents the total limit volume of the structure.

Two problem class are considered. In the first group of problems the dead and working (pay) loads considered deterministic, while in the second one the working (pay) loads follow Gumbel distribution. The structure is subjected to a dead load $\mathbf{P}_d$and two independent, static working loads $\mathbf{P}_1$ and $\mathbf{P}_2$ with multipliers $m_1 \geq 0, m_2 \geq 0$. In the analysis five loading cases ($h = 1, 2, \ldots, 5$) shown in Table 1 are taken into consideration. For each loading case a plastic limit load multiplier m_{ph} can be calculated. Making use of these multipliers a limit curve can be constructed in the m_1, m_2 plane (Figure 1). The structure does not collapse under the action of the loads $m_1\mathbf{P}_1$, $m_2\mathbf{P}_2$ if the points corresponding to the multipliers m_1,m_2 lies inside or on the plastic limit curve, respectively.

Tab. 1. Loading combinations

h	Multipliers	Loads	Load multipliers Plastic limit state
1	$m_2 = 0$	$\mathbf{Q}_1 = \mathbf{P}_1$	m_{p1}
2	$m_1 = 0$	$\mathbf{Q}_2 = \mathbf{P}_2$	m_{p2}
3	$m_1 = 0.5m_2$	$\mathbf{Q}_3 = [0.5\mathbf{P}_1, (0.5\mathbf{P}_1 + \mathbf{P}_2), \mathbf{P}_2]$	m_{p3}
4	$m_1 = m_2$	$\mathbf{Q}_4 = [\mathbf{P}_1, (\mathbf{P}_1 + \mathbf{P}_2), \mathbf{P}_2]$	m_{p4}
5	$m_1 = 2m_2$	$\mathbf{Q}_5 = [2.0\mathbf{P}_1, (2.0\mathbf{P}_1 + \mathbf{P}_2), \mathbf{P}_2]$	m_{p5}

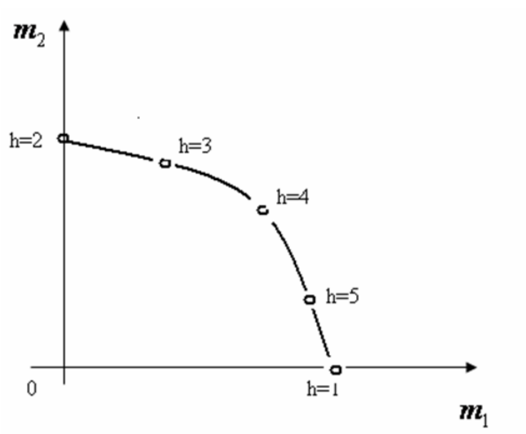

Fig. 1. Limit curve and safe domain

In the second case the working (pay) loads are $\mathbf{P}_1, \mathbf{P}_2$ two random variables with Gumbel distribution. $m_{ph,\psi}(W_{p0})$ is the admissible plastic limit load multiplier, calculated by the use of the assumed W_{p0} and constituting the limit load envelope.

$m_\psi(\mathbf{P})$ is the plastic limit load multiplier depending on the realization of the random vector of load $\mathbf{P}$.

Both limit load multipliers $m_{ph,\psi}(W_{p0})$ and $m_\psi(\mathbf{P})$depend on the direction of load $\mathbf{P}$ defined by angles ψ_i -(0 − 90)-.

2.2 Modelling of the semi-rigid connections

The typical general behaviour of the semi-rigid connection can be illustrated by a moment-rotation relationship shown in Figure 2. In this paper this relationship will be approximated [6,12] by three different elasto-plastic models given in Figure 3. Here $\overline{M}^p$ is the plastic limit moment and $\bar{S}$ is the stiffness of the semi-rigid connection. Their magnitudes can be assumed from the results of experiments. These models are incorporated in the elementary stiffness matrix of the beam elements.

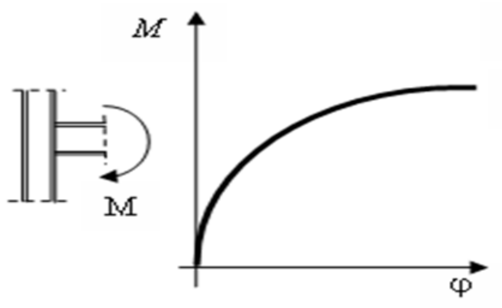

Fig. 2. Real behaviour of the semi-rigid connection

2.3 Reliability-based control of the plastic deformations

Introducing the basic concepts of the reliability analysis and using the force method the failure of the structure can be defined as follows:

$$g(\mathbf{X}_R, \mathbf{X}_S) = \mathbf{X}_R - \mathbf{X}_S \leq 0; \tag{1}$$

where $\mathbf{X}_R$ indicates either the bound for the statically admissible forces $\mathbf{X}_S$ or a bound for the derived quantities from $\mathbf{X}_S$. The probability of failure is given by

$$\mathbb{P}_f = F_g(0); \tag{2}$$

and can be calculated as

$$\mathbb{P}_f = \int_{g(\mathbf{X}_R, \mathbf{X}_S) \leq 0} f(\mathbf{X})\, dx. \tag{3}$$

At the application of the plastic analysis and design methods the control of the plastic behaviour of the structures is an important requirement. Since the limit analysis provides no information about the magnitude of the plastic deformations and residual displacements accumulated before the adaptation of the structure, therefore for their determination several bounding theorems and approximate methods have been proposed. Among others Kaliszky and Lógó [8, 11] suggested that the complementary strain energy of the residual forces could be considered an overall measure of the plastic performance of structures and the plastic deformations should be controlled by introducing a bound for magnitude of this energy:

$$\frac{1}{2}\sum_{i=1}^{n} Q_i^r \mathbf{F}_i Q_i^r \leq W_{p0} \tag{4}$$

Here $\mathrm{W_{p0}}$ is an assumed bound for the complementary strain energy of the residual forces. This constraint can be expressed in terms of the residual moments $M_{i,ai}^r$ and $M_{i,bj}^r$ acting at the ends (*ai* and *bj*) of the finite elements as follows:

$$\frac{1}{6E}\sum_{i=1}^{n}\frac{\ell_i}{I_i}\left[(M_{i,ai}^r)^2 + (M_{i,ai}^r)(M_{i,bj}^r) + (M_{i,bj}^r)^2\right] \leq W_{p0}. \tag{5}$$

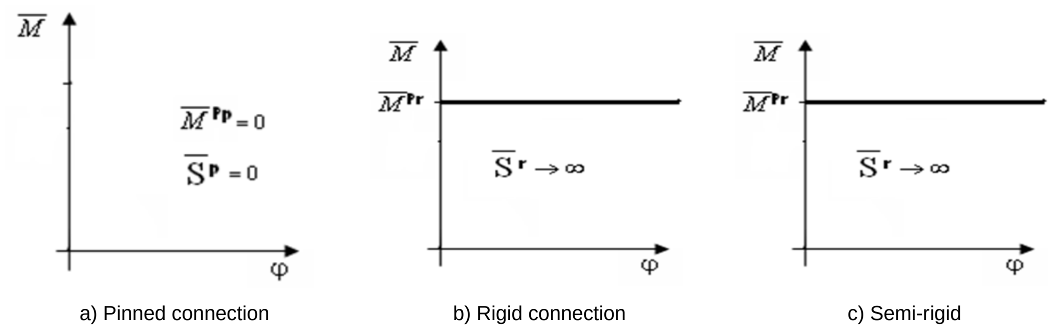

Fig. 3. Models of the semi-rigid connection.

By the use of eq. (5) a limit state function can be constructed:

$$g\left(W_{p0},\mathbf{M}^r\right)=W_{p0}-\frac{1}{6E}\sum_{i=1}^{n}\frac{\ell_i}{I_i}\left[(M^r_{i,ai})^2+(M^r_{i,ai})(M^r_{i,bj})+(M^r_{i,bj})^2\right]. \tag{6}$$

The plastic deformations are controlled while the bound for the magnitude of the complementary strain energy of the residual forces exceeds the calculated value of the complementary strain energy of the residual forces:

$$g\left(W_{p0},\mathbf{M}^r\right)=W_{p0}-\frac{1}{6E}\sum_{i=1}^{n}\frac{\ell_i}{I_i}\left[(M^r_{i,ai})^2+(M^r_{i,ai})(M^r_{i,bj})+(M^r_{i,bj})^2\right]>0. \tag{7}$$

Let assumed that due to the uncertainties the bound for the magnitude of the complementary strain energy of the residual forces is given randomly and for sake of simplicity it follows the Gaussian distribution with given mean value $\bar{W}_{p0}$ and deviation σ_w. Due to the number of the probabilistic variables (here only single) the probability of the failure event can be expressed in a closed integral form:

$$\mathbb{P}_{f,calc}=\int\limits_{g(W_{p0},\mathbf{M}^r)\le 0} f\left(\bar{W}_{p0},\sigma_w\right)dx. \tag{8}$$

By the use of the strict reliability index a reliability condition can be formed:

$$\beta_{target}-\beta_{calc}\le 0; \tag{9}$$

where β_{target} and β_{calc} are calculated as follows:

$$\beta_{target}=-\Phi^{-1}\left(P_{f,target}\right); \tag{10}$$

$$\beta_{calc}=-\Phi^{-1}\left(P_{f,calc}\right). \tag{11}$$

(Due to the simplicity of the present case the integral formulation is not needed, since the probability of failure can be described easily with the distribution function of the normal distribution of the stochastic bound W_{p0}.)

2.3.1 Random loading

The entire problem can be treated alternatively [10]. The limit load multiplier $m_{ph,\psi}(W_{p0})$ represents the structural resistance independently from the real load P. The complementary strain energy W_{p0} expresses all uncertainties characterizing structure that influences on the value of their limit load multiplier $m_{ph,\psi}=m_{ph,\psi}(W_{p0})$ for the fixed direction of the load vector, angle ψ_i -$(0-90)$-. Hence, established the admissible value of failure probability $\mathbb{P}[W_{p0}\le\hat{w}_{p0}]$ ensures the safety level of the structural resistance. Here $\hat{w}_{p0}$ means the boundary value on complementary strain energy. The limit load envelope $\hat{m}_{ph}(W_{p0},\psi)$ for the established value of the complementary strain energy w_{p0} can easily be determined.

Also the random nature of the load can be considered. Vector of the limit loads **P** describes variability of maximum realizations of loads in considered period of time (e.g. lifetime of the structure). The Gumbel distribution with joint probability density function (pdf) $f_{P1,P2\ldots Pn}(p_1,p_2,\ldots p_n)=f_P\,(\mathbf{P})$ is used to model extreme values of loads. Following the paper [3] the vector of random loads **P** can be expressed by means of its length $D=\|\mathbf{P}\|$ and vector of n angles $\Psi=[\Psi_1,\Psi_2,...,\Psi_n]$ linked by the formula $p_i=m cos(\psi_i)$, where m is the load multiplier. The last angle can be calculated in the following way $\psi_n=\cos^{-1}((m^2-p_1^2-...-p_{n-1}^2)^{1/2}/m)$. The relationship $f_{D,\Psi}(m,\psi)=|\mathbf{J}|\,f_p(p))$ comes from the theory of derived distributions, where $|\mathbf{J}|$ is the Jacobian of the transformation. It allows to obtain conditional pdf $f_{D|\Psi}(m,\psi)=f_{D,\Psi}(m,\psi)\,f_\Psi(\psi)$ where $f_\Psi(\psi)$ is joint pdf of random angles Ψ. Limit load multiplier $m_\psi=m_\psi(p)$ is determined by load vector realization p, or by the length and direction of the vector. Deriving from the classical reliability approach, Limit State Function (LSF) can be defined by the formula

$$g_\psi(W_{p0},P)=m_{ph,\psi}(W_{p0})-m_\psi(P) \tag{12}$$

For a given realization of random loads, resistance of the structure represented by load multiplier $m_{ph,\psi}$should be bigger than applied loads represented by load multiplier $m_\psi(\mathbf{p})$ that means safe state $g_\psi(W_{p0},P)\quad>0$, or it is not bigger that means failure $g_\psi(W_{p0},P)\le 0$. It should be noted the LSF is determined for a given loads direction ψ and corresponding probability of failure is given by

$$\mathbb{P}_{f|\psi} = \mathbb{P}[m_{ph,\psi}(W_{p0}) - m_\psi(P) \le 0] = \iint\limits_{m_{ph,\psi} \le m_\psi} f_{D|\Psi}(m,\psi) f_{\omega_{p0|\Psi}}(W_{p0}) d\omega_{p0} dm, \qquad (13)$$

where $f_{\omega_{p0|\Psi}}(W_{p0})$ is conditional pdf of complementary strain energy. Assumed safety level of structural resistance results from fixed $\hat{\omega}_{p0}$ value. Hence, optimization procedure with random constrain yields to limit load multiplier $\hat{m}_{ph,\psi}(\hat{\omega}_{p0}) = \hat{m}_{ph}(\hat{\omega}_{p0},\psi)$ that corresponds to cumulative distribution function (cdf) $F_{W_{p0|\Psi}}(\hat{\omega}_{p0})$, which is equal to the admissible value of failure probability $\mathbb{P}[W_{p0} \le \hat{\omega}_{p0}]$, obtained in optimization procedure for all possible angles

$$F_{W_{p0|\Psi}}(\hat{\omega}_{p0}) = F_{W_{p0}}(\hat{\omega}_{p0}) = \mathbb{P}[W_{p0} \le \hat{\omega}_{p0}]. \qquad (14)$$

Thus, integral (13) can be reformulated

$$\begin{aligned} \mathbb{P}_{f|\psi} &= \int_0^\infty f_{D|\Psi}(m,\psi) \left[\int_0^{\hat{\omega}_{p0}} f_{\omega_{p0|\Psi}}(\omega_{p0})\, d\omega_{p0} \right] dm = \\ &= F_{W_{p0|\Psi}}(\hat{\omega}_{p0}) \int_{\hat{m}_{ph,\psi}(\hat{\omega}_{p0})}^\infty f_{D|\Psi}(m,\psi) dm = \qquad (15) \\ &= F_{W_{p0}}(\hat{\omega}_{p0}) \int_{\hat{m}_{ph,\psi}(\hat{\omega}_{p0})}^\infty f_{D|\Psi}(m,\psi) dm \end{aligned}$$

Probability of failure covering whole variability of ψ can be expressed in the following way

$$\begin{aligned} \mathbb{P}_f &= \int_{\psi_1}^{\psi_2} F_{W_{p0|\Psi}}(\hat{\omega}_{p0}) \int_{\hat{m}_{ph,\psi}(\hat{\omega}_{p0})}^\infty f_{D|\Psi}(m,\psi) dm f_\Psi(\psi) d\psi = \\ &= \mathbb{P}[W_{p0} \le \hat{\omega}_{p0}] \int_{\psi_1}^{\psi_2} \int_{\hat{m}_{ph}(\hat{\omega}_{p0},\psi)}^\infty f_{D|\Psi}(m,\psi) dm d\psi = \\ &= \mathbb{P}[W_{p0} \le \hat{\omega}_{p0}] \int_{\hat{p}_{\hat{\omega}_{p0}}}^\infty f_P(p) dp \\ &= \mathbb{P}[W_{p0} \le \hat{\omega}_{p0}] \mathbb{P}[p_{\hat{\omega}_{p0}} \le \mathbf{P}] = \\ &= \mathbb{P}_{f\omega} \mathbf{P}_{fp} \end{aligned} \qquad (16)$$

where vectors ψ_1 and ψ_2 determinate lower and upper bounds of ψ, envelope $\hat{m}_{ph}(\hat{\omega}_{p0})$ after transformation to the load space can be expressed by the function $\hat{\omega}_{p0}(p)$. As can be seen, in this specific case, failure probability $\mathbb{P}_f$ due to complementary strain energy $\mathbb{P}_{f\omega}$ and loads $\mathbb{P}_{fp}$ can be considered separately. Firstly, limit load multiplier envelope for assumed value of $\mathbb{P}_{f\omega}$ can be determined, and then, $\mathbb{P}_{fp}$ can be calculated. Response surface seems to be best suited method of reliability analysis of this case, but Crude Monte Carlo [10, 16] also can be utilized. It consists in numerical integration over the failure domain.

3 Extended plastic limit design

3.1 Basic design formulations

Determine the maximum load multiplier m_{ph} and cross-sectional dimensions under the conditions that (i) the structure with given layout is strong enough to carry the loads $(\mathbf{P}_d + m_{ph}\mathbf{Q}_h)$, (ii) satisfies the constraints on the limited beam-to-column strength capacity, (iii) satisfies the constraints on plastic deformations and residual displacements, (iv) safe enough and the required amount of material does not exceed a given limit. The design solution method based on the static theorem of limit analysis [11] is formulated as below:

$$\text{Maximize} \quad m_{ph} \qquad (17)$$

Subject to

$$\mathbf{G}^*\mathbf{M}_d^p + \mathbf{P}_d = \mathbf{0}; \qquad (18)$$

$$\mathbf{G}^*\mathbf{M}_h^p + m_{ph}\mathbf{Q}_h = \mathbf{0}; \qquad (19)$$

$$\mathbf{M}_d^e = \mathbf{F}^{-1}\mathbf{G}K^{-1}\mathbf{P}_d; \qquad (20)$$

$$\mathbf{M}_h^e = \mathbf{F}^{-1}\mathbf{G}K^{-1}m_{ph}\mathbf{Q}_h; \qquad (21)$$

$$-2S_{0i}\sigma_y \le (\mathbf{M}_{di}^p + \max \mathbf{M}_{hi}^p) \le 2S_{0i}\sigma_y, (i = 1, 2..., n); \qquad (22)$$

$$-2S_{0i}\sigma_y \le (\mathbf{M}_{di}^p + \min \mathbf{M}_{hi}^p) \le 2S_{0i}\sigma_y, (i = 1, 2..., n); \qquad (23)$$

$$-\overline{\mathbf{M}}_j^p \le (\mathbf{M}_{dj}^p + \max \mathbf{M}_{hj}^p) \le \overline{\mathbf{M}}_j^p, (j = 1, 2..., k); \qquad (24)$$

$$-\overline{\mathbf{M}}_j^p \le (\mathbf{M}_{dj}^p + \min \mathbf{M}_{hj}^p) \le \overline{\mathbf{M}}_j^p, (j = 1, 2..., k); \qquad (25)$$

$$\mathbf{M}_i^r = \left[(\max \mathbf{M}_{hi}^e + \mathbf{M}_{di}^e)\right] - \left[(\max \mathbf{M}_{hi}^p + \mathbf{M}_{di}^p)\right], \quad (i = 1, 2..., n); \qquad (26)$$

$$\beta_{target} - \beta_{calc} \le 0; \qquad (27)$$

$$\sum_i A_i \ell_i - V_0 \le 0. \qquad (28)$$

Here eqs. (18)–(19) are equilibrium equations for the dead loads and for the live (pay) loads, respectively. Eqs. (20)–(21) express the calculations of the elastic fictitious internal forces (moments) from the dead loads and from the live (pay) loads, respectively. Eqs. (22)–(23) are the yield conditions. Eqs. (24)–(25) are used as yield conditions of the semi-rigid connections. Eq. (26) is used to calculate the residual forces. Eq. (27) is the reliability condition. The material redistribution is controlled by eq. (28). The goal is to find the maximum of the statically admissible load multiplier m_{ph}.

This is a nonlinear mathematical programming problem which can be solved by any appropriate solution method (e.g. SPQL method). Selecting one of the semi-rigid connection models for each loading combination $\mathbf{Q}_h$; $(h = 1, 2, \ldots, 5)$ a plastic limit load multiplier m_{ph} can be determined, then the limit curve of the plastic limit state can be constructed with the optimal cross-sectional dimensions. Due to the mathematical nature of problem . (17)–(28) an iterative procedure was elaborated which is governed by solving the equation (27).

3.2 Alternative design formulation

By the use of a simple modification of problem (17)–(28) one can obtain the "classical" minimum volume design problem. Interchanging the objective function -eq. (29)- and the last constraint -eq. (28)- an alternative design formulation can be formulated:

$$\text{Minimize} \quad V = \sum_i A_i \ell_i \tag{29}$$

Subject to

$$\mathbf{G}^* \mathbf{M}_d^p + \mathbf{P}_d = \mathbf{0}; \tag{30}$$

$$\mathbf{G}^* \mathbf{M}_h^p + m_{ph} \mathbf{Q}_h = \mathbf{0}; \tag{31}$$

$$\mathbf{M}_d^e = \mathbf{F}^{-1} \mathbf{G} K^{-1} \mathbf{P}_d; \tag{32}$$

$$\mathbf{M}_h^e = \mathbf{F}^{-1} \mathbf{G} K^{-1} m_{ph} \mathbf{Q}_h; \tag{33}$$

$$-2S_{0i}\sigma_y \leq (\mathbf{M}_{di}^p + \max \mathbf{M}_{hi}^p) \leq 2\S_{0i}\sigma_y, (i = 1, 2..., n); \tag{34}$$

$$-2S_{0i}\sigma_y \leq (\mathbf{M}_{di}^p + \min \mathbf{M}_{hi}^p) \leq 2S_{0i}\sigma_y, (i = 1, 2..., n); \tag{35}$$

$$-\overline{\mathbf{M}}_j^p \leq (\mathbf{M}_{dj}^p + \max \mathbf{M}_{hj}^p) \leq \overline{\mathbf{M}}_j^p, (j = 1, 2..., k); \tag{36}$$

$$-\overline{\mathbf{M}}_j^p \leq (\mathbf{M}_{dj}^p + \min \mathbf{M}_{hj}^p) \leq \overline{\mathbf{M}}_j^p, (j = 1, 2..., k); \tag{37}$$

$$\mathbf{M}_i^r = \left[\left(\max \mathbf{M}_{hi}^e + \mathbf{M}_{di}^e \right) \right] - \left[\left(\max \mathbf{M}_{hi}^p + \mathbf{M}_{di}^p \right) \right], (i = 1, 2..., n); \tag{38}$$

$$\beta_{target} - \beta_{calc} \leq 0; \tag{39}$$

$$m_{ph} - m_0 \leq 0. \tag{40}$$

Here all the equations have the same meanings as it was before in Eqs. (18)–(27). Eq. (40) gives an upper bound for the external loads.

This is also a constrained nonlinear mathematical programming problem which leads to same optimal solution as problem (17)–(28) in the case of the same boundary conditions. The equivalence can be proved by the comparison of the optimality conditions of problems (17)–(28) and (29)–(40).

4 Numerical examples

To demonstrate the theories introduced above, two procedures are elaborated for the limit design problem. The first one is to determine the safe loading domain and cross-sectional dimensions of a simple frame with deterministic loading data with probabilistic bound for the magnitude of the complementary strain energy of the residual forces. The second one presents the safe load multipliers and cross-sectional dimensions for the same steel frame with Gumbel distributed loads.

The application is illustrated by an example shown in Figure 4. At the joints 2 and 4 the portal frame has semi-rigid connection. The working loads are $P_1 = 10$kN, $P_2 = 15$ kN and $P_d = 0$.

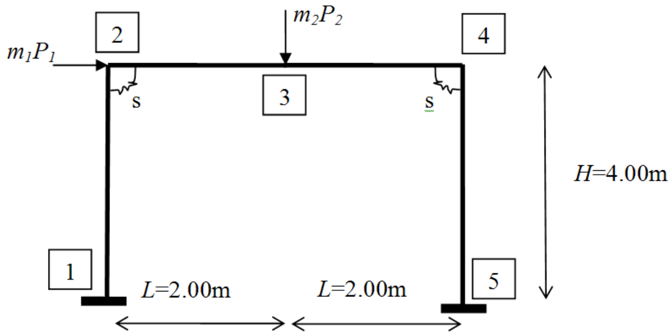

Fig. 4. Portal frame as test problem

The yield stress and the Young's modulus are $\sigma_y = 21\,\text{kN/cm}^2$ and $E = 2.07 \cdot 10^6\,\text{kN/cm}^2$.

The two solution techniques are demonstrated below as example 1 and example 2. The constrained nonlinear mathematical programming problem was solved by SQP (sequential quadratic programming) method. The problem is to determine the maximum load multipliers and cross-sectional dimensions corresponding to a given volume and safety level. Using the problem formulations (17)–(28) and (29)-(40), the results of the solution are illustrated in Figures 5–8 and Figures 9–10, respectively.

4.1 Example 1.

The results of the first solution technique are presented in Figures 5–8 where deterministic loading is considered. The results are in very good agreement with the expectations.

In Figures 5–6 one can see the safe loading domains in function of different expected probability and different limit volume, respectively. In Figure 7 the variation of the base (see Table 1)

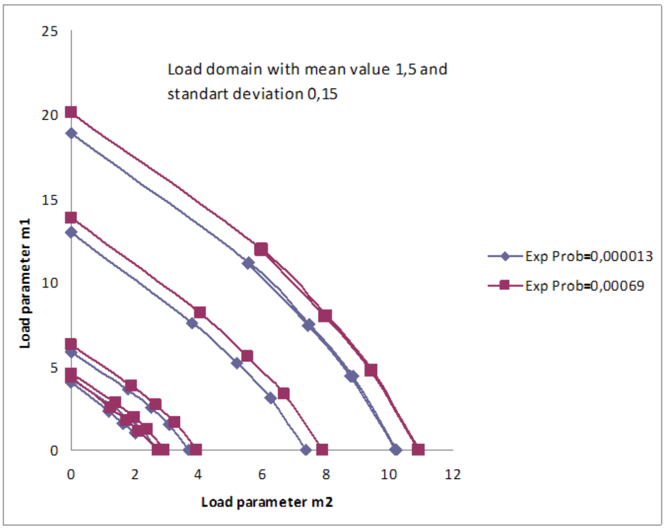

Fig. 5. Safe loading domain for limit design

load-multiplier is presented in function of the connection rigidities and limit volumes. As it is seen the stiffnesses of the semi-rigid connection influence significantly the plastic behaviour of the frame. In Figure 8 the optimal cross-sectional dimensions are presented.

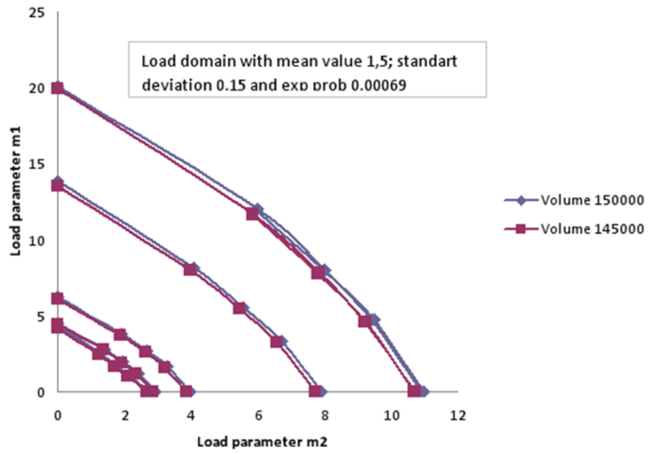

Fig. 6. Safe loading domain for plastic limit design

4.2 Example 2.

Safety level of structural response has been assumed to be $\mathbf{P}_{fw} = 0.00069$ (it corresponds to $\hat{\omega}_{p0} = 1.9798$). Calculated envelopes $\hat{m}_{ph}(\hat{\omega}_{p0})$ are piecewise linear with 10 points calculated for different m_1/m_2 ratios. Gumbel distribution as model of loads has been considered. It is often used in industry to model extreme values associated with environmental loads. It describes variability of maximum realizations of loads in considered period of time (e.g. lifetime of structure). Monte Carlo analysis has been performed for two cases:

Full rigid connections and gradual deterioration of rigidity, see Figure 9. The flexibility of the spring of connections increases from 0.0 (fixed connection) to $7.5 \cdot 10^{-8}$ (semi-rigid). Assumed parameters of random loads are $P_1(\mu_1 = 19\,\text{kN}$, $\sigma_1 = 5\,\text{kN}))$ and $P_2(\mu_2 = 22.5\,\text{kN}$, $\sigma_2 = 6\,\text{kN})$. Sample of

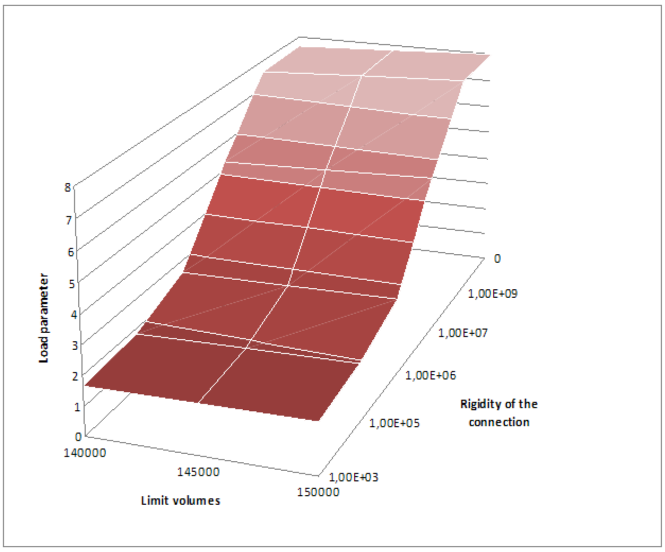

Fig. 7. Variation the base load-multiplier for plastic limit design

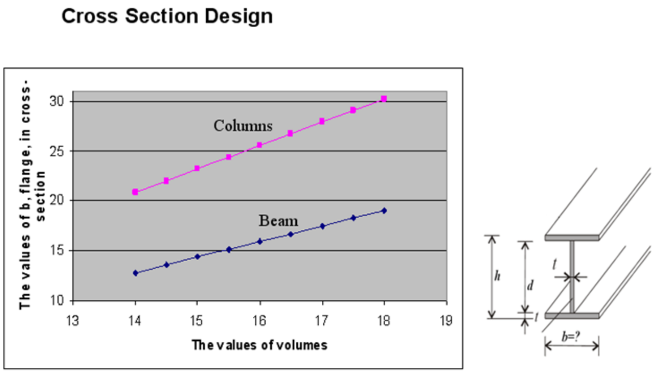

Fig. 8. Variation of the optimal sections

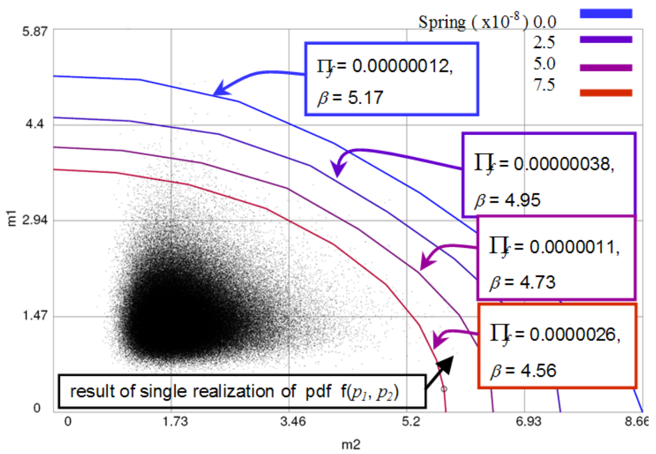

Fig. 9. Monte Carlo analysis, full rigid connections and gradual deterioration of rigidity, sample of n = 250 000 realizations.

$n = 250000$ realizations have been performed accordingly to joint pdf of loads $f_p(p_1, p_2)$.

Zero rigidity connections and gradual increase of rigidity, see Figure 10. The flexibility of the spring of connections increases from 1.5 (representing the semi-rigidity) to $100.0 \cdot 10^{-6}$ (almost fixed connection). Assumed parameters of random loads are $P_1(\mu_1 = 7\,\text{kN}, \sigma_1 = 2\,\text{kN})$ and $P_2(\mu_2 = 10.5\,\text{kN}$, $\sigma_2 = 2.25\,\text{kN})$. Sample of $n = 100000$ realizations have been performed accordingly to joint pdf of loads $f_p(p_1, p_2)$

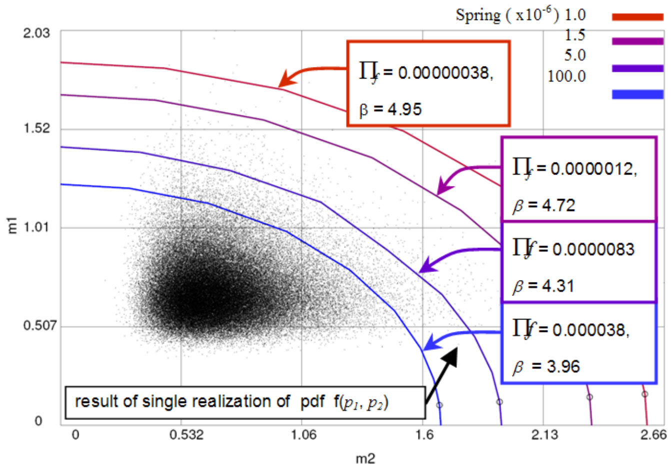

Fig. 10. Monte Carlo analysis, zero rigidity connections gradual increase of rigidity, sample of n = 100 000 realizations.

5 Conclusions

In this paper the semi-rigid behaviour is described by appropriate models and to control the plastic behavior of the structure probabilistically given bound on the complementary strain energy of the residual forces is applied. Fast and accurate failure probability assessment of elastic-plastic frame structures subjected to stochastic loading and random plastic displacement (modelled by means of complementary strain energy) is possible. Limit curves and optimal cross-sections are presented for the plastic limit load. The numerical analysis shows that the stiffness of the semi-rigid connections, the mean value and the standard deviation of the bound of the complementary strain energy of the residual forces can influence significantly the magnitude of the plastic limit load multipliers and in some cases the results are very sensitive on the stiffness of the semi-rigid connections. In the case of designed full rigid connections, deterioration of connections rigidity leads to increase of failure probability (decrease of reliability). In the case of designed zero rigidity connections (means in practice presence of semi-rigid connections) leads to decrease of failure probability (increase of reliability). The presented investigation drowns the attention to the importance of the problem but further investigations are necessary to make more general statements.

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