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Mathematical Modelling of Adaptive Skeletal Structures for Impact Absorption and Vibration Damping

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Abstract

The paper describes mathematical modelling of adaptive skeletal structures, which are equipped with semi-active dissipaters based on smart fluids or fast-operating valves and utilize the paradigm of real-time adaptation to external loading. The proposed approach is based on three subsequent stages: i) exact thermodynamic modelling of a single semi-active dissipater with the use of mass, momentum and energy conservation laws, ii) global description of the entire skeletal structure considered as an assembly of semi-active dissipaters in certain geometrical configuration, iii) real-time control of the fluid flow inside semi-active dissipaters providing instantaneous adaptability to actual dynamic loading. This methodology enables accurate representation of mechanical characteristics of the skeletal structure and reliable analysis of its adaptation capabilities.

Keywords: Adaptive structures, skeletal structures, impact absorption, vibration damping

1. Introduction

The concept of adaptive skeletal structures is based on replacement of standard passive members by semi-active dissipaters with smart fluids or fast-operating valves and application of the paradigm of real-time adaptation to applied dynamic loading [1,2]. Exact mathematical modelling of adaptive skeletal structures and precise simulation of their response to dynamic excitation is still a challenging theoretical and numerical task, which seems not to be fully resolved in the literature. The probable reason is that in case of adaptive skeletal structures with semi-active dissipaters the derivation of the mathematical model cannot be directly based on classical methods of linear dynamics effectively utilizing the formalism of mass, stiffness and damping matrices. Instead, their modelling requires dedicated, highly-specialized approaches taking into account specific properties of semi-active dissipaters and resulting non-classical dynamical models, which often appear to be relatively complex and difficult for the numerical solution.

The paper proposes a complete approach for the exact mathematical modelling of the adaptive skeletal structures based on three subsequent, distinct stages. The first step is exact thermodynamic modelling of a single semi-active dissipater with the use of mass, momentum and energy conservation laws combined with constitutive equation of the fluid taking into account its compressibility and viscosity as well as the analytical model of the controllable

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internal flow. This stage is aimed at accurate representation of dissipative behaviour and controllable characteristics of the applied semi-active devices. In turn, the second step is global description of entire skeletal structure considered as an assembly of semi-active dissipaters in a certain geometrical configuration. This is obtained by constructing global matrix equations of the system, which combine previously derived models of dissipaters and data concerning topology and actual configuration of the structure. Finally, the third step includes finding real-time control of the fluid flow inside semi-active dissipaters providing instantaneous adaptability to actual dynamic loading, which is considered as an inherent feature of adaptive skeletal structures.

The proposed approach seems to provide holistic and universal methodology for the modelling of adaptive skeletal structures composed of various types of semi-active dissipaters and dedicated for diverse engineering applications related to impact absorption and vibration damping.

2. Modelling of semi-active fluid-based dissipaters

The considered fluid-based semi-active dissipater is composed of two chambers filled with fluid and equipped with internal connection which enables controllable fluid flow. The examples such dissipaters are pneumatic dampers equipped with piezoelectric valves or magneto-rheological dampers equipped with magnetic valves (Fig. 1). The type of fluid enclosed in the damper and the type of flow control mechanism (mechanical valve vs. fluid valve) do not constrain generality of the proposed formal description of the system.

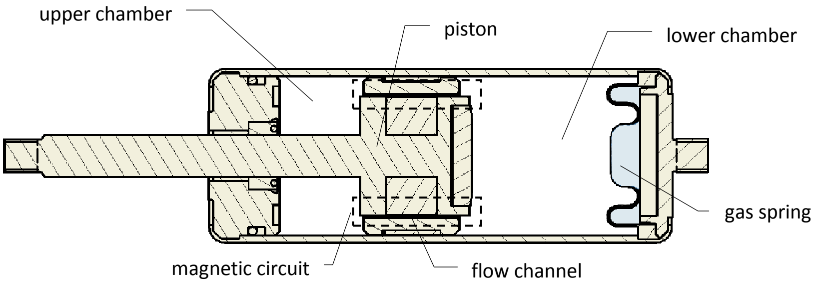


Fig. 1. Magneto-rheological damper: a typical example of fluid-based semi-active dissipater [4]

Mathematical model of fluid-based semi-active damper will be based on three fundamental principles of thermodynamics, i.e. conservation of mass, conservation of momentum and conservation of energy, applied separately for the fluid enclosed in each chamber of the device and supplemented by the constitutive equations of the fluid and the equations describing flow through the valve [3,4]. Since operation of the damper will be considered as relatively slow in comparison to velocity of fluid wave propagation, the classical assumption of the homogeneity of the fluid in each chamber of the damper will be adopted. Consequently, the model of fluid-based semi-active dissipater will comprise:

- two differential equations that govern the balance of the fluid volume for each chamber,
- two differential equations that govern the balance of the fluid energy for each chamber,
- a single algebraic equation that governs the equilibrium of the piston,

which will be complemented by:

- algebraic equation that defines constitutive model of the fluid,
- algebraic equation that describes flow of the fluid between the chambers.

The equations describing balance of fluid volume take the form:

$$\dot{V}_1 + \beta_1 V_1 \dot{p}_1 - \alpha_1 V_1 \dot{T}_1 = Q_V \tag{1}$$

$$\dot{V}_2 + \beta_2 V_2 \dot{p}_2 - \alpha_2 V_2 \dot{T}_2 = -Q_V \rho_1 \rho_2^{-1} \tag{2}$$

where V denotes volume of the fluid, p, T and ρ denote its pressure, temperature and density, while Q_V defines volumetric inflow rate to the first fluid chamber. In addition, β and α denote compressibility and thermal expansion coefficients, which are defined in terms of pressure and temperature of the fluid.

The equation of energy balance combines the energy transferred to the fluid in the form of heat and submitted enthalpy with the internal energy of the fluid and the work done by the fluid. By assuming that the process is adiabatic and by using standard definitions of the remaining thermodynamic quantities we obtain the equation:

$$\dot{m}c_pT_{(v)} + Q_V p_{(v)} - Q_V \alpha T_{(v)} p_{(v)} = \dot{m}c_pT + \dot{m}\rho^{-1}\left(-\alpha pT + \beta p^2 - \alpha Tp\right) \\ + mc_p\dot{T} - \alpha pV\dot{T} + \beta pV\dot{p} - \alpha TV\dot{p} + p\dot{V} \tag{3}$$

which has to be applied for both chambers of the device. In the above equation, the index v indicates the parameters of the fluid flowing through the valve, which are equal to the parameters of the fluid in the outflow chamber. Consequently, the energy balance equation for the outflow chamber takes a simplified form:

$$Q_V p = \dot{m}\rho^{-1}\left(-\alpha pT + \beta p^2\right) + mc_p\dot{T} - \alpha pV\dot{T} + \beta pV\dot{p} - \alpha TV\dot{p} + p\dot{V} \tag{4}$$

The equation of system equilibrium is reduced to the definition of the force generated by the damper which is expressed basically as difference of forces generated by fluid enclosed in both chambers of the damper:

$$F_{damper} = p_1A_1 - p_2A_2 \tag{5}$$

Other types of forces, such as the friction forces generated by the sealing or the delimiting forces generated at the end of the stroke can be easily taken into account by adding proper additional terms into Eq. 5.

The constitutive equation describing the fluid defines a relation between its pressure, temperature, mass and volume (or alternatively density):

$$f(p,T,m,V) = 0 \quad \text{or} \quad f(p,T,\rho) = 0 \tag{6}$$

Finally, the definition of the volumetric (or mass) flow rate of the fluid between both chambers of the device can be obtained by integration of the differential equations defining steady fluid flow. Usually, the procedure can be performed fully analytically and leads to algebraic equation defining volumetric flow rate as a function of pressures in both chambers and temperature in the outflow chamber. In such equation mechanical or non-mechanical operation of the valve is described by the presence of additional scalar time-dependent coefficient:

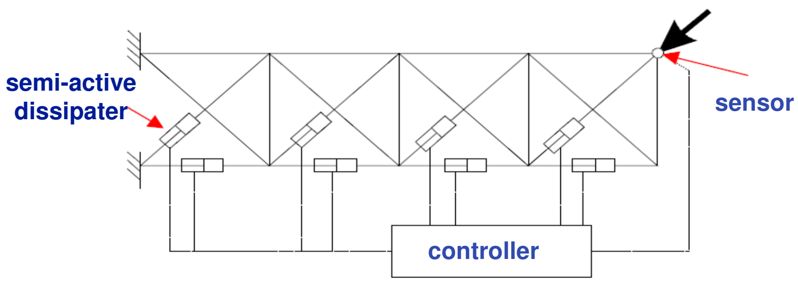
$$Q_V = Q_V(p_1, p_2, T_1, C(t)) \tag{7}$$

Regardless of the physical interpretation of the coefficient C, it provides control of the valve operation and control of the mechanical response of the semi-active dissipater.

The specification of the above equations for the case of pneumatic damper with controllable mechanical valve and the possibilities of semi-active control are presented in papers [5, 6]. Small scale applications of such devices as micro actuators are described in [7], while large scale applications for earthquake mitigation are proposed in [8]. In turn, specification of the above equations for the case of magnetorheological damper equipped with magnetic valve and confirmation that the proposed approach allows to obtain correct mechanical properties of MR damper including characteristic force-velocity hysteresis loops are presented in papers [3, 4].

3. Modelling of adaptive skeletal structure

Adaptive skeletal structure under consideration can be considered as an assembly of semi-active fluid-based dissipaters in a certain geometrical configuration. The example of such structure is depicted in Fig.2 [9].



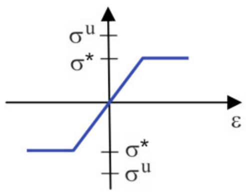


Fig. 2. Adaptive structure equipped with fluid-based semi-active dissipaters [9]: a) general scheme, b) exemplary characteristics of the dissipater

The main equation governing global response of the adaptive skeletal structure is a matrix equation describing equilibrium of its joints. It defines the balance of inertia forces resulting from structure motion, internal forces generated by semi-active dissipaters and possible external loads applied at joints. Consequently, the first term of the above equation is expressed in a classical way, with the use of mass matrix and actual accelerations of the nodes. In turn, the term which indicates internal forces exerted by semi-active dissipaters can not be described with the use of damping and stiffness terms representing dissipative and elastic response, respectively. Instead, it has to be expressed in non-classical manner with the use of previously derived definitions of forces generated by semi-active dissipaters and geometrical matrix defining their mutual connection and actual configuration. The most general form of this equation reads:

$$\mathbf{M}\frac{\mathrm{d}^2\mathbf{u}}{\mathrm{d}t^2}+\mathbf{G}(\mathbf{u})\mathbf{F_d}\left(\mathbf{u},\dot{\mathbf{u}},\int_0^t \mathbf{u}(\bar{t})d\bar{t}\right)=\mathbf{P}(\mathrm{t}) \qquad (8)$$

$$\mathbf{u}(0)=\mathbf{u_0},\ \dot{\mathbf{u}}(0)=\mathbf{v_0}$$

where $\mathbf{M}$ denotes the mass matrix derived in a classical manner, $\mathbf{G}$ denotes geometrical matrix defining topology and actual configuration of dissipaters, $\mathbf{u}$ is the vector collecting displacements of the joints, $\mathbf{F_d}$ is the vector collecting forces generated by semi-active dissipaters and $\mathbf{P}$ is the vector of external forces applied at joints.

Let us note that although matrix $\mathbf{M}$ depends exclusively on the initial configuration of the structure, the matrix $\mathbf{G}$ depends explicitly on actual configuration of the skeletal structure. Therefore, it has to be continuously updated during deformation of the structure. Moreover, the forces generated by semi-active dissipaters, which are collected in the vector $\mathbf{F_d}$, depend nonlinearly not only on displacements and velocities of the adjacent joints but also on the entire history of deformation which is symbolically written in Eq. 8 as the integral of the displacements over time.

The problem solved is to find the time-history of the equilibrium states of the skeletal structure expressed by displacements, velocities and accelerations of the nodes, as well as, the corresponding forces generated by semi-active dissipaters. Due to dependence on the time-history of the process the above problem has to be solved incrementally. Moreover, in order to accurately fulfil the equilibrium conditions, the iterative solution procedure at each time step is required. In general, the above problem can be solved both with implicit and explicit solution methods. The preference of the former ones in the case of relatively small-size problem is stimulated by solution accuracy, while the preference of the latter ones in case of large-scale problems is motivated by solution efficiency.

4. Real-time control of adaptive skeletal structure

The design of the adaptive skeletal structure is inherently associated with introduction of the real-time control strategy. Due to the fact that considered structure is assumed to be adaptive and it is composed of purely dissipative devices, the control is introduced in a semi-active way and it is executed by controlling the fluid flow inside the dissipaters. The flow can be controlled either by proper operation of the mechanical valve or by changing properties of the fluid flowing through the orifice. Independently from the assumed option, the control is always reflected in the mathematical model by change of the coefficient C present in the flow equation (Eq. 7). In case of open loop control the change of coefficient C in terms of time is predetermined before the process. In turn, in case of close loop control the coefficient C depends on actual dynamic response of the system including kinematics of selected nodes and forces generated by selected dissipaters.

The objective of applied control depends on the type of problem under consideration. In particular, different control objectives should be formulated for the problems of impact absorption and vibration damping. Hereafter, let us focus on problem of impact absorption, which usually involves not only the skeletal structure but also an impacting rigid object of a certain mass and initial velocity. In such case, the corresponding control problem may be aimed either at protection of the impacting object or protection of the impacted structure. The control problems related to protection of the impacting object may embrace: i) minimisation of impacting object deceleration during impact and ii) minimization of the final kinetic energy (and rebound) of the impacting object.

The first control problem can be mathematically formulated as:

$$\text{Find } C(t) \text{ such that } E_k(t_{end})\cong 0 \text{ and } J=\max_t|\ddot{u}(t)| \text{ is minimal} \qquad (9)$$

In the above formulation the min-max problem is supplemented with an additional condition, which enforces dissipation of the entire kinetic energy of the impacting object. It indicates that velocity of the impacting object has to be reduced to zero and that rebound of the impacting object can not occur. Obviously such condition is not always possible to be fulfilled. Thus, the second formulation of the control problem is directly related to minimization of final kinetic energy of the impacting object:

$$\text{Find } C(t) \quad \text{such that} \quad J = E_k(t_{end}) \quad \text{is minimal} \tag{10}$$

The above formulation covers both the case when impacting object velocity is not completely reduced and the case when rebound of the impacting object occurs. The control problems defined by (Eq. 9) and (Eq.10) can be considered together as a joint problem of minimisation of final kinetic energy and maximal value of deceleration:

$$\text{Find } C(t) \quad \text{such that} \quad J = E_k(t_{end}) + \max_t |\ddot{u}(t)| \quad \text{is minimal} \tag{11}$$

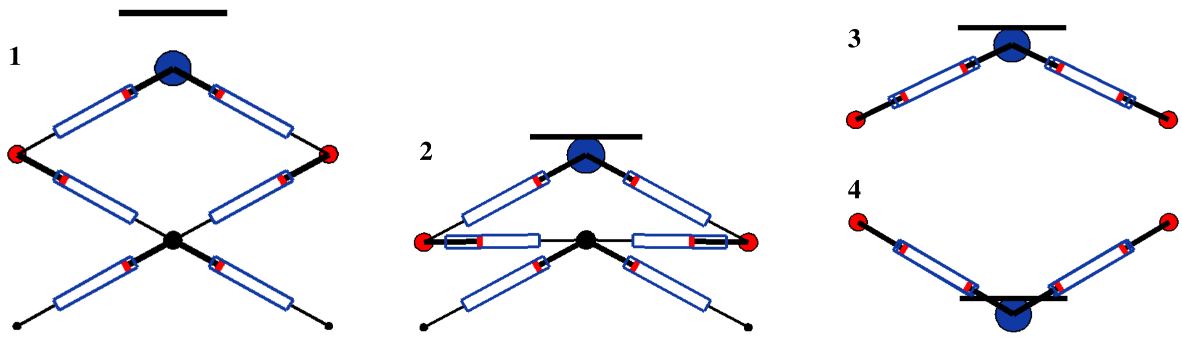
More general version of this formulation is obtained by applying weighting coefficients to both included terms.

5. Numerical example – adaptive multifolding structure

Exemplary adaptive skeletal structure considered in this section is adaptive multifolding structure. Elastic structures of this type were previously considered in the context of bifurcation analysis [10], while dissipative structures composed of magnetorheological dampers were proposed as one of the special technologies for adaptive impact absorption [11]. Here, the multifolding structure is composed of six adaptive double-chamber pneumatic cylinders arranged in three adjacent layers (Fig. 3). Each cylinder is equipped with controllable valve which governs the flow of the gas between two pressure chambers and allows to control mechanical characteristics of the device. The two bottom nodes are fixed, while the displacements of remaining joints are constrained in the horizontal direction. Moreover, we focus exclusively on the deformation modes symmetric along the vertical axis.

Mathematical model of the analyzed adaptive skeletal structure includes equations describing mechanical response of the pneumatic absorbers, matrix equation governing global equilibrium of the structure and the control law defining operation of the controllable valves. The equations governing mechanical response of the pneumatic absorbers are directly based on previously derived general equations presented in Sect. 2 (Eqs 1-7) and they include constitutive model specified for the case of ideal gas by setting proper definitions of compressibility and thermal expansion coefficients. The matrix equation of global equilibrium is now reduced to three differential equations defining dynamic equilibrium of the nodes, which take into account both physical and geometrical nonlinearity of the system. The values of forces acting on the nodes are obtained from the model of the pneumatic absorbers, while their actual orientations result from the actual inclination angles between the dissipaters. The model includes the possibility of controlling the operation of internal valves by using open and closed control loops. The important fact is that valve opening can be modified between closed position totally restricting the fluid flow and fully open position enabling immediate equalization of pressure. This allows to model both the response with snap-through effect characteristic for elastic elements and the smooth response characteristic for dissipative elements.

The presented numerical example reveals the influence of the opening of the valves located at particular level of the structure on its global dynamic response (Fig. 3). At this stage of the research the previously formulated control objectives were not considered and implemented. Instead, it was proved that change of valve opening leads to qualitative change of the skeletal structure response.



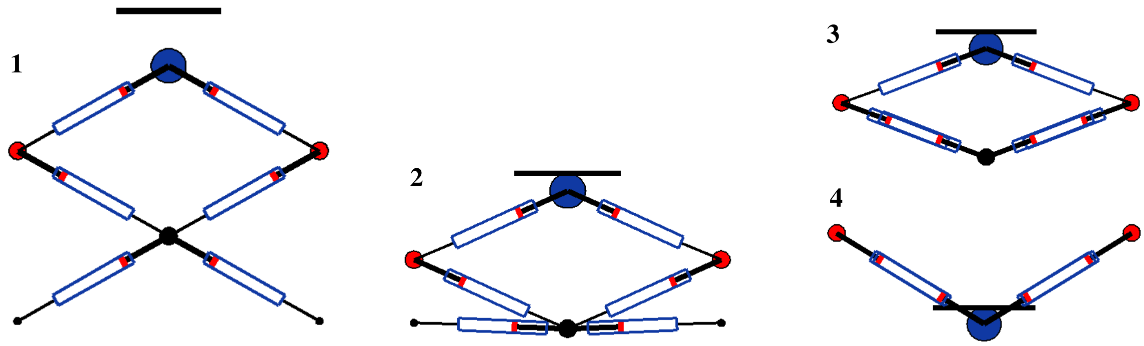


Fig. 3. Various folding sequences of adaptive multifolding skeletal structure

The system with closed valves is characterized by elastic response involving subsequent snap-through effects, which occur at selected layers of the structure. By increasing opening of the valves in pairs of adjacent absorbers we increase compliance of the selected layer of the structure and we initiate the local process of plastic folding. As a result, two different, non-trivial folding patterns characterized by different change of reaction force and different energy dissipation can be obtained (Fig. 3).

6. Conclusions

The proposed approach seems to provide holistic and universal methodology for the modelling of the adaptive skeletal structures composed of various types of semi-active dissipaters and dedicated for diverse engineering applications. Its main advantage is exact representation of the mechanical characteristics of adaptive skeletal structure and reliable analysis of the possibilities of its real-time control aimed at impact absorption and vibration damping. More comprehensive numerical examples illustrating the above described modelling methodology are currently under preparation and they are intended to be presented at the conference.

Acknowledgements

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