Anisotropic friction and wear rules with account for anisotropy evolution
Zenon Mróz, Stanisław Kucharski

Institute of Fundamental Technological Research
Warsaw, Poland. zmroz@ippt.pan.pl

Summary: In the paper the effect of anisotropy and of variation of surface topography on friction and wear process has been investigated theoretically and experimentally. The evolution of roughness and friction coefficient during wear process is analysed experimentally. The model predicting evolution of wear rate and friction coefficient has been proposed.

Introduction
The anisotropic friction response in sliding along contact surfaces is usually associated with distribution of surface asperities in the form of oriented striation patterns induced by machining or polishing processes. Such response is also observed in fibre-reinforced composite materials where the significant effect of fiber orientation on frictional behaviour and wear has been observed, cf. [1]. The friction anisotropy occurs also when sliding proceeds along the mono-crystalline surface.

The present study is related to analysis of coupled friction and wear process in sliding along the rough surface with an anisotropic asperity pattern, characterized by mutually orthogonal striations. Due to wear process the initial anisotropic response evolves with the asperity distribution tending to a steady-state pattern, usually corresponding to the isotropic response. The friction response is also affected and reaches its steady state.

Orthotropic friction model
Following the previous papers by Michalowski and Mróz [2], Mróz and Stupišewicz [3] consider an orthotropic surface roughness with the parallel layout of asperities, Fig.1

![Diagram showing orthotropic friction model](image)

Fig.1 Loci of limit friction condition $F=0$ and of constant dissipation rate, $D=const$ for an orthotropic roughness profile

The limit friction condition is assumed in the form:

$$F(T_1, T_2) = \left[ \frac{T_1}{\mu_1} \right]^2 + \left[ \frac{T_2}{\mu_2} \right]^2 - N = 0$$  \hspace{1cm} (1)

where $T_1, T_2$ are the tangential contact tractions parallel to the orthotropy axes $1 \parallel 2$. The normal contact traction $N$ is assumed positive, $N>0$, for the case of compression. The non-associated sliding rule was derived in [2, 3] but for the simplicity the associated rule is
assumed, thus
\[ v_1 = \dot{\lambda} \frac{\partial f}{\partial T_1} = \frac{\dot{\lambda}}{\mu_1}, \quad v_2 = \dot{\lambda} \frac{\partial f}{\partial T_2} = \frac{\dot{\lambda}}{\mu_2} \]  
(2)

In view of (1), (2), there is
\[ \dot{\lambda} = \dot{D} = N \sqrt{(\mu_1 v_1)^2 + (\mu_2 v_2)^2} = T_1 v_1 + T_2 v_2 \]
(3)

and the dissipation rate \( \dot{D} \) is expressed by the multiplier \( \dot{\lambda} \). Here \( \mu_1 \) and \( \mu_2 \) are the principal values of the friction tensor \( M \) with principal directions coinciding with the orthotropy axes 1, 2. Now we have the inverse relations

\[ T_1 = \frac{\mu_1^2 v_1}{\sqrt{(\mu_1 v_1)^2 + (\mu_2 v_2)^2}}, \quad T_2 = \frac{\mu_2^2 v_2}{\sqrt{(\mu_1 v_1)^2 + (\mu_2 v_2)^2}} \]

and
\[ \frac{T_1}{T_2} = \tan \alpha = \frac{\mu_2^2 v_2}{\mu_1^2 v_1} = \frac{\mu_2^2}{\mu_1^2} \tan \beta \]

where \( \alpha \) and \( \beta \) denote the angles of the tangential force \( T \) and the sliding velocity vector \( v \) to the orthotropy axis 1.

**Wear rule accounting for the asperity evolution**

Assume the wear rate to be proportional to the friction dissipation rate, thus
\[ \dot{w}_w = k_w D = k_w N \sqrt{(\mu_1 v_1)^2 + (\mu_2 v_2)^2} = k_w N \sqrt{(\mu_1 \cos \beta)^2 + (\mu_1 \sin \beta)^2} \]

(6)

where \( k_w \) is the wear factor and \( v \) is the sliding vector modulus. Such wear rule was used in [4] in numerical analysis. However, due to wear process, the friction moduli \( \mu_1 \) and \( \mu_2 \) are assumed to vary, thus
\[ \mu_1 = \mu_{\infty} - (\mu_{\infty} - \mu_0) e^{-\lambda t}, \quad \mu_2 = \mu_{\infty} - (\mu_{\infty} - \mu_0) e^{-\lambda t} \]

(7)

where \( \mu_{\infty}, \mu_0 \) are the asymptotic principal values of the friction tensor \( M \) and \( \mu_{\infty} \), \( \mu_0 \) are the initial values. \( \lambda \) denotes the material parameter. Relations (7) can be expressed as
\[ M = \kappa (M_{\infty} - M) e^{-\lambda t}, \quad M_{\infty} = M_{\infty} - (M_{\infty} - M) e^{-\lambda t} \]

(8)

where \( \lambda = \int \dot{\lambda} dt = \int \dot{D} dt \) is the total frictional dissipation. In particular, it can be assumed that \( \mu_{\infty} = \mu_{\infty} = \mu \), and then the asperity pattern tends to isotropic distribution in the wear process.

**Experimental verification**

The wear tests have been executed by means of a ball-on-disk wear tester by applying the reciprocal ball motion (stroke length 3 mm, frequency of ball oscillation \( \omega = 4.461 \text{ s}^{-1} \)) in unlubricated wear conditions against flat specimen, (cf. tests for isotropic friction [5]). The sapphire (AL2O3) ball of diameter \( D = 6 \text{ mm} \) was used. The wear of ground surface characterised by long grooves was tested. Three directions of ball motion with respect to grooves generated in surface finishing process have been tested: longitudinal, oblique and transverse directions. Different values of normal load were applied: 0.05N, 0.1N and 0.5N. The evolution of friction coefficient in the initial stage of wear is presented in Fig. 2 for longitudinal and transverse sliding directions.

One can observe that initially, the friction coefficient corresponding to the transverse direction is greater than that corresponding to the longitudinal direction. In this stage