THE EFFECT OF STRUCTURE ON DYNAMIC THERMAL CHARACTERISTICS OF MULTILAYER WALLS

E. KOSSECKA (Warszawa)

The effect of internal thermal structure on dynamic characteristics of multilayer walls is analyzed. Mathematical basis constitute the integral formulae for the heat flow across the surfaces of the wall. The notion of structure factors is introduced and the conditions they impose on response factors are derived, using the Laplace transform method. Simple examples of walls, representing different types of thermal resistance and capacity distribution, are analyzed to illustrate general relations between the structure factors and the response factors.

1. INTRODUCTION

The reason for consideration presented below was primarily the problem of taking into account thermal bridges in exterior walls of buildings, in energy simulation in building design, avoiding detailed simulation of three-dimensional heat transfer processes. The question was: How to modify response factors for plane walls used in programs such as DOE-2?

The simplest method to be suggested here is of course just to calculate - solving the steady state heat transfer problem - the overall resistance of a wall with imperfections and simply multiply response factors by the resulting correction factor. This would be satisfactory for light walls, for which storage effects are insignificant. It is also possible to calculate separately response factors for wall elements with thermal bridges, but it would be troublesome to include them into existing programs for computerized energy calculations. The question posed reduces thus to the following: Is it possible to find another method, which would be relatively simple but at the same time accurate enough?

Imperfections in plane walls not only change their resistance but also modify their thermal dynamics - the profiles of thermal impulses penetrating through them - represented by response factors. To have an idea of possible relationships between static and dynamic thermal characteristics of structures, in which three-dimensional heat flow occurs, one should analyze, with great attention, relationships of this kind for a plane multi-layer wall. The important mathematical tool, to be used here, are the integral formulae for the heat flow across the wall surfaces, in a finite time interval [1, 2, 3, 4].

Those formulae lead to the idea of thermal structure factors for a wall, which govern the storage effects in transient heat flow. They have their counterparts in three-dimensional heat transfer problems [2]. One should just find the conditions they impose on response factors and make use of analogies between one- and three-dimensional problems, and then try to replace three-dimensional structure by an equivalent but much simpler - from the mathematical point of view - plane wall.

These conditions - which represent the relationships between structure factors and response factors - are derived in this work. Similarly as the integral formulae for the heat flow, they have their three-dimensional counterparts; this will be presented in [5, 6]. They lead to the conclusion that structure factors themselves indicate whether a wall, of given resistance and capacity, is relatively penetrable to thermal impulses or whether it is delaying them.

Although the occasion to study the relations between structure factors and response factors was the problem of thermal bridges, I realized that results for plane walls are so interesting that it is reasonable to prepare a separate paper devoted to this problem. This work is thus an introduction to the study on the effects of internal structure on the thermal dynamics of walls and possibilities to represent them by another, equivalent structures.

Simple examples of structures, representing various types of thermal design, are analyzed to demonstrate general relations between their static /structural/ and dynamic thermal characteristics.

2. STORAGE EFFECTS IN TRANSIENT HEAT FLOW THROUGH A WALL

Consider heat transfer through an exterior building wall of thickness L, separating a room at temperature T_i from environment at temperature T_e. Assume that thermophysical properties of the wall: thermal conductivity λ, specific heat c and density ρ are constant in time.

Assume that one-dimensional heat transfer conditions are satisfied. The temperature in the wall is represented by the function $T(x,t)$ of the spatial coordinate x and time t. The heat flux is represented by the function $q(x,t) = -\lambda \partial T/\partial x$. Let one-dimensional Fourier's heat conduction equation be satisfied [8]:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[\lambda \frac{\partial T}{\partial x}\right]. \tag{2.1}$$

In the case of a multilayer wall composed of materials with different thermal properties, Equation (2.1) has to be understood symbolically, as a set of heat conduction equations written for the separate layers and the conditions of continuity of temperatures and heat flow rates at the interfaces.

The coordinate system is assumed in which the wall surface facing temperature T_i corresponds to the plane $x=0$ and the surface facing temperature T_e to $x=L$; surface temperatures and heat fluxes are thus denoted by $T(0)$, $T(L)$, $q(0)$ and $q(L)$ respectively; surface fluxes in the directions of the outward normals are denoted by q_{ni} and q_{ne}:

$$q_{ni} = -q(0), \qquad q_{ne} = q(L). \tag{2.2}$$

Heat transfer from the wall surfaces to the room and environment is expressed by the Newton's law; the surface heat balance equations take the form:

(2.3) $$q_{ni} = \frac{1}{R_i}\left[T(0) - T_i\right], \qquad q_{ne} = \frac{1}{R_e}\left[T(L) - T_e\right],$$

where R_i and R_e are the constant surface film resistances.

Let $R_{0\text{-}x}$ and $R_{x\text{-}L}$ denote the thermal resistances of the wall layers enclosed in the intervals $[0, x]$ and $[x, L]$ respectively, and $R_{i\text{-}x}$, $R_{x\text{-}e}$ the resistances for heat transmission from the point x in the wall to the internal and external environment, respectively. R is the total thermal resistance of the wall, surface to surface, R_k is the total resistance for heat transmission through the wall, environment to environment:

(2.4) $$R_{0-x} = \int_0^x \frac{dx'}{\lambda(x')}, \qquad R_{x-L} = \int_x^L \frac{dx'}{\lambda(x')}, \qquad R = \int_0^L \frac{dx'}{\lambda(x')},$$

(2.5) $$R_{i-x} = R_i + R_{0-x}, \qquad R_{x-e} = R_{x-L} + R_e, \qquad R_k = R_i + R + R_e\,.$$

$R_{i\text{-}x}$, $R_{x\text{-}e}$ and R_k satisfy the identity:

(2.6) $$\frac{R_{i-x}}{R_k} + \frac{R_{x-e}}{R_k} \equiv 1$$

Multiplying the heat conduction equation (2.1) by the functions $R_{x\text{-}e}/R_k$, $R_{i\text{-}x}/R_k$, and integrating the expressions appearing on its left and right side first with respect to x from 0 to L, taking into account the boundary conditions (2.3), and then with respect to time from t_0 to t, with the assumption of constant resistances and heat capacities, we obtain the following integral relations between the total amounts of heat flow across the surfaces in the finite time interval $[t_0, t]$, Q_{ni} and Q_{ne},

(2.7) $$Q_{ni}(t_0,t) = \int_{t_0}^{t} dt' q_{ni}(t')\,, \qquad Q_{ne}(t_0,t) = \int_{t_0}^{t} dt' q_{ne}(t')\,,$$

as well as the temperatures $T_i(t)$ and $T_e(t)$ and temperature differences inside the wall between t and t_0:

(2.8) $$Q_{ni}(t_0,t) = \frac{1}{R_k}\int_{t_0}^{t} dt'\left[T_e(t') - T_i(t')\right] - \int_0^L dx \frac{R_{x-e}}{R_k}\rho c\left[T(x,t) - T(x,t_0)\right],$$

(2.9) $$Q_{ne}(t_0,t) = \frac{1}{R_k}\int_{t_0}^{t} dt'\left[T_i(t') - T_e(t')\right] - \int_0^L dx \frac{R_{i-x}}{R_k}\rho c\left[T(x,t) - T(x,t_0)\right].$$

The relations (2.8), (2.9) may be rewritten as follows:

$$Q_{ni} = -Q_{ie} - Q_{ci} \ , \qquad Q_{ne} = -Q_{ie} - Q_{ce} \ , \tag{2.10}$$

where:

$$Q_{ie}(t_0,t) = \Delta t \frac{1}{R_k}\left[\overline{T}_i - \overline{T}_e\right]; \qquad \Delta t = t - t_0 \ , \tag{2.11}$$

$$Q_{ci}(t_0,t) = \int_0^L dx \frac{R_{x-e}}{R_k} \rho c\left[T(x,t) - T(x,t_0)\right], \tag{2.12}$$

$$Q_{ce}(t_0,t) = \int_0^L dx \frac{R_{i-x}}{R_k} \rho c\left[T(x,t) - T(x,t_0)\right]. \tag{2.13}$$

$\overline{T}_i$ and $\overline{T}_e$ are the time averages of the temperatures T_i and T_e over the interval $[t_0,t]$; Q_{ie} represents the total heat flow through the wall due to the difference of the ambient temperatures. Summing up the expressions for Q_{ni} and Q_{ne}, and taking into account the identity (2.6), gives the heat balance equation for the wall:

$$Q_{ni} + Q_{ne} = -\left[Q_{ci} + Q_{ce}\right] = -Q_c \ , \tag{2.14}$$

where Q_c is the difference in the amount of heat stored in the wall element of unit cross-sectional area:

$$Q_c(t_0,t) = \int_0^L dx \rho c\left[T(x,t) - T(x,t_0)\right]. \tag{2.15}$$

Q_{ci} and Q_{ce} thus represent the components of the total heat flow across the wall surfaces contributing to Q_c. Under the assumption made previousely, about the independence of the thermophysical parameters of the materials on time, the quantities Q_{ci} and Q_{ce} depend exclusively on the temperature difference between the final and the initial state; if there is no temperature difference, Q_{ci} and Q_{ce} vanish. For prescribed T_i and T_e, they are the only unknown components of the heat flows across the wall surfaces. For equal T_i and T_e (symmetric heating or cooling), they coincide with Q_{ni} and Q_{ne}.

Formulae analogous to (2.8), (2.9) for wall elements of complex structure were derived by E.Kossecka in [2].

3. THERMAL STRUCTURE FACTORS FOR PLANE WALLS

Evaluation of Q_{ci}, Q_{ce} and Q_c for prescribed temperatures $T_i(t)$ and $T_e(t)$ necessitates, in general, the solution of the transient heat conduction problem. Explicit expressions, however, can be written down immediately for the heat conduction process, for which the initial and final temperature difference converges asymptotically to temperature difference between the two steady states of heat flow /see [1, 2, 3, 4]/:

(3.1) $$T(x,t)-T(x,t_0) \Rightarrow \frac{R_{i-x}}{R_k}\Delta T_e + \frac{R_{x-e}}{R_k}\Delta T_i ,$$

$$\Delta T_i = T_i(t)-T_i(t_0), \qquad \Delta T_e = T_e(t)-T_e(t_0) .$$

For such a process, it follows from (2.8), (2.9), that Q_{ni}, Q_{ne} and Q_c converge to:

(3.2) $$Q_{ni}(t_0,t) \Rightarrow \frac{\Delta t}{R_k}\left[\overline{T}_e - \overline{T}_i\right] - C\varphi_{ii}\Delta T_i - C\varphi_{ie}\Delta T_e ,$$

(3.3) $$Q_{ne}(t_0,t) \Rightarrow \frac{\Delta t}{R_k}\left[\overline{T}_i - \overline{T}_e\right] - C\varphi_{ei}\Delta T_i - C\varphi_{ee}\Delta T_e ; \qquad \Delta t = t - t_0 ,$$

(3.4) $$Q_c(t_0,t) \Rightarrow C\gamma_i \Delta T_i + C\gamma_e \Delta T_e ,$$

where the quantities φ_{ii}, φ_{ie}, φ_{ee}, and γ_i, γ_e are given by:

(3.5) $$\varphi_{ii} = \frac{1}{C}\int_0^L dx\, \rho c \frac{R_{x-e}^2}{R_k^2}, \qquad \varphi_{ee} = \frac{1}{C}\int_0^L dx\, \rho c \frac{R_{i-x}^2}{R_k^2} ,$$

(3.6) $$\varphi_{ie} = \frac{1}{C}\int_0^L dx \rho c \frac{R_{i-x}R_{x-e}}{R_k^2} ,$$

(3.7) $$\gamma_i = \varphi_{ii} + \varphi_{ie} = \frac{1}{C}\int_0^L dx\, \rho c \frac{R_{x-e}}{R_k}, \qquad \gamma_e = \varphi_{ie} + \varphi_{ee} = \frac{1}{C}\int_0^L dx\, \rho c \frac{R_{i-x}}{R_k} .$$

C is the total thermal capacity of the wall element of unit cross sectional area:

(3.8) $$C = \int_0^L dx \rho c .$$

The following identities are satisfied - as a consequence of the identity (2.6):

(3.9) $$\gamma_i + \gamma_e = 1 ,$$

(3.10) $$\varphi_{ii} + 2\varphi_{ie} + \varphi_{ee} = 1 .$$

The dimensionless quantities φ_{ii}, φ_{ie}, φ_{ee}, and γ_i, γ_e, in what follows, are called the thermal structure factors for a wall. They constitute, together with the total resistance R_k and capacity C, the basic wall thermal characteristics and have their counterparts for structures in which three-dimensional heat flow occurs. They can be determined experimentally in the heat transfer processes with steady initial and final state of heat flow.

The structure factors multiplied by C are called by B.R.Anderson the thermal mass factors /see [7]/. They have an essential influence on storage effects in a wall. Their magnitudes should be taken into account in data analysis of in situ measurements of thermal resistances and thermal transmittances /see also [3,4]/.

4. RELATIONSHIPS BETWEEN THERMAL STRUCTURE FACTORS AND RESPONSE FACTORS

The response factors for multilayer walls, used in building energy modelling, may be calculated by applying the Laplace transform method [9, 10]. The Laplace transforms of the surface heat fluxes q_{ni}, q_{ne}, and ambient temperatures T_i, T_e, denoted here as $\bar{q}_{ni}(s)$, $\bar{q}_{ne}(s)$, $\bar{T}_i(s)$, $\bar{T}_e(s)$ respectively, are related by:

$$\bar{q}_{ni}(s) = \frac{1}{B(s)}\bar{T}_e(s) - \frac{D(s)}{B(s)}\bar{T}_i(s)\,, \tag{4.1}$$

$$\bar{q}_{ne}(s) = \frac{1}{B(s)}\bar{T}_i(s) - \frac{A(s)}{B(s)}\bar{T}_e(s), \tag{4.2}$$

where $A(s)$, $B(s)$ and $D(s)$ are the elements of the transition matrix for the structure.

The Laplace transform of a surface heat flux $q(t)$, due to boundary temperature $T(t)$, thus has the general form:

$$\bar{q}(s) = (-1)^p \frac{N(s)}{B(s)}\bar{T}(s)\,, \qquad p = 0, 1\,. \tag{4.3}$$

For $q = q_{ni}$ $p = 0$, $N = 1$ for $T = T_e$ and $p = 1$, $N = D$ for $T = T_i$, whereas for $q = q_{ne}$ $p = 0$, $N = 1$ for $T = T_i$ and $p = 1$, $N = A$ for $T = T_e$. Therefore $p = 0$ for the (ie) and (ei) modes, whereas for the (ii) and (ee) modes $p = 1$.

The Laplace transform of the heat flow $Q(t)$, corresponding to (4.3), is of the form:

$$\bar{Q}(s) = (-1)^p \frac{N(s)}{sB(s)}\bar{T}(s), \qquad p = 0, 1\,. \tag{4.4}$$

Consider now the heat transfer process with the function $T(t)$, in the interval $(0, n\Delta)$, linear in every interval $[(m\text{-}1)\Delta, m\Delta]$, $m = 1, 2, ...n$, and equal to $T(m\Delta)$ at $t = m\Delta$, with $T(0) = 0$, represented as the sum of triangular pulses /see [9, 10]/:

$$T(t) = \sum_{m=1}^{n} T(m\Delta)P_\Delta(t - m\Delta)\,, \tag{4.5}$$

$$P_\Delta(t - m\Delta) = \frac{1}{\Delta}\{r[t-(m-1)\Delta] - 2r[t - m\Delta] + r[t-(m+1)\Delta]\}\,. \tag{4.6}$$

$r(t)$ in (4.6) denotes the ramp function: $r(t) = t\eta(t)$, where $\eta(t)$ is the Heaviside unit step function.

The Laplace transform of $T(t)$, given by (4.5), is the sum:

$$\bar{T}(s) = L\left\{\sum_{m=1}^{n} T(m\Delta)P_\Delta(t - m\Delta)\right\} = \sum_{m=1}^{n} T(m\Delta)\bar{P}_\Delta(s, m\Delta)\,. \tag{4.7}$$

The expression for the Laplace transform of the shifted pulse function is to be obtained using the expression for the Laplace transform of the ramp function $\bar{r}(s) = 1/s^2$ and the general property of the Laplace transform of a shifted function /see [11]/:

(4.8) $$L\{g(t-t_0)\eta(t-t_0)\} = e^{-st_0}\bar{g}(s),$$

(4.9) $$\bar{P}_\Delta(s, m\Delta) = L\{P_\Delta(t-m\Delta)\} = \frac{1}{\Delta s^2}\left[e^{-s(m-1)\Delta} - 2e^{-sm\Delta} + e^{-s(m+1)\Delta}\right].$$

The Laplace transform of the heat flux density corresponding to (4.7) is given as:

(4.10) $$\bar{q}(s) = (-1)^p \sum_{m=1}^{n} T(m\Delta)\frac{N(s)}{B(s)}\bar{P}_\Delta(s, m\Delta).$$

The responses of the structure to a temperature pulse, after time periods equal to the multiples of Δ, are called the response factors [9, 10, 12]. The m-th response factor corresponding to a triangular pulse, denoted here as $X(m\Delta)$, is defined as:

(4.11) $$X(m\Delta) = L^{-1}\left\{\frac{N(s)}{B(s)}\,\bar{P}_\Delta(s)\right\}\Big|_{t=m\Delta}.$$

The representation of $q(t)$ at $t = n\Delta$ in terms of response factors $X(m\Delta)$ is as follows:

(4.12) $$q(n\Delta) = (-1)^p \sum_{m=1}^{n} T(m\Delta)X[(n-m)\Delta] = (-1)^p \sum_{m=0}^{n-1} T[(n-m)\Delta]X(m\Delta)$$

For the compatibility of (4.12) with the steady state heat flow solution, it is necessary that response factors satisfy the condition:

(4.13) $$\sum_{m=0}^{\infty} X(m\Delta) = \frac{1}{R_k}$$

Consider now the heat transfer process with temperature $T(t)$ in the form of the unit step function:

(4.14) $$T(t) = \eta(t), \qquad \bar{T}(s) = \frac{1}{s}$$

For such a process the heat flow in time interval equal to the multiple of Δ, $Q(n\Delta)$, can be represented exactly in terms of response factors $X(m\Delta)$ corresponding to triangular pulses. According to (4.4), (4.14), the Laplace transform of the heat flow corresponding to $T(t)$ in the form of the unit step function is given as:

(4.15) $$\bar{Q}(s) = (-1)^p \frac{1}{s^2}\frac{N(s)}{B(s)}$$

With the help of the identity:

$$1 \equiv 1 + \sum_{m=1}^{n} \left\{ (m+1) e^{-sm\Delta} - 2me^{-sm\Delta} + (m-1) e^{-sm\Delta} \right\} = \tag{4.16}$$
$$= \sum_{m=1}^{n} m \left\{ e^{-s(m-1)\Delta} - 2e^{-sm\Delta} + e^{-s(m+1)\Delta} \right\} + (n+1) e^{-sn\Delta} - ne^{-s(n+1)\Delta}$$

the Laplace transform of the ramp function, in the interval (0, $n\Delta$), may be represented as:

$$\frac{1}{s^2} = \sum_{m=1}^{n} m\Delta \overline{P}_\Delta (s, m\Delta) + \frac{1}{s^2} \left[(n+1) e^{-sn\Delta} - ne^{-s(n+1)\Delta} \right]. \tag{4.17}$$

(4.17) combined with (4.15) gives the following expression for $Q(n\Delta)$:

$$Q(n\Delta) = L^{-1} \left\{ (-1)^p \frac{1}{s^2} \frac{N(s)}{B(s)} \right\} \Big|_{t=n\Delta} = (-1)^p \sum_{m=1}^{n} m\Delta X \left[(n-m)\Delta \right] \tag{4.18}$$
$$= (-1)^p \sum_{m=0}^{n-1} (n-m)\Delta X (m\Delta).$$

The last two terms in (4.17), by virtue of (4.8), give no contribution to (4.18). For sufficiently large number n, the sum of response factors in (4.18) may be, according to (4.13), replaced by $1/R_T$; thus the following asymptotic relationship takes place:

$$\lim_{n\to\infty} \left[Q(n\Delta) - (-1)^p n\Delta \sum_{m=0}^{n-1} X(m\Delta) \right] = \lim_{n\to\infty} \left[Q(n\Delta) - (-1)^p \frac{n\Delta}{R_k} \right] \tag{4.19}$$
$$= -(-1)^p \Delta \sum_{m=1}^{\infty} mX(m\Delta).$$

On the other hand, the asymptotic relationship between the heat flow due to the boundary temperature excitation in the form of a unit step, as follows from the asymptotic relations (3.2), (3.3) and definition of the thermal structure factors (3.11), has the form:

$$\lim_{n\to\infty} \left[Q(n\Delta) - (-1)^p \frac{n\Delta}{R_k} \right] = -C\varphi, \qquad p = 0, 1. \tag{4.20}$$

p in (4.20) is the same as p in (4.3) and (4.4). If Q in (4.20) is to be identified with Q_{ni} then $p = 0$, $\varphi = \varphi_{ie}$ for $T = T_e$, $p = 1$, $\varphi = \varphi_{ii}$ for $T = T_i$, whereas for $Q = Q_{ne}$ $p = 0$, $\varphi = \varphi_{ei}$ for $T = T_i$, $p = 0$, $\varphi = \varphi_{ee}$ for $T = T_e$.

Equating the right hand sides of (4.19) and (4.20) gives the following relation between the response factors $X(m\Delta)$, $m \geq 1$, for a given heat transfer mode, appropriate structure factor and thermal capacity of a wall:

(4.21) $$(-1)^p \Delta \sum_{m=1}^{\infty} mX(m\Delta) = C\varphi, \qquad p = 0, 1 .$$

Denote by $X_{ii}(m\Delta)$, $X_{ie}(m\Delta)$ and $X_{ee}(m\Delta)$ the response factors corresponding to the (ii), (ie) and (ei), and (ee) heat transfer modes respectively. In the usual notation /see[6, 7]/ $X_{ii}(m\Delta) = X(m\Delta)$, $X_{ie}(m\Delta) = Y(m\Delta)$, $X_{ee}(m\Delta) = Z(m\Delta)$. The heat fluxes q_{ni} and q_{ne}, across the wall surfaces in the direction of the outside normal, are represented in terms of the response factors $X_{ii}(m\Delta)$, $X_{ie}(m\Delta)$ and $X_{ee}(m\Delta)$ in the following way:

(4.22) $$q_{ni}(n\Delta) = \sum_{m=0}^{n-1} T_e[(n-m)\Delta] X_{ie}(m\Delta) - \sum_{m=0}^{n-1} T_i[(n-m)\Delta] X_{ii}(m\Delta),$$

(4.23) $$q_{ne}(n\Delta) = \sum_{m=0}^{n-1} T_i[(n-m)\Delta] X_{ie}(m\Delta) - \sum_{m=0}^{n-1} T_e[(n-m)\Delta] X_{ee}(m\Delta).$$

From (4.21), we obtain thus the set of equations, which represent the relationships between the response factors $X_{ii}(m\Delta)$, $X_{ie}(m\Delta)$, $X_{ee}(m\Delta)$, thermal capacity of the wall C and structure factors φ_{ii}, φ_{ie}, φ_{ee}:

(4.24) $$-\Delta \sum_{n=1}^{\infty} nX_{ii}(n\Delta) = C\varphi_{ii}, \qquad -\Delta \sum_{n=1}^{\infty} nX_{ee}(n\Delta) = C\varphi_{ee} ,$$

(4.25) $$\Delta \sum_{n=1}^{\infty} nX_{ie}(n\Delta) = C\varphi_{ie} .$$

The addition of the above equations gives:

(4.26) $$\Delta \sum_{n=1}^{\infty} n\left[-X_{ii}(n\Delta) + X_{ie}(n\Delta)\right] = C\left[\varphi_{ii} + \varphi_{ie}\right] = C\gamma_i ,$$

(4.27) $$\Delta \sum_{n=1}^{\infty} n\left[X_{ie}(n\Delta) - X_{ee}(n\Delta)\right] = C\left[\varphi_{ie} + \varphi_{ee}\right] = C\gamma_e ,$$

(4.28) $$\Delta \sum_{n=1}^{\infty} n\left[-X_{ii}(n\Delta) + 2X_{ie}(n\Delta) - X_{ee}(n\Delta)\right] = C\left[\gamma_i + \gamma_e\right] = C .$$

The last equation, (4.28), could also be obtained directly from (4.22), (4.23), (4.21) and the heat balance equation (2.18).

Equations (4.24 - 4.28) represent the relationships between the structural and dynamic thermal characteristics of the wall: φ_{ii}, φ_{ie}, φ_{ee}, γ_i, γ_e, capacity C and the response factors $X_{ii}(m\Delta)$, $X_{ie}(m\Delta)$ and $X_{ee}(m\Delta)$ calculated for the given time interval Δ.

Equations (4.24), (4.25) for $X_{ii}(m\Delta)$, $X_{ie}(m\Delta)$ and $X_{ee}(m\Delta)$ must be satisfied simultaneousely with (4.13). Response factors $X(m\Delta)$, with $m \geq 1$, which appear in (4.24 - 28), describe the storage effects - heat fluxes after the time of duration of the triangular temperature impulse. Their magnitudes increase with the product $C\varphi$. However, at the same time, the sum of all response factors must be equal to $1/R_k$, therefore the larger are the values of $X(m\Delta)$ for $m \geq 1$, the smaller is the value $X(0)$ and vice versa.

All these equations do not determine the response factors in a unique way, but rather play the role of constraints conditions. One may expect however, that walls with the same total thermal resistance R_k, capacity C and structure factors have also similar dynamic characteristics - specifically response factors. This leads to the concept of the „equivalent wall" - a simple structure which has the same type of dynamic thermal behaviour as a more complex structure and may be used as its substitute in energy simulations in building design. This problem is discussed in [5, 6]. An example of a two-layer wall, equivalent to the four-layer one, is presented in the next section.

5. RELATIONS BETWEEN STRUCTURE AND THERMAL DYNAMICS FOR MULTILAYER WALLS

The form of the expressions under the integral signs in (3.5), (3.6) indicates that φ_{ii} is comparatively large if most of the thermal mass is located near the interior surface $x = 0$, whereas most of the total resistance resides in the outer part of the wall, located near the surface $x = L$; the opposite holds for φ_{ee}. The following estimates are obvious:

(5.1) $$0<\varphi_{ii}<1, \qquad 0<\varphi_{ee}<1.$$

φ_{ie} is comparatively large if most of the thermal mass is located in the centre of a wall and the resistance is symmetrically distributed on both sides of it.

The thermal mass and structure factors of a wall with surface film resistances R_i and R_e differ from the factors of a „naked" wall. For homogeneous walls, with R_i and R_e neglected, $\varphi_{ii} = \varphi_{ee} = 1/3$, $\varphi_{ie} = 1/6$, $\gamma_i = \gamma_e = 1/2$. For walls with internal symmetry planes (different values of R_i and R_e upset the symmetry in general), $\varphi_{ii} = \varphi_{ee}$ and thus $\gamma_i = \gamma_e = 1/2$.

Structure factors for a wall composed of n plane homogeneous layers, numbered from 1 to n with layer 1 at the interior surface, are given as:

(5.2) $$\varphi_{ii} = \frac{1}{R_k^2 C}\sum_{m=1}^{n} C_m\left[\frac{R_m^2}{3} + R_m R_{m-e} + R_{m-e}^2\right],$$

(5.3) $$\varphi_{ie} = \frac{1}{R_k^2 C}\sum_{m=1}^{n} C_m\left[-\frac{R_m^2}{3} + \frac{R_m R_T}{2} + R_{i-m} R_{m-e}\right],$$

(5.4) $$\varphi_{ee} = \frac{1}{R_k^2 C}\sum_{m=1}^{n} C_m\left[\frac{R_m^2}{3} + R_m R_{i-m} + R_{i-m}^2\right],$$

(5.5) $$\gamma_i = \frac{1}{R_k C}\sum_{m=1}^{n} C_m\left[\frac{R_m}{2} + R_{m-e}\right], \qquad \gamma_e = \frac{1}{R_k C}\sum_{m=1}^{n} C_m\left[\frac{R_m}{2} + R_{i-m}\right],$$

where R_m and C_m denote the thermal resistance and capacity of the layer m, whereas $R_{i\text{-}m}$ and $R_{e\text{-}m}$ denote the resistances for the heat transfer from its surfaces to inner and outer surroundings:

$$R_{i-m} = R_i + \sum_{s=1}^{m-1} R_s, \qquad R_{m-e} = R_e + \sum_{s=m+1}^{n} R_s \ . \tag{5.6}$$

Examining the above expressions, one can prove that for a two-layer wall the following estimate on φ_{ie} holds: $0 < \varphi_{ie} < 3/16$. For a three-layer wall, and also for n-layer with $n \geq 3$, the estimate is different: $0 < \varphi_{ie} < 1/4$. The upper limit of φ_{ie} for a two-layer wall is lower than for a three-layer one, because with two layers only, the internal symmetry, with thermal mass at the centre and resistance outside, cannot be realized.

Structure factors of multilayer walls are affected by differentiation of thermal parameters of individual layers and their arrangement.

To demonstrate the effect of arrangement on thermal structure factors φ_{ii}, φ_{ie} and φ_{ee}, six simple examples are examined (rather „academic"), of walls with the same total thermal resistance and capacity, composed in six different ways of two layers of heavyweight concrete /λ = 1.73 W/(mK), ρ = 2240 kg/m^3, c= 0.838 kJ/(kgK)/ and two layers of insulation /λ = 0.043 W/(mK), ρ = 91 kg/m^3, c = 0.838 kJ/(kgK)/ of the same thickness 0.076 m. The arrangement of layers in structures of different types, numbered from 1 to 6, is presented in Figure 1. The structure factors are given in Table 1. In case (a) the surface film resistances R_i and R_e are neglected, in (b) they are taken into account /R_i = 0.12 m^{2}K/W, R_e = 0.05 m^{2}K/W; see [11]/. Results for the homogeneous wall are added for comparison.

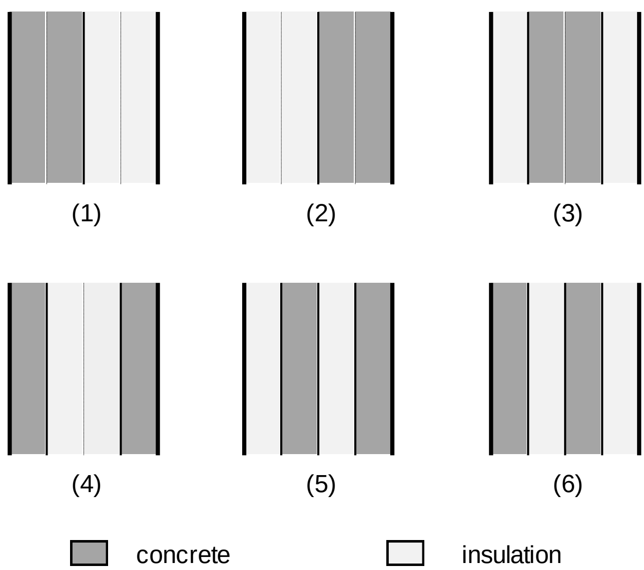

Fig. 1 Different types of four-layer structures composed of concrete and insulation /structure factors represented in Table 1/

Rys. 1 Różne typy struktur czterowarstwowych utworzonych z betonu i izolacji /współczynniki strukturalne przedstawione w Tabeli 1/

The structures of the type (1) and (2), which represent de facto two-layer walls, are highly asymmetric. The values of φ_{ii} and φ_{ee} are close to 0 or to 1. For symmetric structures (3) and (4), without R_i and R_e , they are equal.

For the exterior walls of rooms with stabilized temperature T_i, the most interesting are the values of φ_{ie}, which, as indicated by equation (4.25), determine the character of their thermal responses to variations of the ambient temperature T_e. Here the structure of the type (3) - a heavy center covered with insulation on both sides - is expected to be the most delaying thermal impulses penetrating through it. For structures of the type (1), (2) or (4), the remote effects of the ambient temperature fluctuations on heat flux at the inner surface are small; either because the inner heavy layers are protected by the external insulation, or because the outer heavy layers of small resistance may comparatively quickly exchange heat with the surroundings. Structures of the type (5) and (6) behave rather as homogeneous. Surface resistances R_i and R_e, corresponding to massless layers, increase the φ_{ie} values.

To demonstrate the effect of relative differentiation of thermal parameters on structure factors and response factors, two simple examples were analyzed of walls described below. They have the same total resistance R = 1 m^{2}K/W and capacity C = 180 kJ/(m^{2}K) per unit surface area, time constant RC = 50 h, are composed in the same way of layers with the same thermal diffusivity $a = \lambda/\rho c$, but different resistances and capacities; the surface resistances R_i, R_e being neglected. Transfer functions in both examples have a simple form; their poles may be found analytically.

Table 1

Thermal structure factors for different types of four-layer walls represented in Figure 1
Surface film resistances neglected (a) or taken into account (b)
Termiczne współczynniki strukturalne dla różnych typów ścian dwuwarstwowych przedstawionych na Rysunku 1. Powierzchniowe opory przejmowania ciepła pominięte (a) lub uwzględnione (b).

Wall No	γ_i	γ_e	φ_{ii}	φ_{ie}	φ_{ee}
1a	0,968	0,032	0,950	0,018	0,014
2a	0,032	0,968	0,014	0,018	0,950
3a	0,500	0,500	0,253	0,247	0,253
4a	0,500	0,500	0,488	0,012	0,488
5a	0,266	0,734	0,136	0,130	0,605
6a	0,734	0,266	0,605	0,130	0,136
homogen.	0,500	0,500	0,333	0,167	0,333
1b	0,940	0,060	0,895	0,045	0,015
2b	0,046	0,954	0,013	0,032	0,922
3b	0,492	0,508	0,245	0,247	0,262
4b	0,492	0,508	0,457	0,034	0,474
5b	0,269	0,731	0,132	0,137	0,594
6b	0,715	0,285	0,570	0,144	0,141
homogen.	0,492	0,508	0,317	0,174	0,334

First one is the wall composed of two layers of the same thickness /structures of the type (1) and (2)/, with resistances, capacities, diffusivity and structure factors depending on the structural parameter δ in the following way:

(5.7) $$\frac{R_1}{R} = \delta, \quad \frac{R_2}{R} = 1-\delta, \quad \frac{C_1}{C} = 1-\delta, \quad \frac{C_2}{C} = \delta \quad \Rightarrow \quad R_1C_1 = R_2C_2 \,,$$

(5.8) $$\alpha = \frac{L^2}{4RC\delta(1-\delta)}\,, \qquad \varphi_{ie} = \frac{2}{3}\delta(1-\delta)\,, \qquad \varphi_{ii} = \left(1-\frac{2}{3}\delta\right)(1-\delta)\,.$$

Maximum differentiation is attained at δ close to 0; δ = 1/2 means homogeneity of the wall.

The second one is the symmetric three-layer wall, with thicknesses in the proportion 1:2:1 /structures of the type (3) and (4)/, with resistances and capacities depending on the structural parameter δ in the following way:

(5.9) $$\frac{R_1}{R} = \frac{R_3}{R} = \delta, \quad \frac{R_2}{R} = 1-2\delta, \quad \frac{C_1}{C} = \frac{C_3}{C} = \frac{1}{2}-\delta, \quad \frac{C_2}{C} = 2\delta \Rightarrow 4R_1C_1 = R_2C_2 \,,$$

(5.10) $$\alpha = \frac{L^2}{8RC\delta(1-2\delta)}, \qquad \varphi_{ie} = \frac{5}{6}\delta - \frac{2}{3}\delta^2, \qquad \varphi_{ii} = \frac{1}{2} - \varphi_{ie}\,.$$

Maximum differentiation is attained at δ close to 0 and 1/2; δ = 1/4 means homogeneity.

Structure factors for different values of δ, together with values of the decrement factor *df* and time lag τ of the heat flux for harmonic oscillations of time period 24 h, are collected in Tables 2, 3; normalized response factors H_{ie} are represented in Figures 2, 3. /Decrement factor *df* is defined as the ratio of the amplitudes of the heat flux for the harmonic, of given frequency, and quasistationary oscillations/. The smallest value of δ in Table 2 corresponds to structures close to (1) and (2) in Table 1 whereas the extreme values of δ in Table 3 correspond to structures close to (3) and (4) in Table 1.

Table 2

Structure factors, decrement factors and time lags for two-layer walls of time constant RC = 50 h and different values of the structural parameter δ

Współczynniki strukturalne, współczynniki tłumienia i przesunięcia czasowe dla ścian dwuwarstwowych o tej samej stałej czasowej *RC*=50 h i różnych wartościach czynnika strukturalnego δ

δ	φ_{ie}	φ_{ii}	*df*	τ[h]
0.026	0.017	0.958	0.991	0.832
0.100	0.060	0.840	0.894	2.875
0.300	0.140	0.560	0.636	5.921
0.500	0.167	0.333	0.562	6.751

Table 3

Structure factors, decrement factors and time lags for three-layer walls of time constant RC=50 h and different values of the structural parameter δ

Współczynniki strukturalne, współczynniki tłumienia i przesunięcia czasowe dla ścian trójwarstwowych o tej samej stałej czasowej *RC*=50 h i różnych wartościach czynnika strukturalnego δ

δ	φ_{ie}	φ_{ii}	*df*	τ [h]
0.013	0.011	0.489	0.998	0.530
0.050	0.040	0.460	0.972	1.989
0.150	0.110	0.390	0.777	5.129
0.250	0.167	0.333	0.562	6.751
0.350	0.210	0.290	0.419	6.922
0.487	0.248	0.252	0.301	5.143

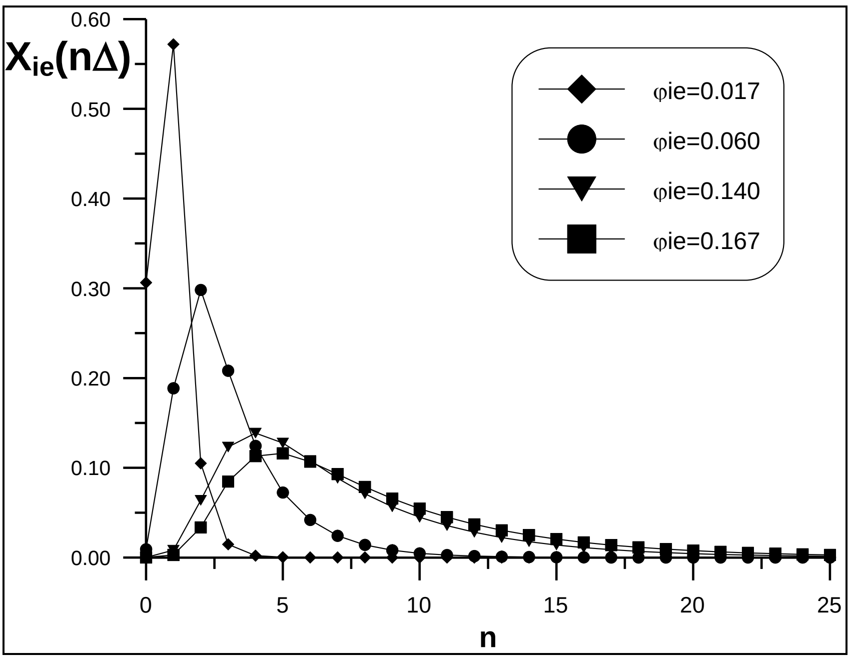

Fig. 2. Response factors $X_{ie}(n\Delta)$ for two-layer walls of the same time constant *RC*=50h and different structure factors

Rys. 2. Współczynniki odpowiedzi $X_{ie}(n\Delta)$ dla ścian dwuwarstwowych o tej samej stałej czasowej *RC*=50h i różnych współczynnikach strukturalnych

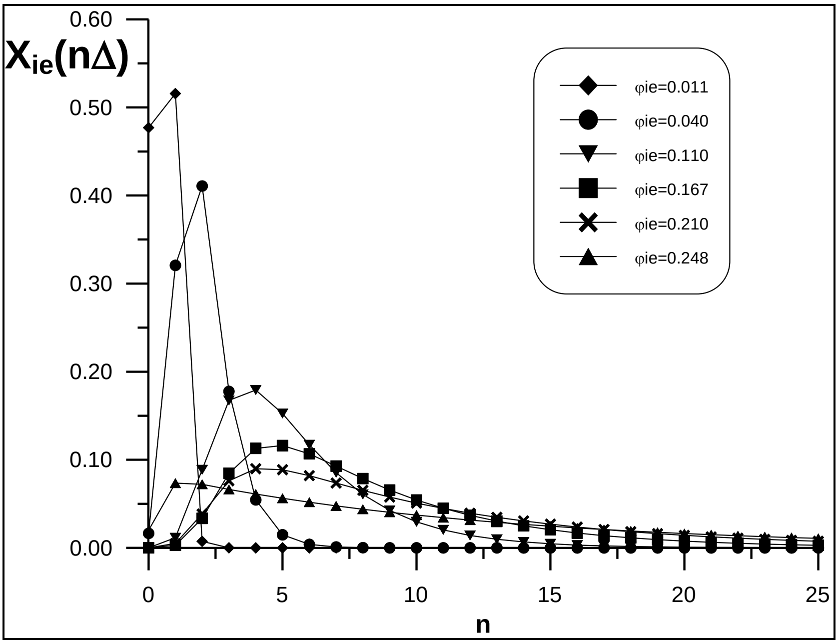

Fig. 3 Response factors $X_{ie}(n\Delta)$ for three-layer walls of the same time constant RC=50h and different structure factors

Rys. 3 Współczynniki odpowiedzi $X_{ie}(n\Delta)$ dla ścian trójwarstwowych o tej samej stałej czasowej RC=50h i róznych współczynnikach strukturalnych.

The plots of response factors in Figures 2, and 3 clearly illustrate the fact that structure factors have an essential influence on dynamic thermal behavior of walls. They confirm the supposition that walls characterized by small values of the structure factor φ_{ie} comparatively quickly transfer thermal responses, whereas those with larger values of φ_{ie} delay thermal responses. The magnitudes of decrement factors and time lags of the heat flux for harmonic oscillations indicate the same. The response, in the form of the heat flux at the surface, to thermal impulse at the opposite surface, in the case of a wall with φ_{ie} close to zero, is comparatively large, increases and disappears comparatively quickly. In the case of a wall with φ_{ie} close to the maximum possible value of 1/4, such a response is smaller and slowly decreases - however it increases more quickly than for a homogeneous wall.

Figure 4 gives the comparison between response factors $X_{ie}(n\Delta)$ of the four-layer wall, represented in Figure 1 as (5), and the equivalent two-layer wall. Four-layer wall is composed of the two layers of concrete /λ = 0.4 W/(mK), ρ = 1200 kg/m^3, c= 0.840 kJ/(kgK)/ and two layers of mineral wool /λ = 0.04 W/(mK), ρ = 100 kg/m^3, c= 0.750 kJ/(kgK)/, of the same thickness 0.04 m. Total resistance R = 2.2 m^{2}K/W, capacity C = 86.64 kJ/(m^{2}K), per unit cross sectional area; time constant RC = 52.9 h. Thermophysical characteristics of the layers of the two-layer wall, of thickness 0.08 m, are as follows: λ_1 = 0.123 W/(mK), ρ_1 = 922.3 kg/m^3, c_1 = 0.840 kJ/(kgK), λ_2 = 0.052 W/(mK), ρ_2 = 367 kg/m^3, c_2 = 0.840 kJ/(kgK). Response factors of both structures do not differ much from response factors for the homogeneous wall, of the same resistance and capacity.

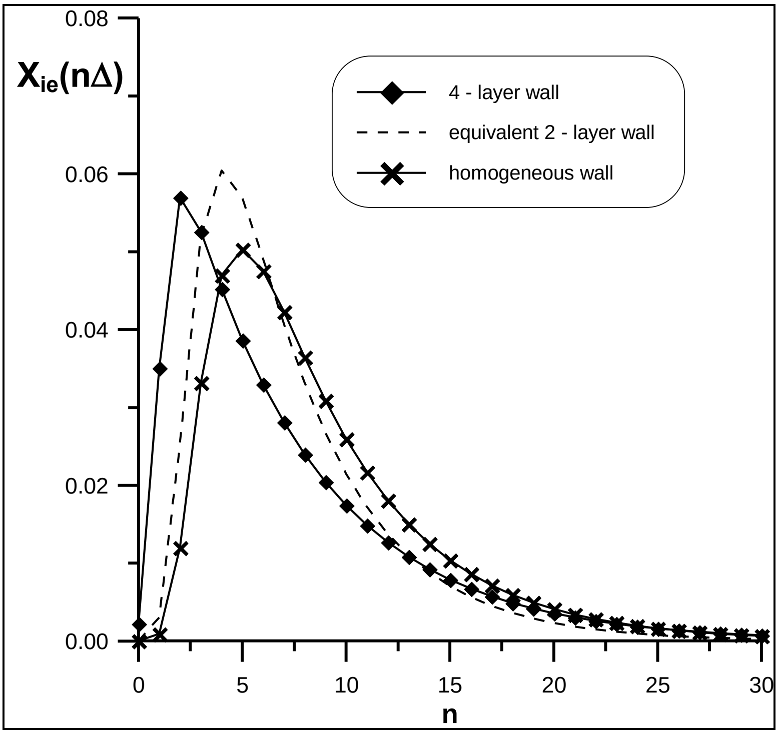

Fig. 4 Response factors $X_{ie}(n\Delta)$ for the 4-layer and equivalent 2-layer wall, of RC = 52.9 h
Rys. 4 Współczynniki odpowiedzi $X_{ie}(n\Delta)$ dla ściany 4-warstwowej i równoważnej 2-warstwowej o stałej czasowej RC = 52.9 h

6. CONCLUSIONS

Thermal structure factors, defined by the integrals (3.5 – 3.7) and for a multilayer wall given by the expressions (5.2 – 5.5), together with total resistance and capacity, determine the thermal dynamics of the wall - through the conditions they impose on response factors. At the same time damping effects, for harmonic oscillations of heat flow, are strongly affected by structure factors.

Structure factors of multilayer walls are affected by differentiation of thermal parameters of individual layers and their arrangement.

Large value of the structure factor, corresponding to a given heat flux response mode, indicates that response factors with number $m \geq 1$, representing storage effects, are comparatively large; on the contrary, small value of the structure factor indicates that they are comparatively small.

Structures with thermal mass outside, on one or both sides, are characterized by small values of structure factor φ_{ie}, what means that they allow for a quick transfer of the thermal response. The response factors $X_{ie}(m\Delta)$, which represent the surface heat flux due to a triangular temperature impulse at the opposite surface, decay relatively quickly, however those with smallest index m assume relatively great values. Structures with thermal mass inside, covered with light insulation, characterized by large values of structure factor φ_{ie} /up to 1/4/, delay thermal responses; the response factors $X_{ie}(m\Delta)$ are relatively small but decay relatively slowly.

APPENDIX

In what follows, the relationships between the thermal structure factors and the values of the residues and poles of transfer functions are derived.

The Laplace transforms of the heat flux density $q(t)$ and heat flow $Q(t)$, across the surfaces of a plane, multilayer wall, in the directions of the outside normals, due to boundary temperature $T(t)$, for zero initial conditions, have the general form /see [6,7] and (4.1), (4.2), (4.3)/:

(A1) $$\bar{q}(s) = (-1)^p \frac{N(s)}{B(s)} \bar{T}(s), \qquad \bar{Q}(s) = (-1)^p \frac{N(s)}{sB(s)} \bar{T}(s), \qquad p = 0, 1\,,$$

where $N(s)$, $B(s)$ are the elements of the transition matrix for the structure, p=0 for the (ie) and (ei) modes of heat transfer and p=1 for (ii) and (ee) modes.

Consider the heat transfer process with $T(t)$ in the form of the unit step function $T(t)=\eta(t)$. The Laplace transform of T(t) is equal to 1/s. Inversion of the Laplace transforms, by applying the residue theorem, yields /see [6,7,8]:

(A2) $$q(t) = (-1)^p \left\{ \frac{1}{R_k} - \sum_{k=1}^{\infty} \frac{\mu_k}{\beta_k} e^{-\beta_k t} \right\},$$

(A3) $$Q(t) = (-1)^p \left\{ \frac{t}{R_k} + \frac{d}{ds}\left[\frac{N(s)}{B(s)}\right]_{s=0} + \sum_{k=1}^{\infty} \frac{\mu_k}{\beta_k^2} e^{-\beta_k t} \right\},$$

where the quantities $(-\beta_k)$ are the poles of the transfer function $N(s)/B(s)$, that is the negative real roots of the equation $B(s)$=0, and μ_k the residues at the poles:

(A4) $$\mu_k = \left[\frac{N(s)}{B'(s)}\right]_{s=-\beta_k}.$$

The limit conditions at t = 0: $q(0)$=0 and $Q(0)$=0 require that:

(A5) $$\sum_{k=1}^{\infty} \frac{\mu_k}{\beta_k} = \frac{1}{R_k},$$

(A6)
$$\sum_{k=1}^{\infty}\frac{\mu_k}{\beta_k^2}=-\frac{d}{ds}\left[\frac{N(s)}{B(s)}\right]_{s=0}.$$

For $t\to\infty$ the exponential terms in (A2), (A3) tend to zero, which gives the limit conditions:

(A7)
$$\lim_{t\to\infty} q(t)=\frac{(-1)^p}{R_k},\qquad \lim_{t\to\infty}\left[Q(t)-(-1)^p\frac{t}{R_k}\right]=(-1)^p\frac{d}{ds}\left[\frac{N(s)}{B(s)}\right]_{s=0}.$$

On the other side, for this asymptotically steady heat transfer process, corresponding to unit boundary temperature jump, as follows from the general asymptotic relationships (3.2), (3.3) /see also (4.20)/:

(A8)
$$\lim_{t\to\infty}\left[Q(t)-(-1)^p\frac{t}{R_k}\right]=-C\varphi.$$

Compatibility of the above asymptotic relationships at infinity requires the following equality:

(A9)
$$(-1)^p\frac{d}{ds}\left[\frac{N(s)}{B(s)}\right]_{s=0}=-C\varphi.$$

which, for multilayer walls, may easily be verified by direct calculation. (A8) combined with (A6) gives the following condition for the poles ($-\beta_k$) and residues μ_k:

(A10)
$$\sum_{k=1}^{\infty}\frac{\mu_k}{\beta_k^2}=(-1)^p C\varphi$$

The above equality is nontrivial and may be verified by direct calculation only for simplest cases, as the homogeneous wall with $R_i = 0$ and $R_e = 0$.

REFERENCES

1. E. KOSSECKA, *Heat balance of simple solar heating systems and building walls within long time intervals*, Arch. Civ. Engn., **37**(2), 249-264, 1991, [in Polish].
2. E. KOSSECKA, *Heat transfer through building wall elements of complex structure*, Arch. Civ. Engn., **38**, 1-2, 117-126, 1992.
3. E. KOSSECKA, *Problem of the measured R-value error estimation*, Arch. Civ. Engn., **39**(4), 413-427, 1993.
4. E. KOSSECKA, J. KOŚNY, J.E. CHRISTIAN, *A Simple Algorithm for Determining the Thermal Resistance of a Wall*, Build. Res. Journ., **3**(1), 57-66, 1994.
5. E. KOSSECKA, J. KOŚNY, *Equivalent wall as a dynamic model of the complex thermal structure*, submitted to Journal of Thermal Insulation and Buildings Envelopes.

6. E. KOSSECKA, J. KOŚNY, *Relations between structural and dynamic thermal characteristics of building walls*, 1996 International Symposium of CIB W67 „Energy and Mass Flows in the Life Cycle of Buildings", Vienna, 4-10 August.
7. ISO/DIS 9869.2, *Draft International Standard, Thermal Insulation - Building elements - In situ measurement of thermal resistance and thermal transmittance,* International Organization for Standarization, 1991.
8. H.S CARSLAW, J.C. JAEGER, *Conduction of Heat in Solids*, Oxford University Press, Oxford 1959.
9. T. KUSUDA, *Thermal Response Factors for Multi-Layer Structures of Various Heat Conduction Systems*, ASHRAE Trans. **75**(1), 241-271, 1969.
10. J.A. CLARKE, *Energy Simulation in Building Design*, Adam Hilger Ltd, 1985.
11. M.G. SMITH, Laplace Transform Theory, D.Van Nostrand Company Ltd., 1966.
12. *ASHRAE Procedures for Determining Heating and Cooling Loads for Computerising Energy Calculations*, 1976.
13. ASHRAE Handbook 1989: Fundamentals; I-P Edition.

WPŁYW STRUKTURY NA DYNAMIKĘ CIEPLNĄ ŚCIAN WARSTWOWYCH

S t r e s z c z e n i e

Analizowany jest wpływ wewnętrznej struktury termicznej ścian warstwowych na ich charakterystyki dynamiczne. Podstawę matematyczną stanowią formuły całkowe dla przepływu ciepła przez powierzchnie ścian. Wprowadzone jest pojęcie czynników strukturalnych i wyprowadzone są warunki, przy wykorzystaniu metody transformacji Laplace'a, które nakładają one na współczynniki odpowiedzi. Zbadane są proste przykłady ścian, reprezentujących różne typy rozkładów oporności i pojemności, które ilustrują ogólne zależności między współczynnikami strukturalnymi i współczynnikami odpowiedzi.

INSTITUTE OF FUNDAMENTAL TECHNOLOGICAL RESEARCH
POLISH ACADEMY OF SCIENCES

Received May 26, 1996

Remarks on the paper should be
sent to the Editorial Board
not later than December 30, 1996