Sedimentation of small particles

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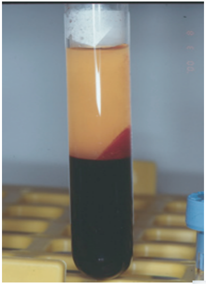
Sedimentation problems

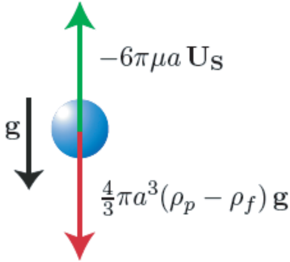


Summary

Background

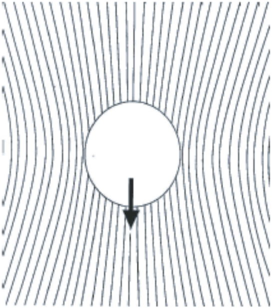
- Sedimentation velocity of a single sphere, Stokes 1851.

Sedimentation of a single sphere



Stokes' velocity

$$\mathbf{U_S} = 2(\rho_p - \rho_f)a^2\mathbf{g}/9\mu$$

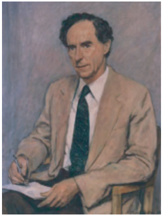


Pozrikidis 1997

Long-range interactions

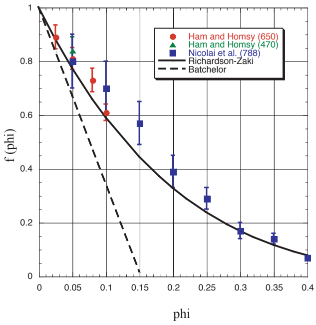
$$u \sim O(\frac{aU_S}{r})$$

Background

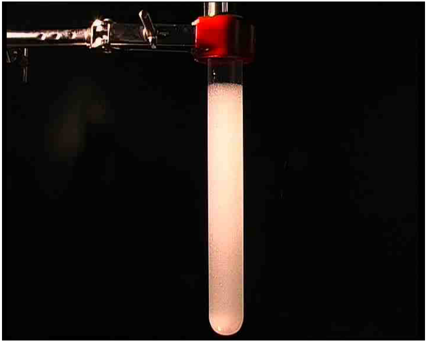


- Average sedimentation velocity of a suspension of spheres, Batchelor 1972.

Mean velocity

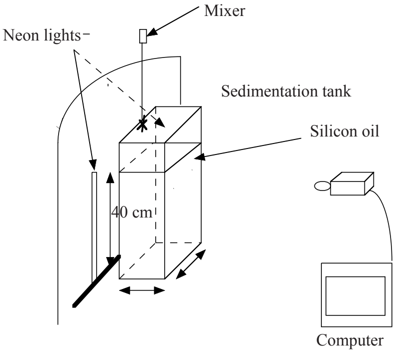


- Hindered settling: $\langle U \rangle = U_S f(\phi)$ Richardson-Zaki 1954: $f(\phi) = (1-\phi)^5$
- Main effect = Back-flow
- Batchelor 1972: $f(\phi) = 1 - 6.55\phi + O(\phi^2)$ assuming uniformly dispersed spheres.
- Cichocki and Sadlej 2005: stationary-state sedimentation coefficient $= -3.87$

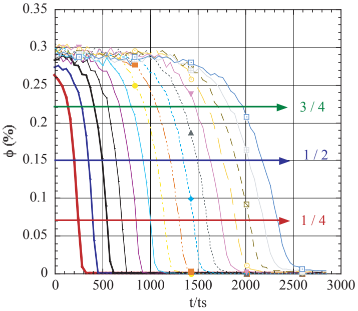


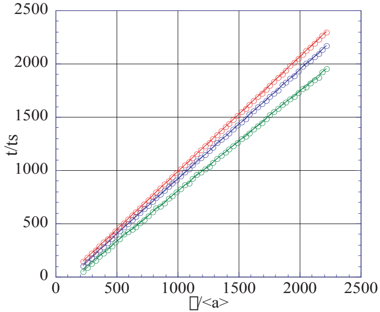
- $Re = aU/\nu \ll 1 \quad \Rightarrow$ Stokes' flow
- $Pe = aU/D \gg 1 \quad \Rightarrow$ Only hydrodynamics
- Dilute suspensions
- Rigid sphere particles

Experimental set-up

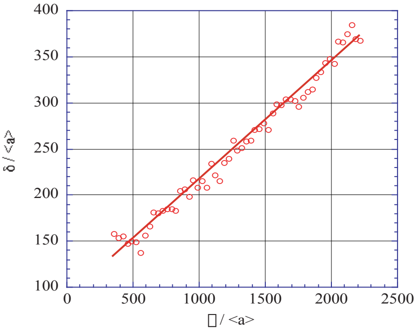


Front measurements





Front measurements

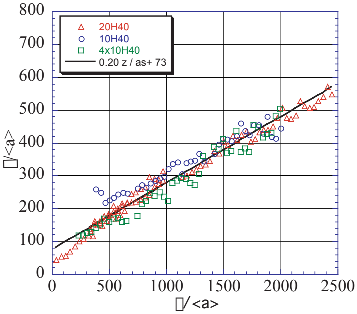


$$\delta = t_{1/2}(\xi/t_{3/4} - \xi/t_{1/4})$$

Front spreading

- Geometry
- Polydispersity
- Concentration

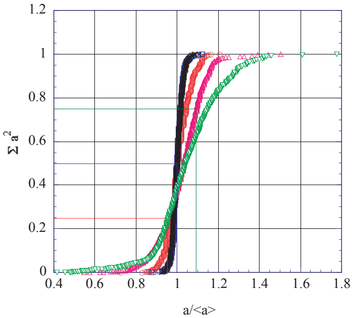
Geometry



Particles $\langle a \rangle$ 0.0149(cm). Sections 20×20 cm^2 ($\triangle$), 10×10 cm^2 ($\circ$) and 4×10 cm^2 ($\Box$). $\phi_0 = 0.3\%$.

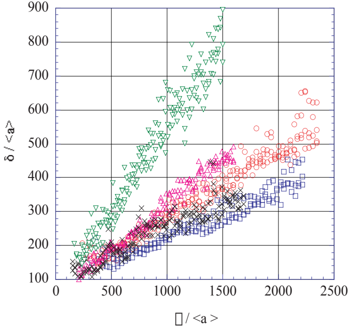
Size does not matter!!!

Polydispersity



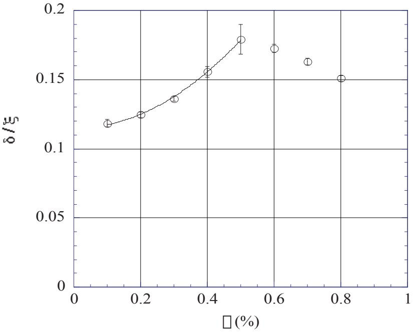
Four different poydispersities were tested: 2%, 5%, 11% and 18%.
Spreading predicted $\delta^p/\xi = (a^2_{3/4} - a^2_{1/4})/a^2_{1/2}$

Polydispersity



For very polydisperse particles, prediction and experiments agree!

Concentration

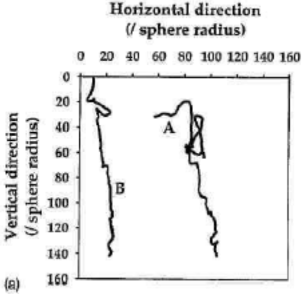


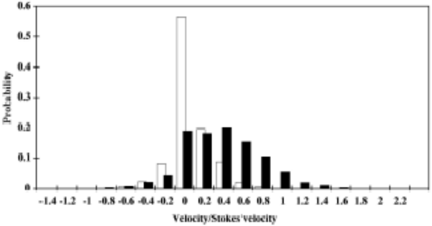
δ^p/ξ predicted: 0.06

Conclusions

1. The spreading of the sedimentation front:
   - Is a convective process.
   - Does not depend on geometry.
   - Is explained only by polydispersity for a certain range of values
2. Front sharpening is observed for $\phi \geq 0.5\%$.

Velocity fluctuations

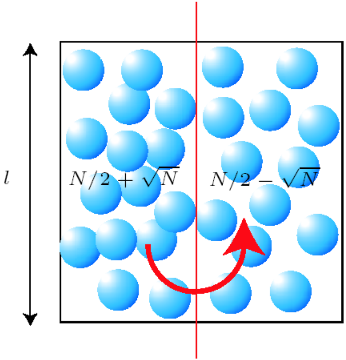




Large anisotropic fluctuations

Sphere-tracking in an index-matched suspension
Ham & Homsy 1988, Nicolai *et al.* 1995

Divergence of velocity fluctuations?



- Randomly distributed particles
- Box of size $a\phi^{-1/3} < l < L$
- Statistical fluctuations$\sqrt{N} \rightarrow$
  $\Delta U_{\parallel} \sim \frac{\sqrt{N}\frac{4}{3}\pi a^3(\rho_s - \rho)g}{6\pi\mu l} \sim U_S\sqrt{\phi\frac{l}{a}}$
- Large-scale fluctuations are dominant
  $\Delta U_{\parallel} \sim U_S\sqrt{\phi\frac{L}{a}}$ diverges!
  Caflisch & Luke 1985, Hinch 1988

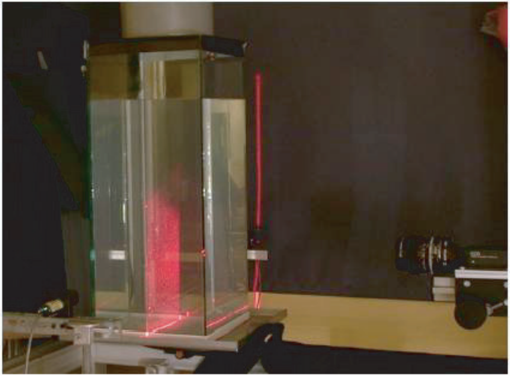
BUT no such divergence seen in experiments

Nicolai & Guazzelli 1995, Segrè *et al.* 1997, Guazzelli 2001.

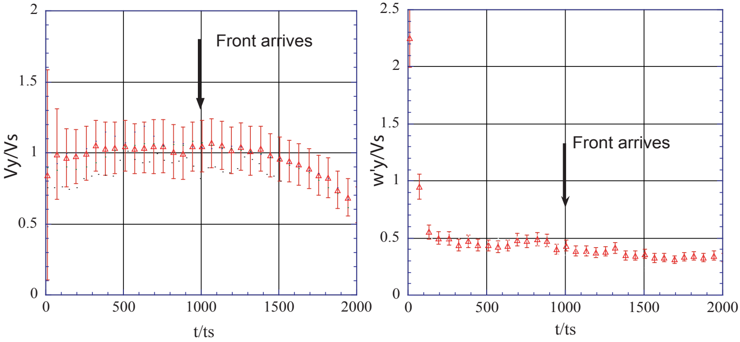
Other theories . . .

- Koch & Shaqfeh 1991: a non-random microstructure
- Tong & Ackerson 1998: turbulent convection analogy
- Levine *et al.* 1998: stochastic model
- da Cunha 1995, Ladd 2002: impenetrable bottom
- Brenner 1999: wall effect
- Luke 2000: stratification → fluctuation decay
- Tee *et al.* 2002, Mucha *et al.* 2003-04: diffusive spreading of the front → stratification → fluctuation decay
- Nguyen & Ladd 2005: polydispersity → stratification
- Hinch 1985, Asmolov 2004, Luke 2005: bottom and top = sink of large-scale disturbances

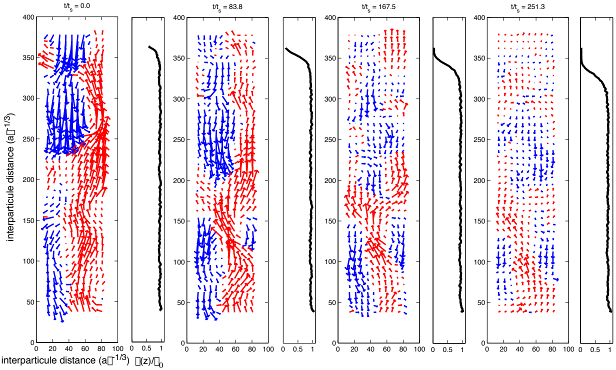
Experimental set-up



Mean sedimentation velocity and fluctuations

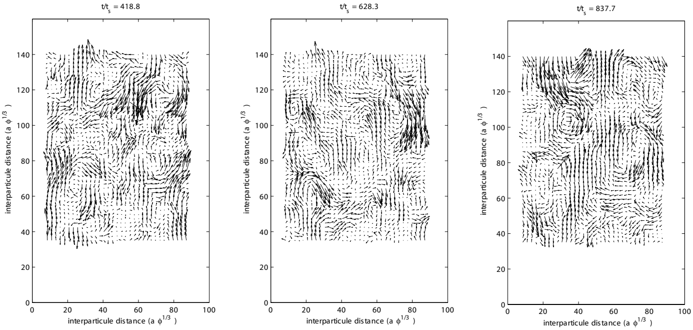


Relaxation of large-scale fluctuations



Initially, the large-scale fluctuations dominate the dynamics. But, they are transient as the heavy parts settle to the bottom and light parts raise to the top.

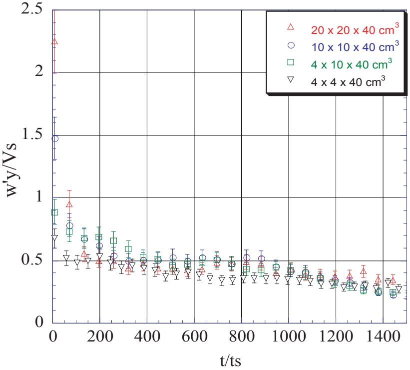
Left with smaller-scale fluctuations



Then, smaller-scale fluctuations are dominant until the arrival of the upper sedimentation front.

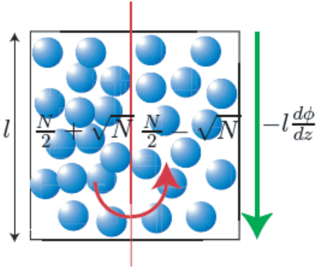
Chehata, Bergougnoux, Guazzelli, & Hinch 2006

Recipient size



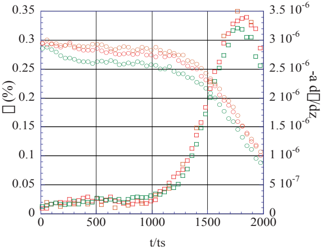
- Effect on the initial fluctuations.

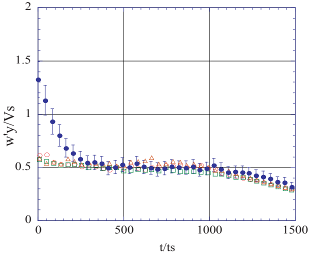
Stratification?



- Effect **size of the box** limited to the size where $\sqrt{N}\frac{a^3}{l^3} = \sqrt{\phi\frac{a^3}{l^3}} \sim -l\frac{d\phi}{dz}$
- Largest fluctuations on this scale $l_s \sim a\phi^{1/5}(-a\frac{d\phi}{dz})^{-2/5}$ with velocity fluctuations $\Delta U_{||}(l_s) \sim U_S\phi^{3/5}(-a\frac{d\phi}{dz})^{-1/5}$
- Critical stratification for $l_s = L$ $(-a\frac{d\phi}{dz})_c \sim (\frac{a}{L})^{5/2}\phi^{1/2}$

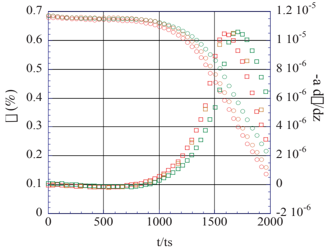
Stratification and fluctuations

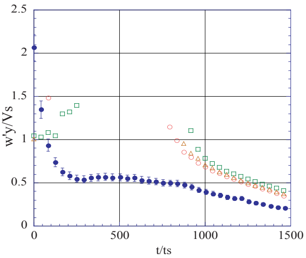




Using prediction $\frac{\Delta U_{\parallel}}{U_S} = \phi^{3/5}(-2a\frac{d\phi}{dz})^{-1/5}$
Mucha *et al.* 2003.

Stratification and fluctuations

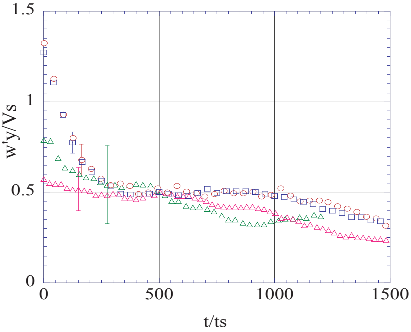




Conclusions

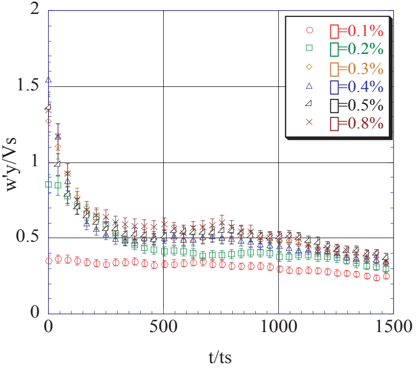
1. Initial fluctuations depend on size of the container.
2. For the steady state, fluctuations do not depend on size of the container.
3. Stratification theory:
   3.1 Does not describes initial fluctuation decay.
   3.2 Fails to always describe the velocity fluctuations.

Polydispersity



Effect only in large scale fluctuations.

Concentration



$\phi(\%)$	$\langle a\rangle/\phi^{-1/3}(cm)$
0.1	0.15
0.2	0.12
0.3	0.11
0.4	0.10
0.5	0.09
0.8	0.08

Table: Interparticle distance

Perspectives

- Scaling velocity fluctuations
- What is the initial distribution of particles?