A network model of self organization in geochemical flows

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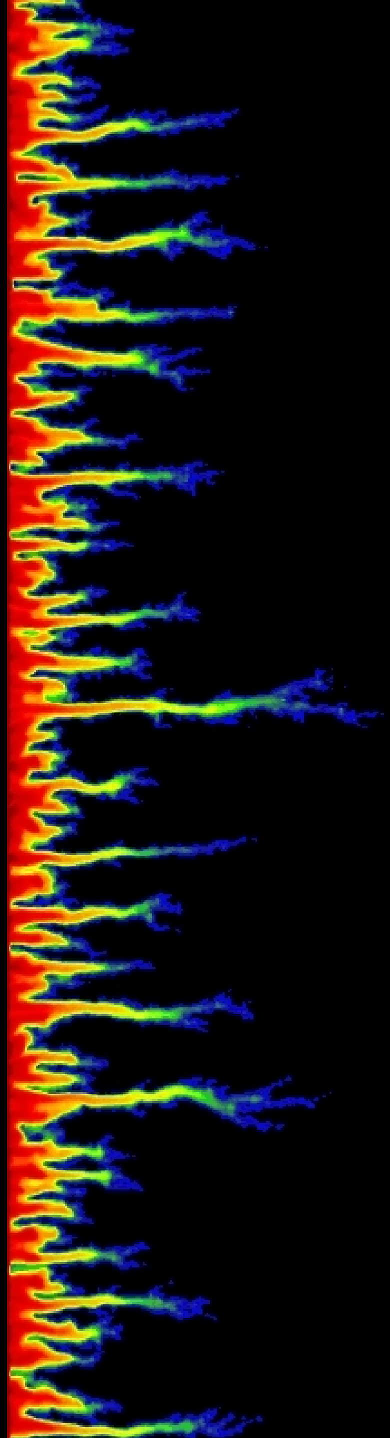
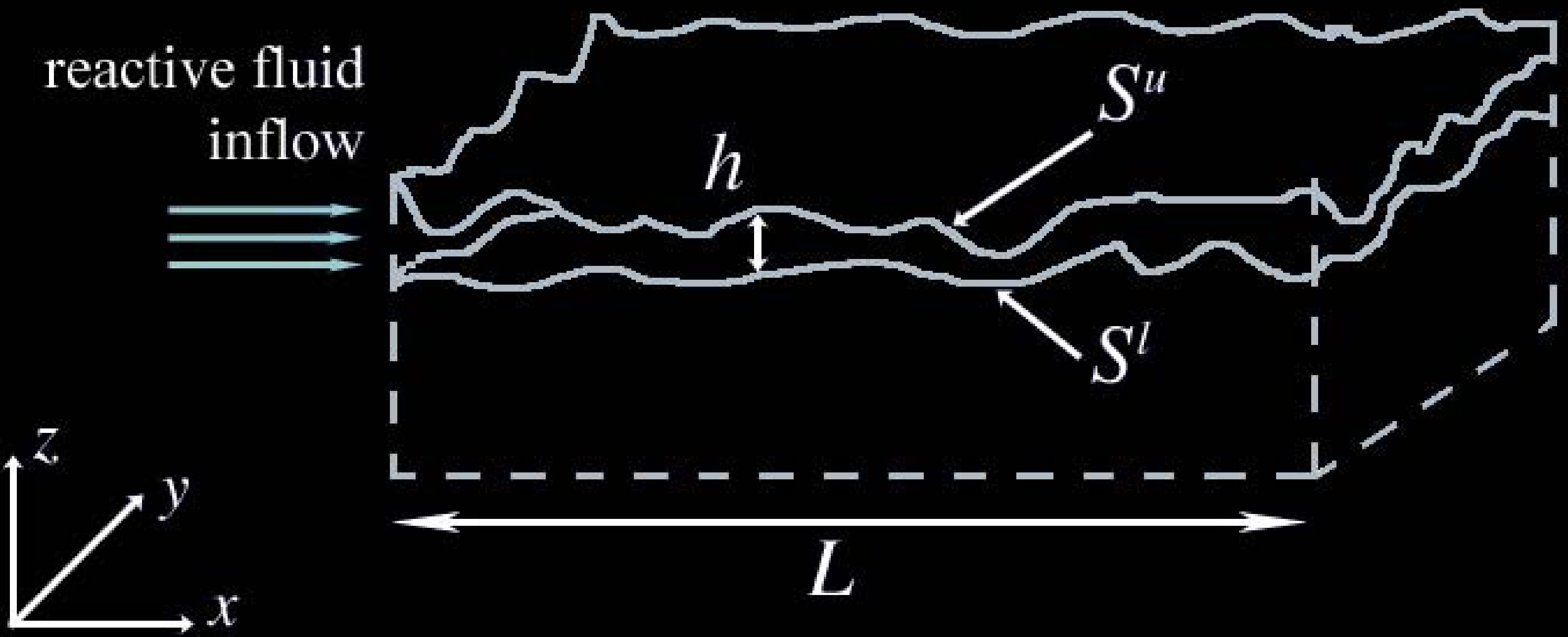


Dissolution of rock fractures



S^l, S^u – fracture surfaces

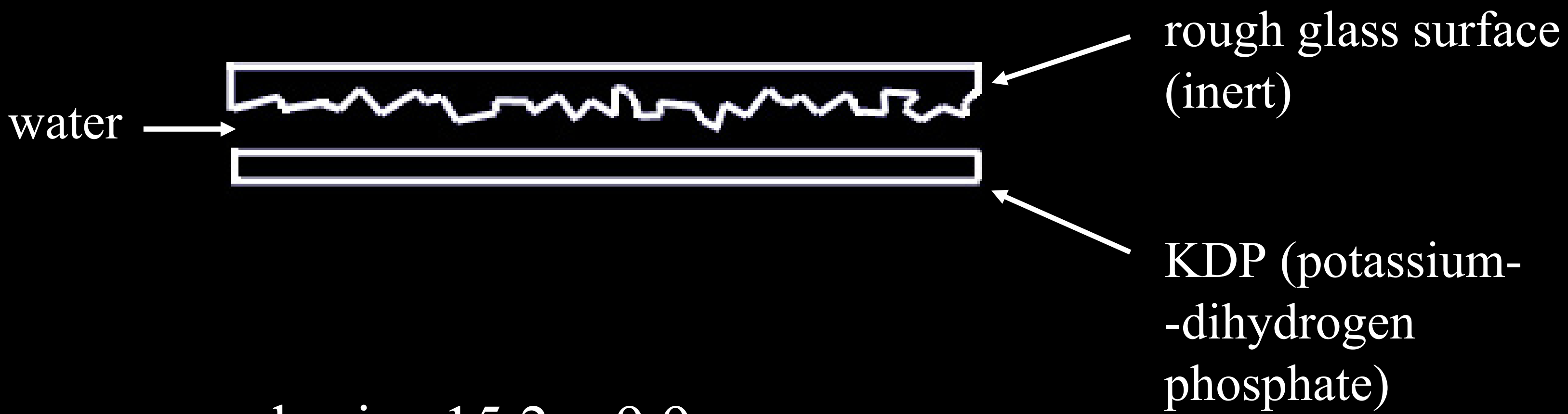
h – aperture, $h/L \ll 1$

Fracture dissolution is a complex process…

- Fluid flow in a complicated geometry
- Reactant and product transport to and from fracture surfaces
- Chemical kinetics
- Geometry evolution

Experiment: KDP fracture

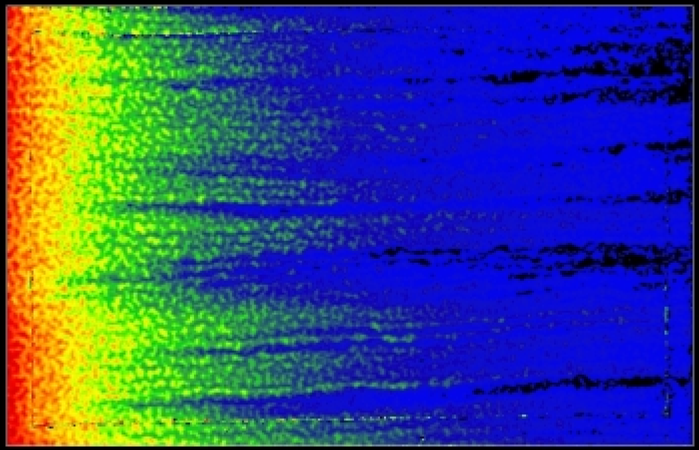
(Russell Detwiler et al., LLNL, 2003)



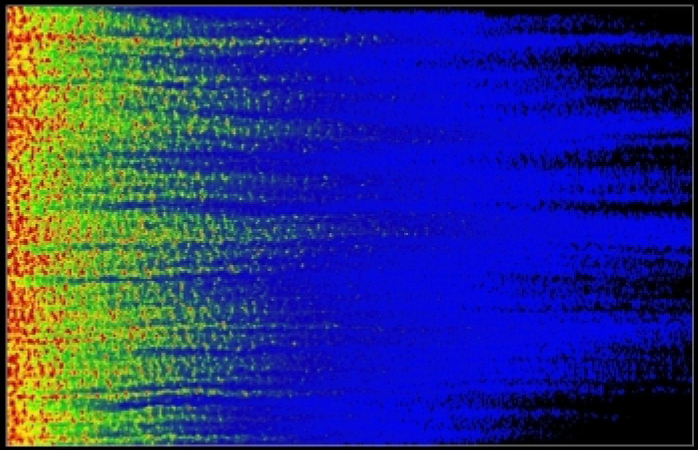
- sample size 15.2 × 9.9 cm
- initial mean aperture $\langle h_0 \rangle = 0.126$ mm
- dissolved until $\langle h \rangle = 2\langle h_0 \rangle$ at Pe = 54 and Pe = 216
- high resolution data on fracture topography

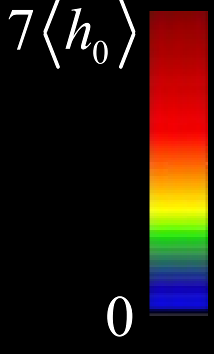
Aperture growth at Pe = 216 for $\langle h \rangle = 2\langle h_0 \rangle$

experiment



simulation

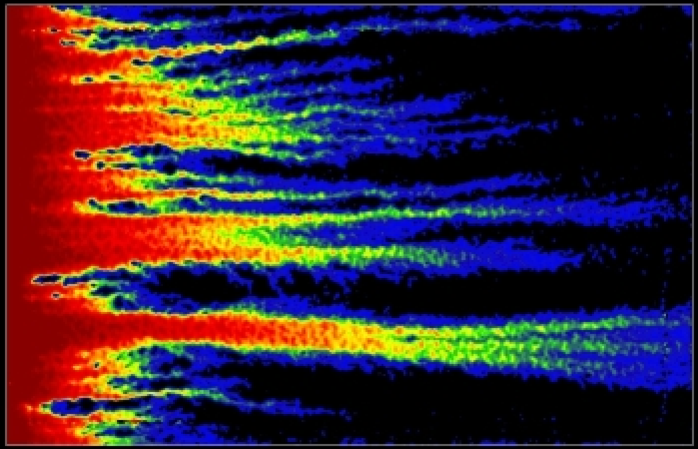




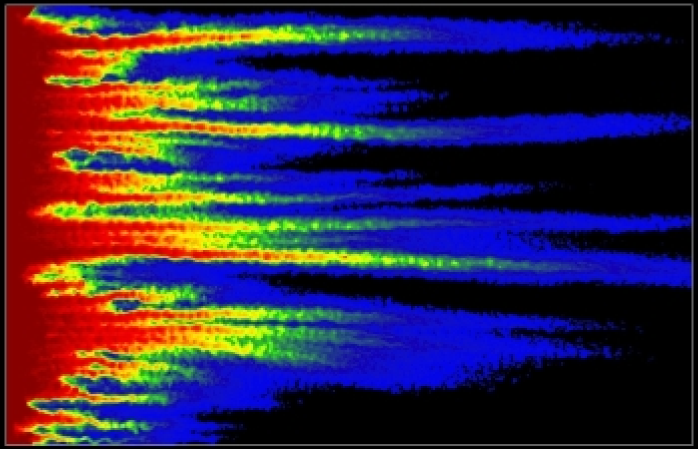
- unsaturated fluid penetrates deep inside the fracture
- uniform dissolution
- lack of pronounced channels

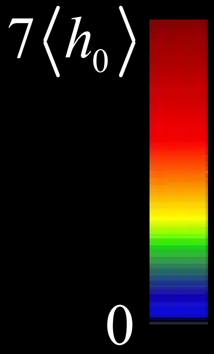
Aperture growth at Pe = 54 for $\langle h \rangle = 2\langle h_0 \rangle$

experiment



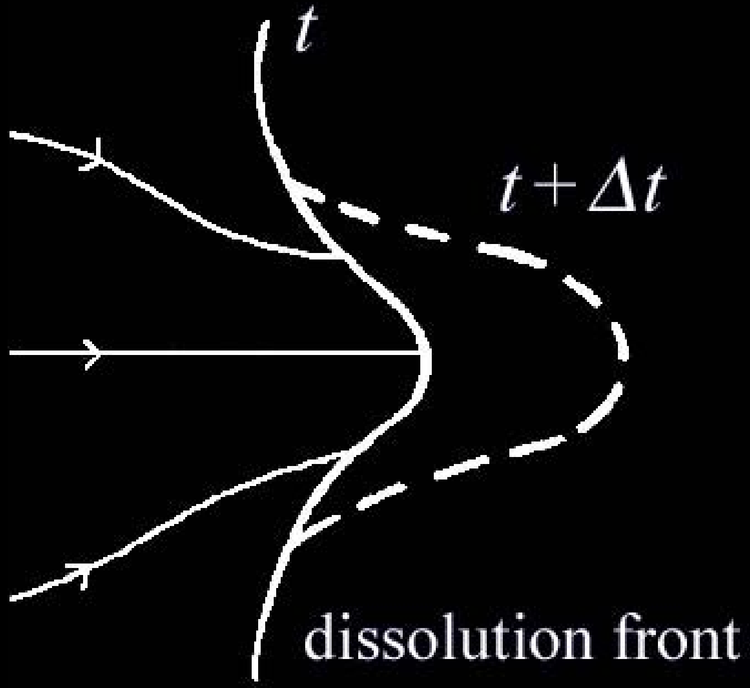
simulation





- channels form, grow, compete for the flow
- only few channels survive at the end

Channeling instability of the dissolution front

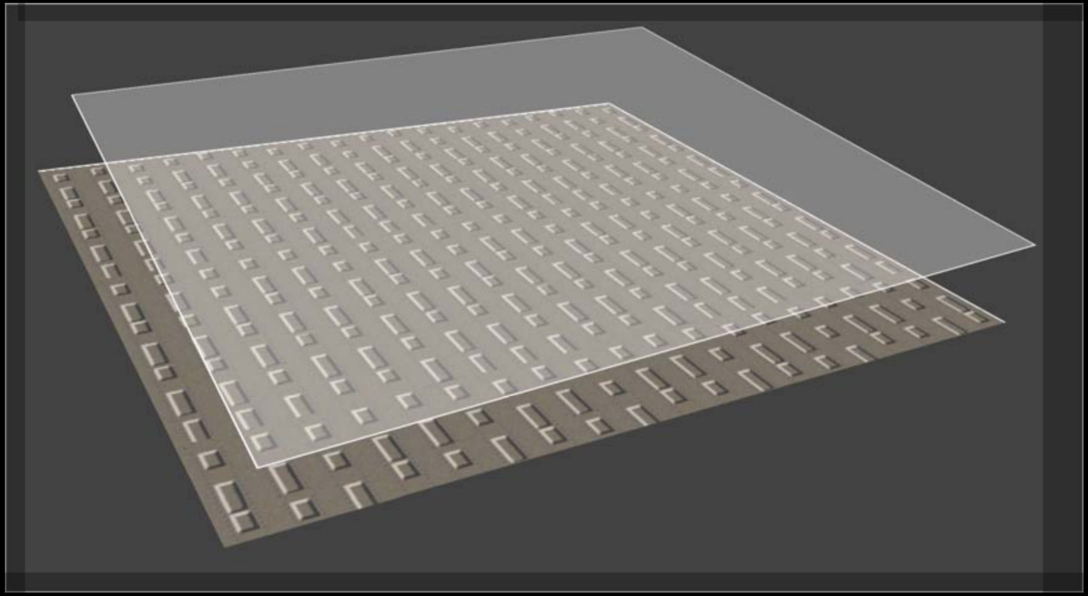


A small perturbation to the dissolution front is unstable

The locally increased flow rate leads to increased dissolution, amplifying the perturbation (Ortoleva, 1987)

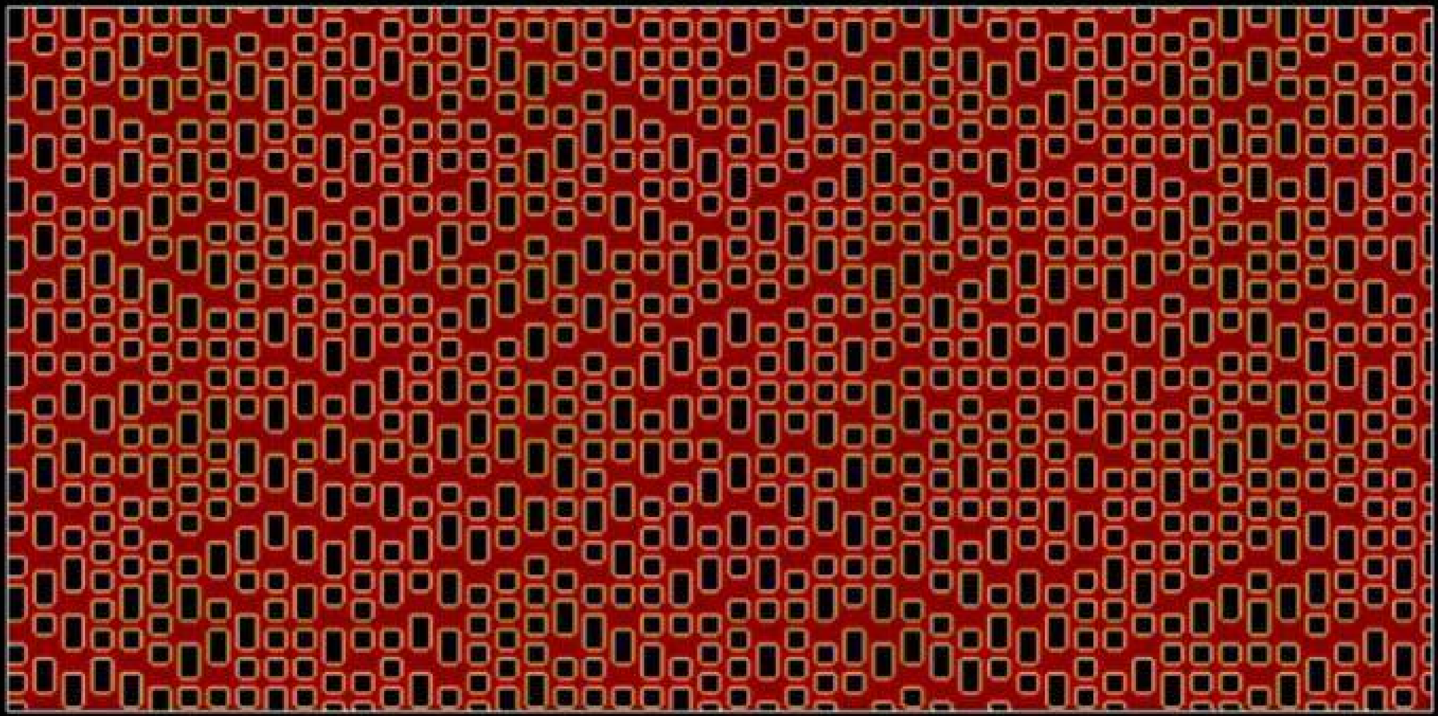
How do these perturbations develop in space and time?

Numerical study of dissolution in an idealized "fracture" geometry



- Pore-scale numerical simulations
- Simple initial fracture topography: plane channel with random obstacles
- No long-range correlations
- Transport-limited regime (large reaction rate)

Initial geometry

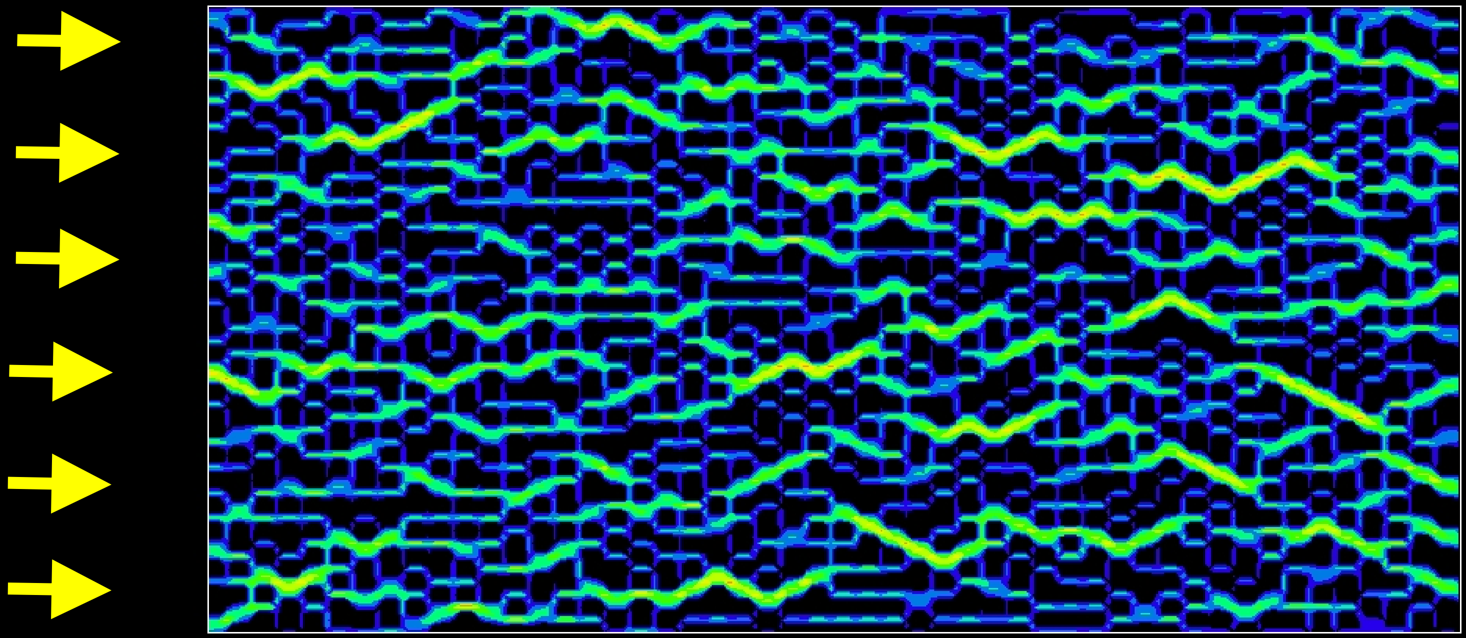


(fragment)

Initial flow field

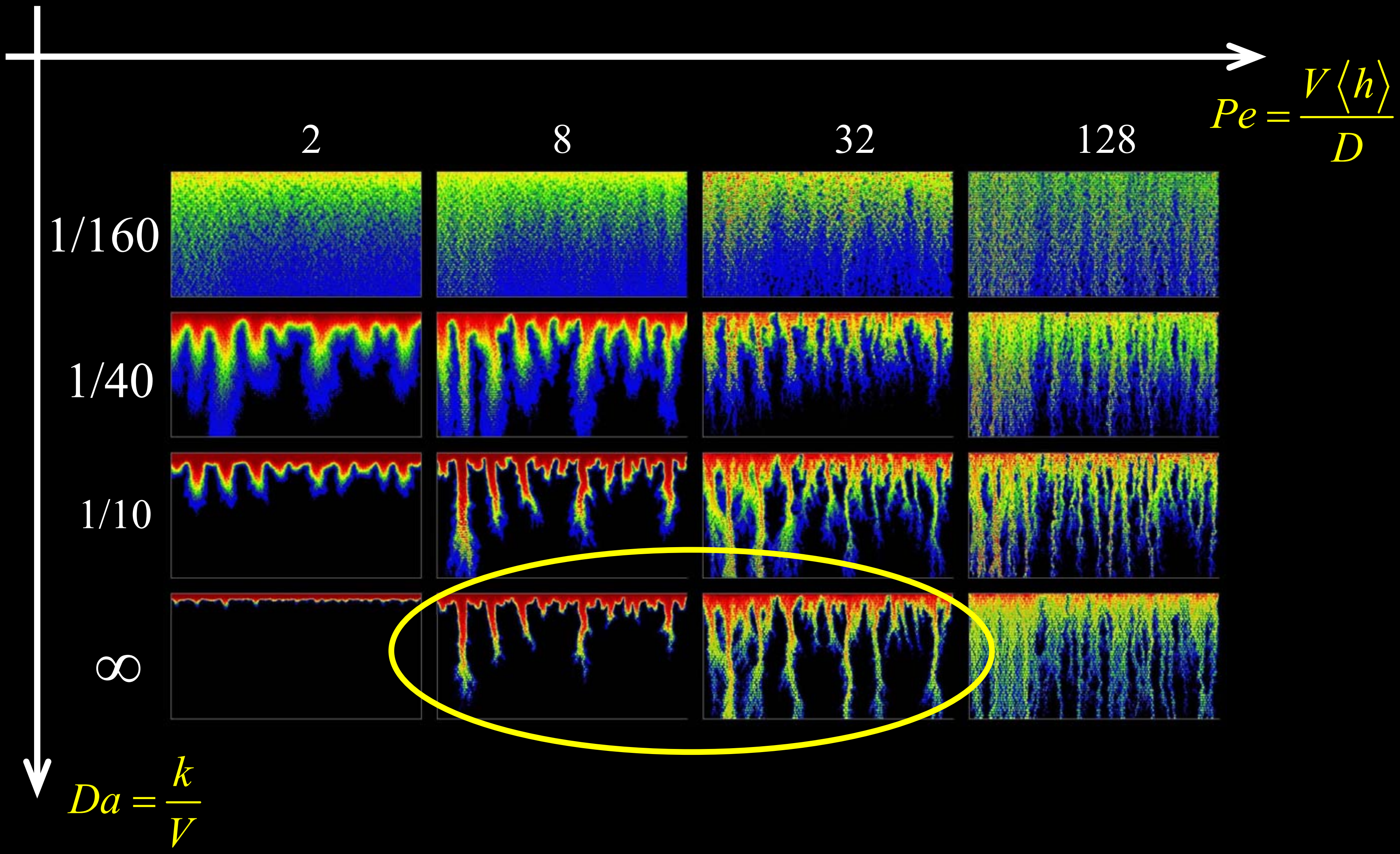
$$V(x,y)=\int_{S_l}^{S_u}\sqrt{v_x^2+v_y^2}\,dz$$

(total in-plane velocity flux)

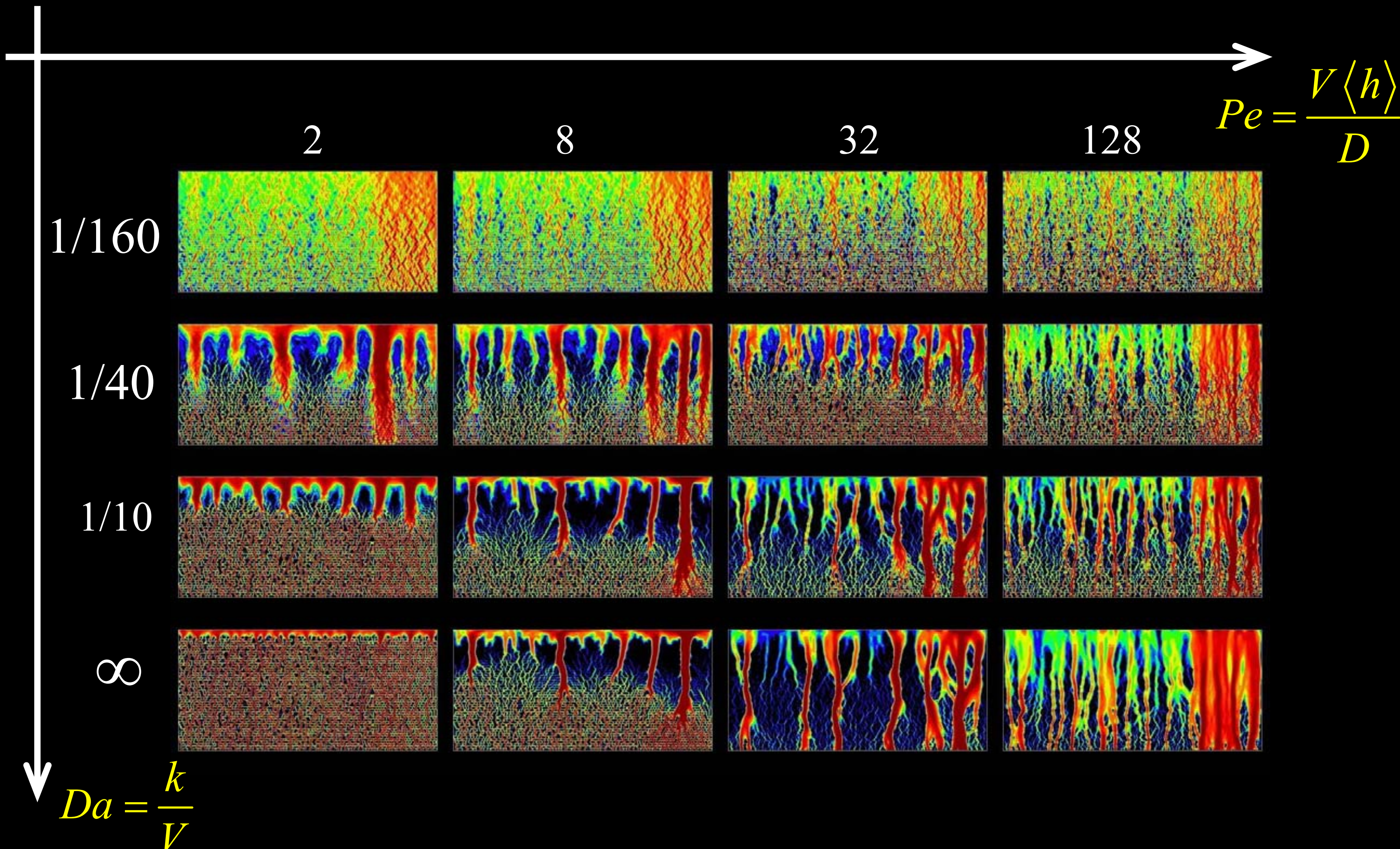


(fragment)

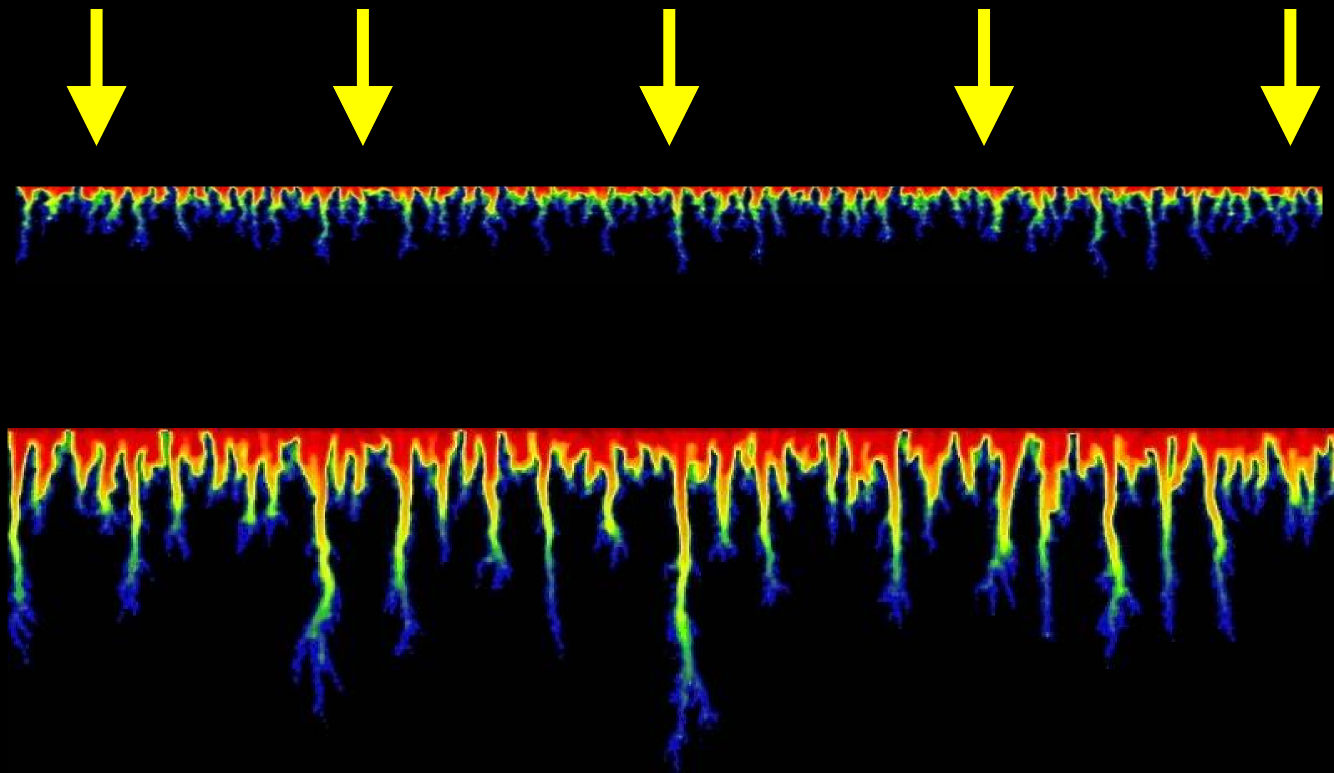
Geometry evolution



Flow maps



Late-stage dissolution is dominated by channel competition

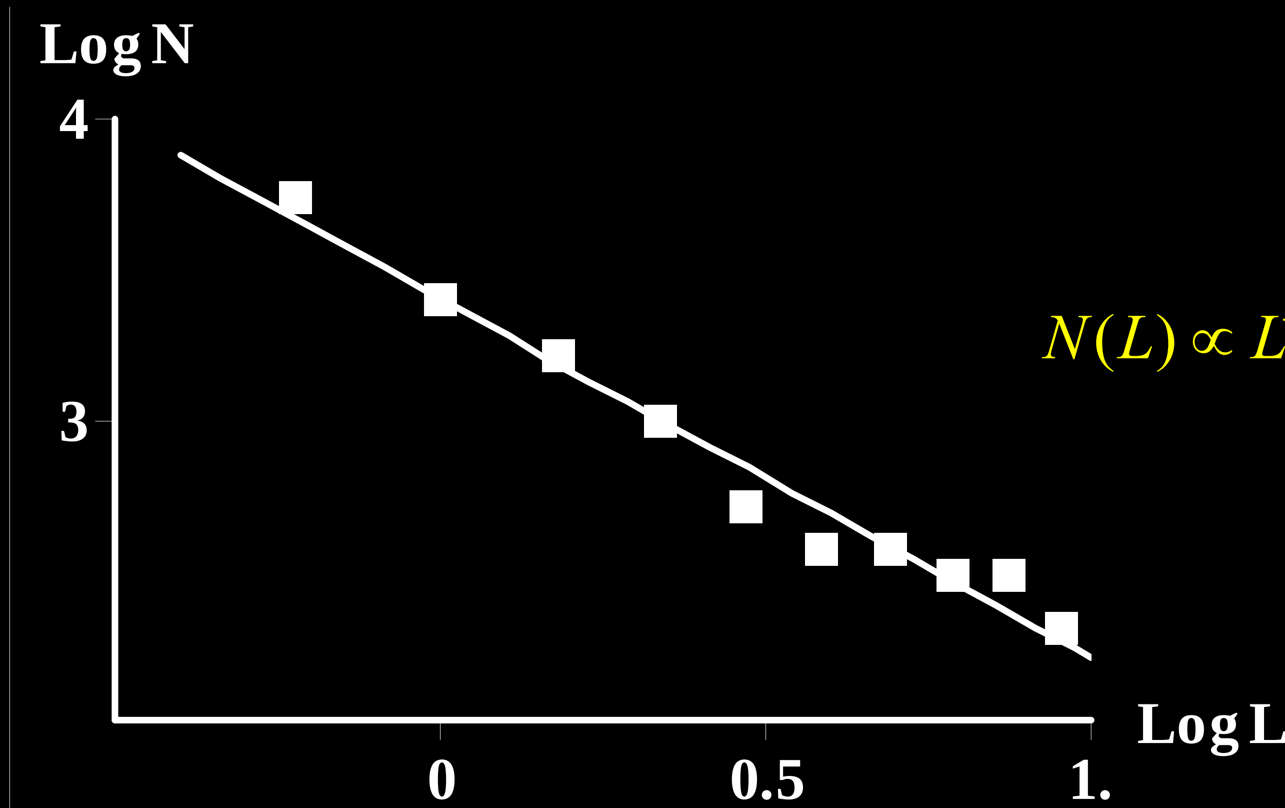


$\langle \Delta h \rangle = 0.15 \langle h \rangle_0$

$\langle \Delta h \rangle = 0.5 \langle h \rangle_0$

characteristic length between the channels is increasing

Scale invariance

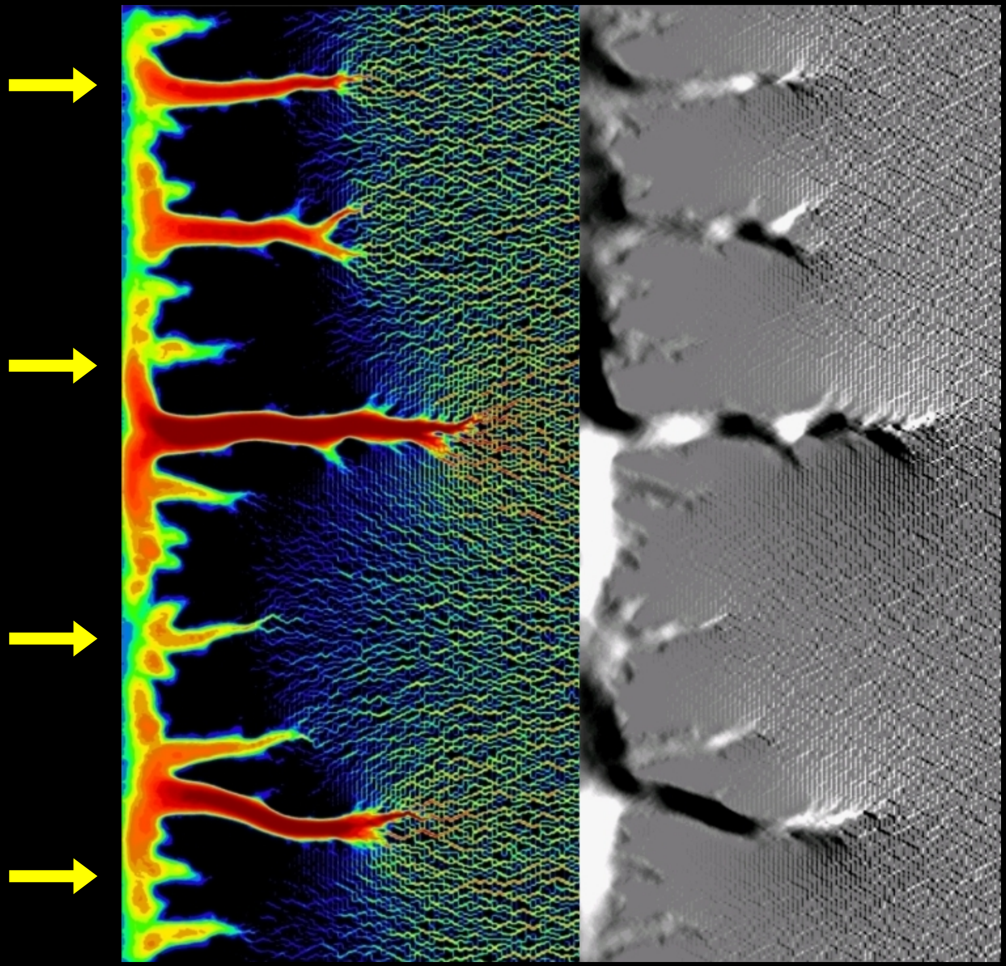


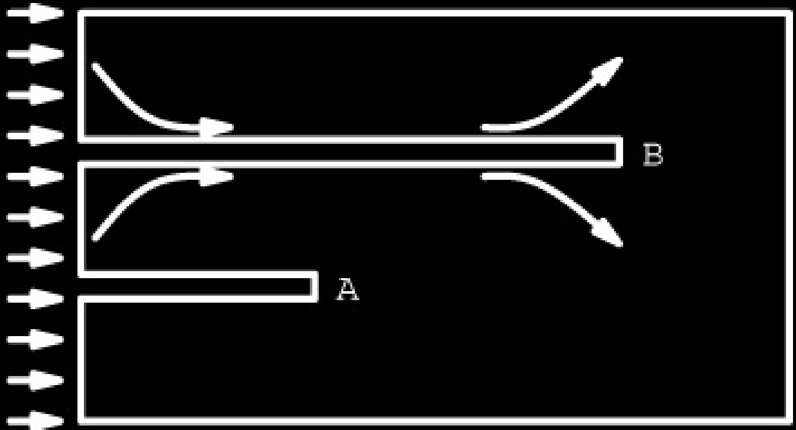
$$N(L) \propto L^{-\alpha}; \quad \alpha \approx 1.2$$

$$Pe = \frac{V\langle h \rangle}{D} = 32$$

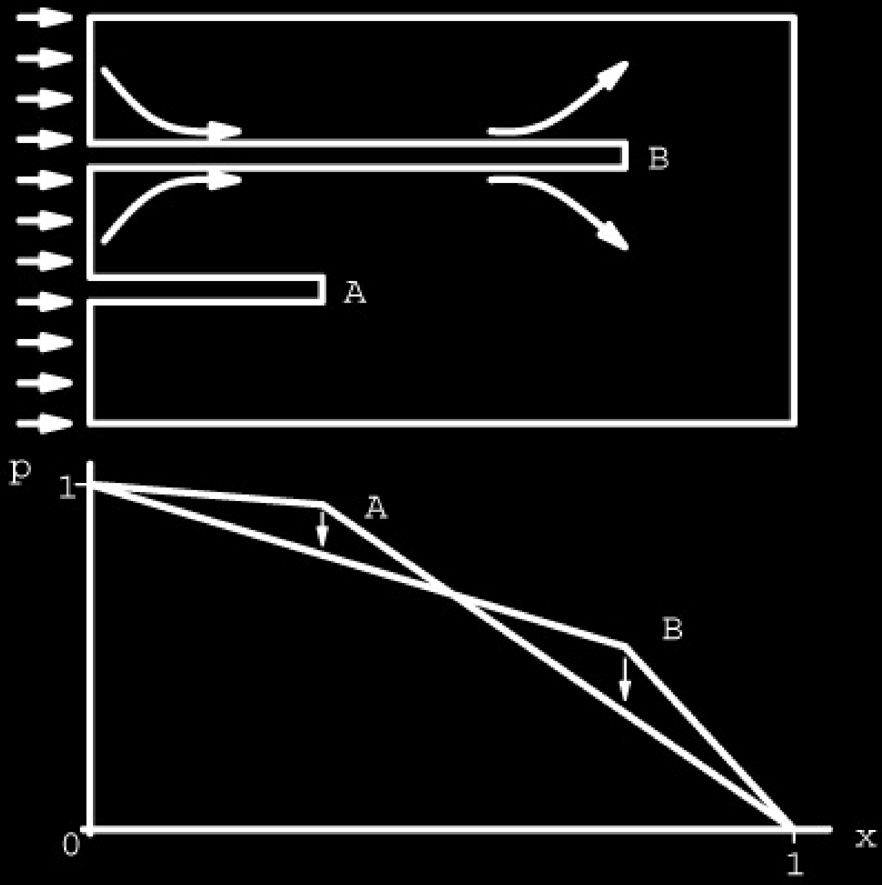
The cumulative distribution of channel lengths

Channel competition: flow capturing





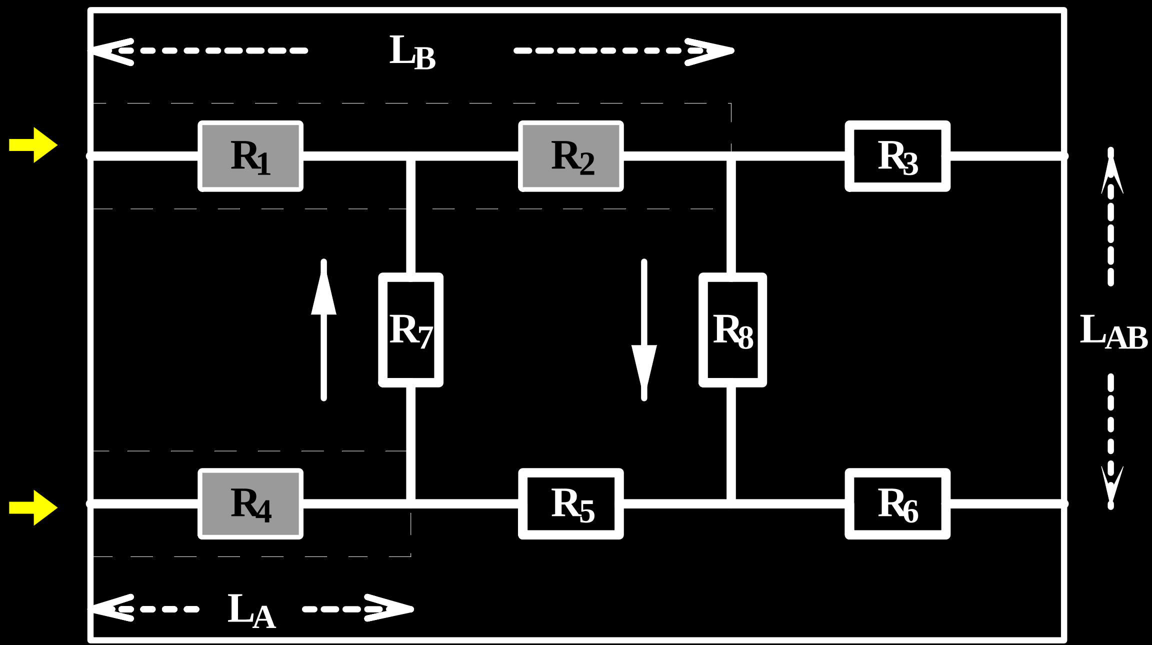
Channel competition: flow capturing



Upstream: towards long channel
Downstream: away from long channel

Pressure gradient in long channel is steeper (higher flow rate)
Pressure at long channel is lower than the short channel near the inlet
But higher near the outlet

Resistor network model



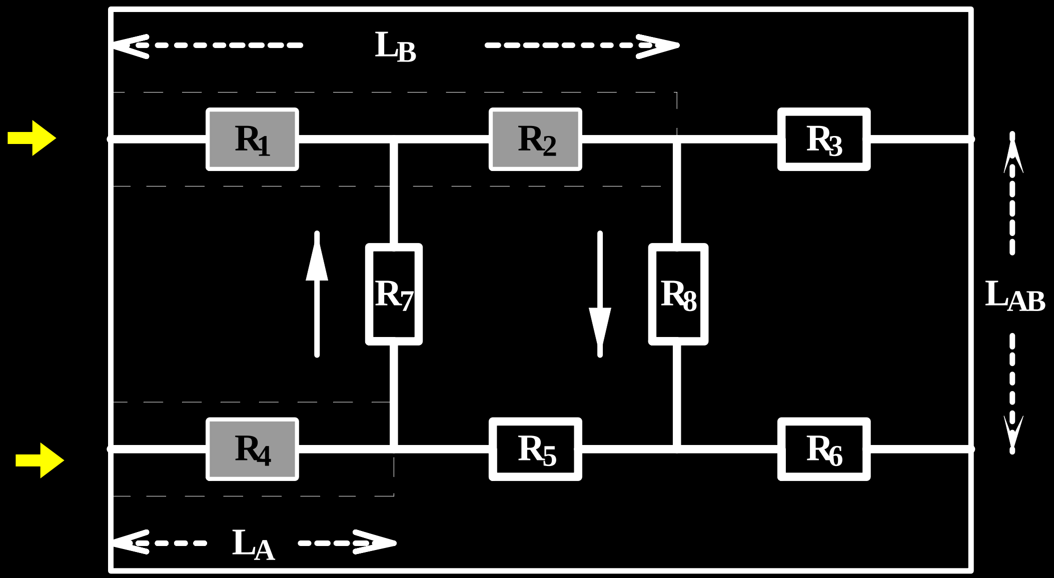
The undissolved medium has high resistivity (ρ_U)

Channels have low resistivity (ρ_C)

resistance proportional to length

$$R_1 = R_4 = \rho_{\mathbf{U}} L_A, \qquad R_5 = \rho_{\mathbf{M}}(L_B - L_A)\ldots$$

Evolution of the resistor network

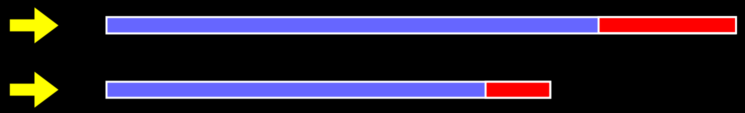


dissolution rate proportional to flow rate

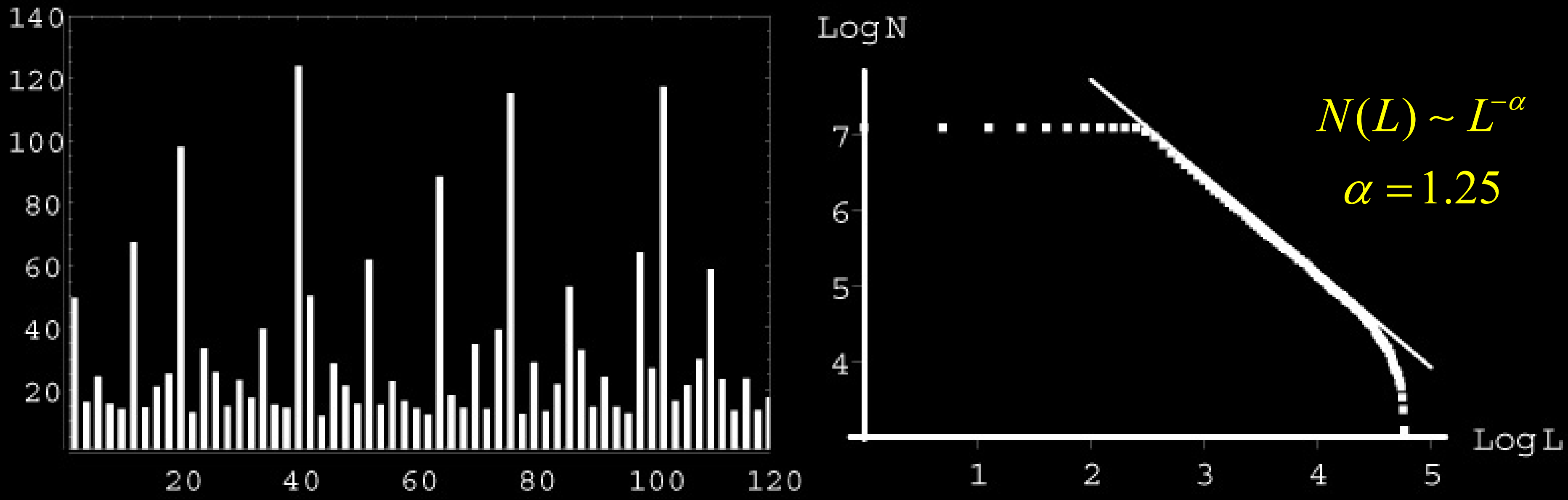
$$\frac{dL_{\mathbf{B}}}{dt} = I_{\mathbf{2}}$$

$$\frac{dL_{\mathbf{A}}}{dt} = I_{\mathbf{4}}$$

needle-like channels growing only at the top

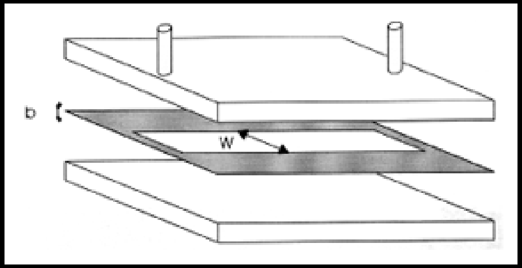


Multi-channel model~1200 channels

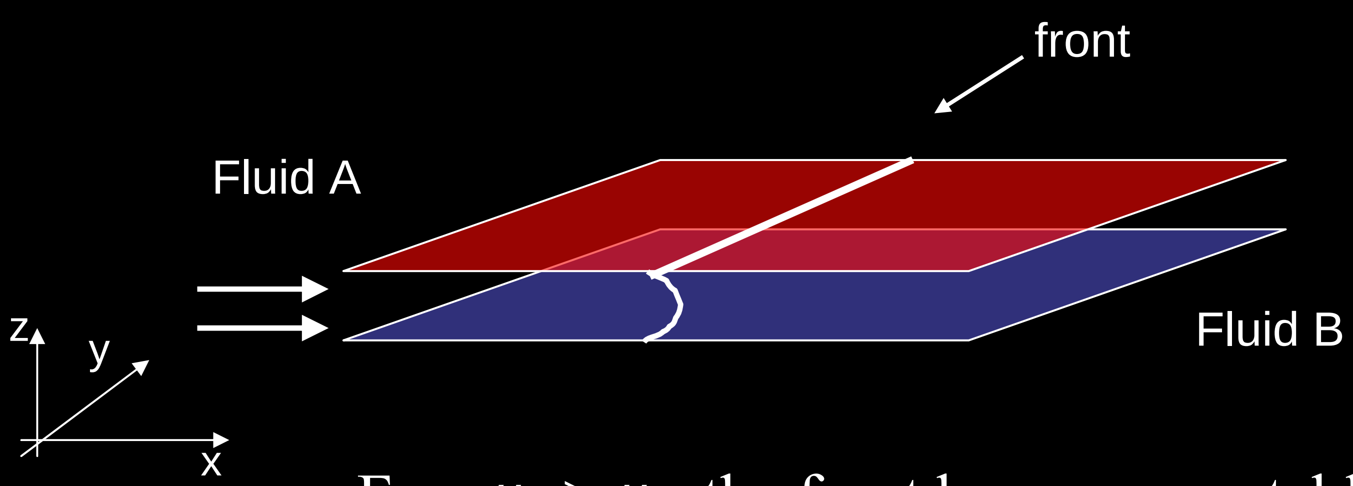


Network model shows the same scaling properties as the dissolving fracture system

Viscous fingering

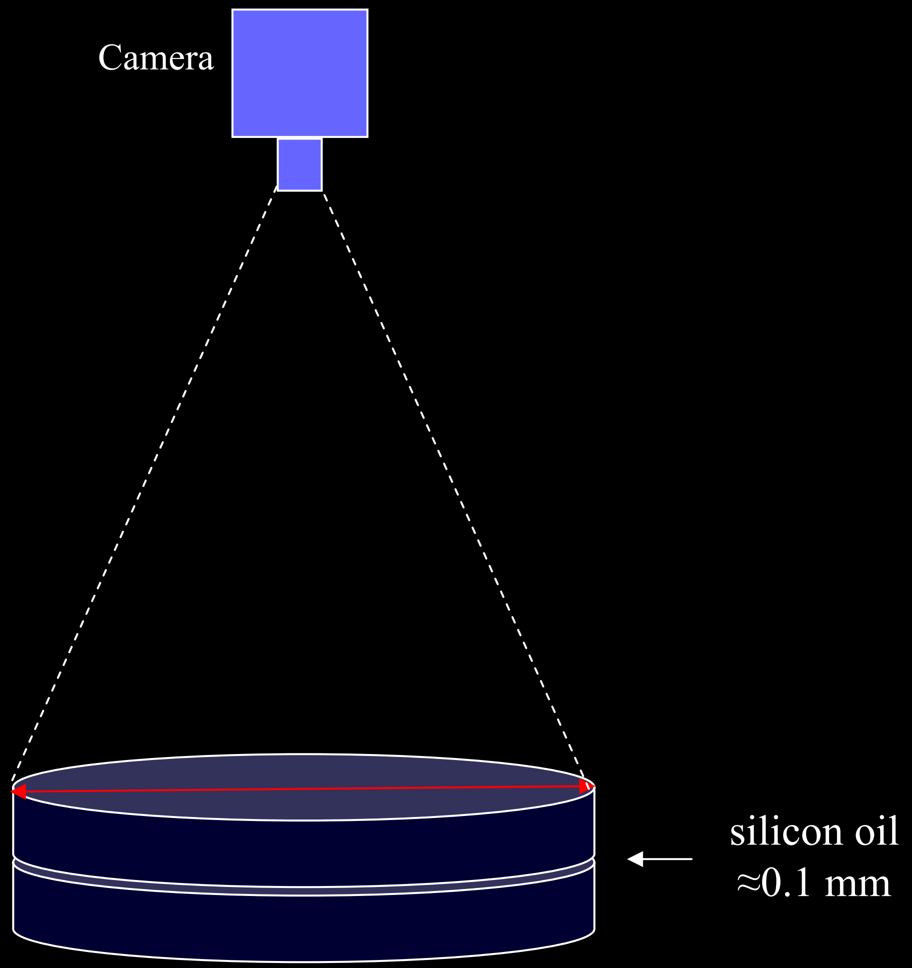


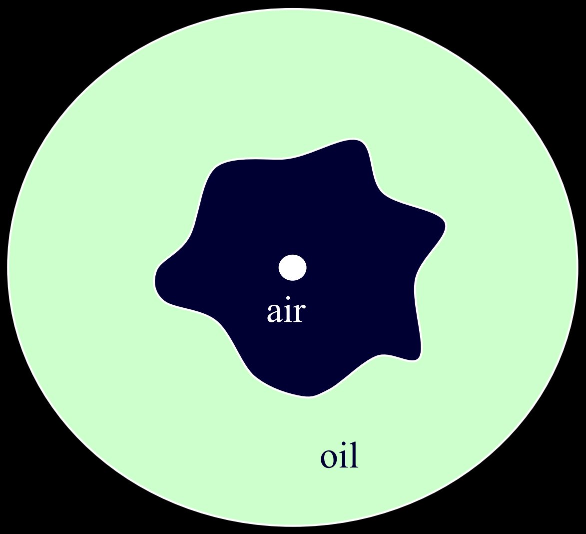
Hele-Shaw cell



For $\mu_B > \mu_A$ the front becomes unstable

Experiment

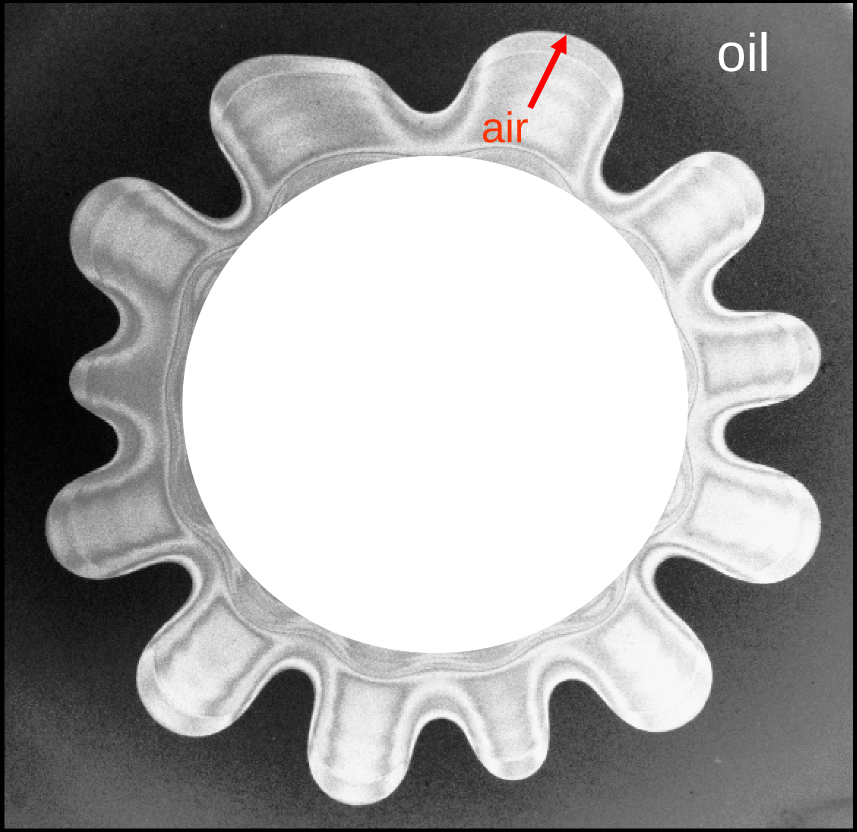




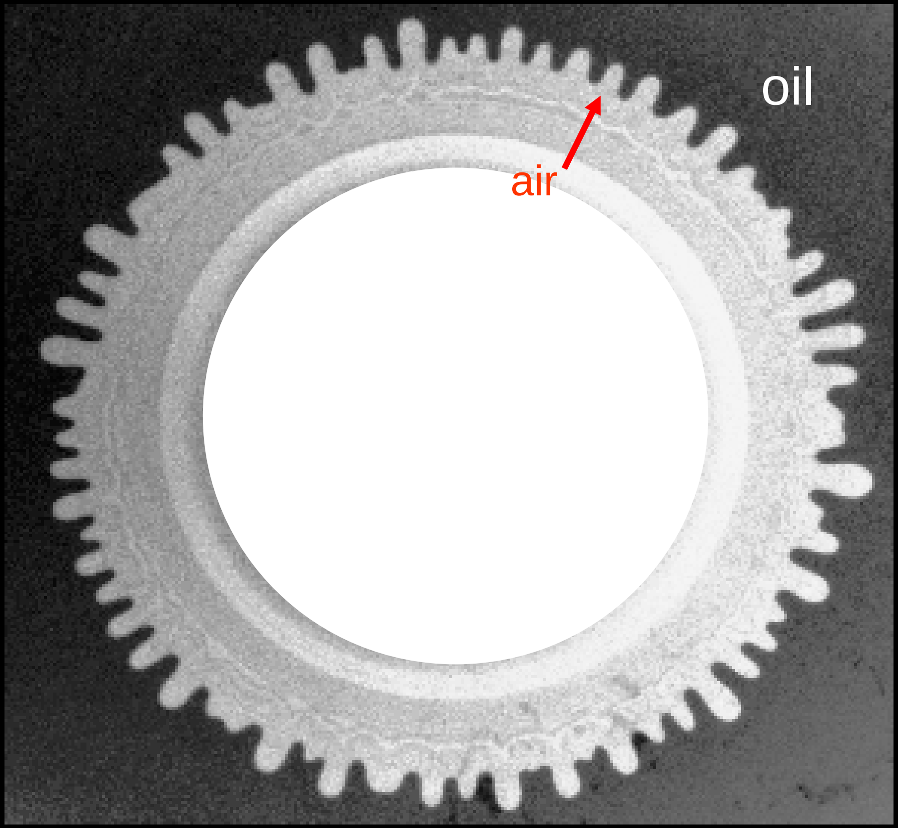
radial Hele-Shaw cell

Front instability

slow pumping

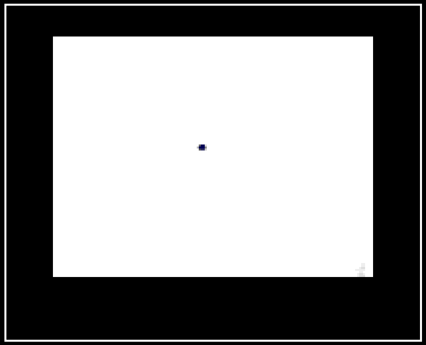


fast pumping



Swinney et al. (2002)

Late stages

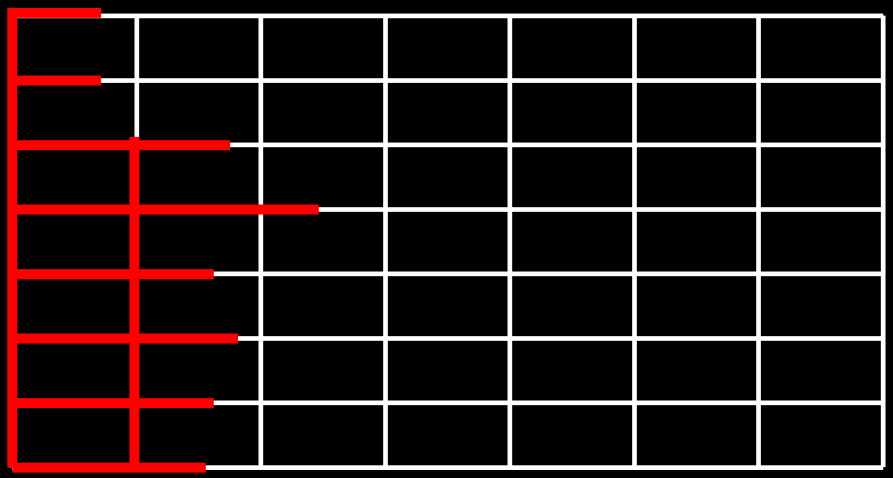


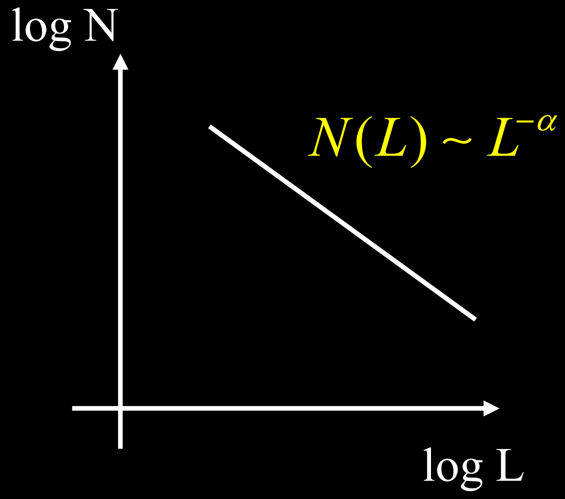
Swinney et. al (2002)

Hertzberg, Sweetman (2005)

Experiment needed...

Viscous fingering in a network of channels





scale invariance?

$\alpha = ?$

Conclusions

- The dissolution patterns in a porous medium show scale-invariant properties
- Core of the interaction between the channels is capture of the flow form the shorter by the longer ones
- A model of interaction between dissolving channels was constructed by mapping the system into an evolving resistor network
- Network model shows the same nontrivial scaling features as the dissolving fracture system
- A similar scaling should be observed in the viscous fingering phenomena in the network of channels