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ADAPTIVE ESTIMATION OF PENALTY FACTORS IN CONTACT MODELLING

***T. Bednarek*[1,2] *and P. Kowalczyk*[1]**
[1]*Institute of Fundamental Technological Research, Warsaw, Poland*
[2]*Kazimierz Wielki University, Bydgoszcz, Poland*

1. Introduction

The main objective of this paper is to improve stability conditions, uniqueness and convergence of numerical analysis of metal forming processes with contact constraints enforced by the penalty method. A commonly known drawback of this approach is the choice of penalty factor values. When assumed too low, they result in inaccurate fulfillment of the constraints while when assumed too high, they lead to bad conditioning of the equations system which affects stability and uniqueness of the solution. The proposed modification of the penalty algorithm consists in adaptive estimation of the penalty factor values for the particular system of finite element equations and for the assumed allowed inaccuracy in fulfillment of the contact constraints. The algorithm is tested on realistic examples of sheet metal forming. The finite element code based on flow approach formulation [1, 2] (for rigid-plastic and rigid-viscoplastic material model) has been used.

2. Main idea of penalty algorithm modification

The main idea is to estimate the penalty factors , adjusting their values to current stiffness and load conditions of the model and to an assumed accuracy of contact modelling. It is assumed that the penalty factors ϵ_k differ at different locations (for different discrete node-to-surface contact constraints $k = 1, \ldots, M$) and at different time steps or even equilibrium iterations.

The idea is first explained on a 1D example. The model shown in figure 1 is considered; k denotes stiffness of the spring, q is the exciting force, $\hat{u}$ is the assumed value of displacement (restriction resulting from the contact constraint), ϵ is the penalty factor and δ is the allowed inaccuracy of contact modelling (limit penetration depth). It is clear that, in order to preserve the desired accuracy of the solution, the penalty ϵ must at least equal $[k(\hat{u} - \delta) - q]/\delta$.

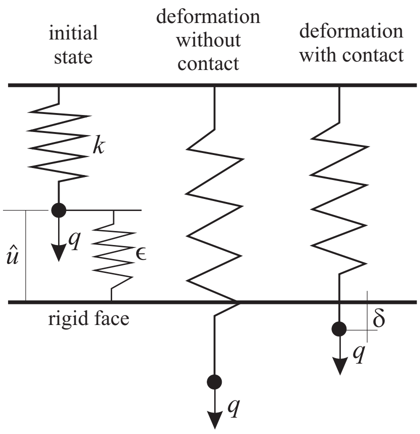

Figure 1. Elastic spring with contact constraint

Let us now pass to the general 3D case, i.e. consider a FE-discretized structure, with a non-linear system of equations solved by the Newton–Raphson scheme for the unknown displacement vector $\mathbf{u}$. Our goal is now to estimate the penalty factor values ϵ_k as large enough to preserve the desired accuracy of contact constraints but not larger so that the conditioning of the system matrix is not significantly worsened.

The allowed inaccuracy of contact modeling (penetration) is now a vector $\boldsymbol{\delta} = \{\delta_k\}$. Thus, in the worst case we allow $\mathbf{Du} - \hat{\mathbf{u}} = \boldsymbol{\delta}$, where $\mathbf{D}$ is a geometric matrix of directional cosines of rigid surfaces at the contact points. Substituting this to the general contact-penalized system of equations [4]

(1) $$\left(\mathbf{K} + \mathbf{D}^T \boldsymbol{\epsilon} \mathbf{D}\right) \mathbf{u}' = \mathbf{q}' + \mathbf{D}^T \boldsymbol{\epsilon} \hat{\mathbf{u}}', \qquad \boldsymbol{\epsilon} = \mathrm{diag}(\epsilon_k),$$

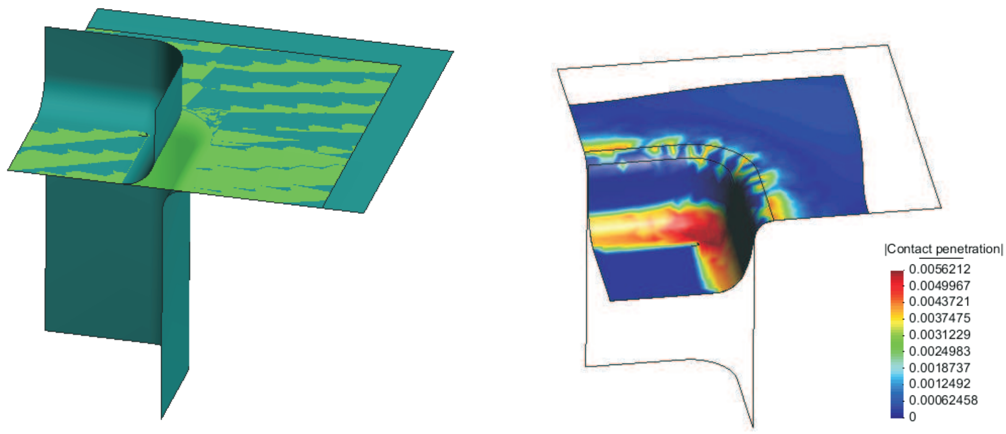

Figure 2. Benchmark test of deep drawing: initial geometry and deformed shape with contact penetration map.

(where $\mathbf{u}'$ is the solution corrector at the current iteration and $\mathbf{q}'$ the vector of residual forces), one can derive after a series of transformations the formula $\boldsymbol{\epsilon}\boldsymbol{\delta} = \mathbf{D}(\mathbf{q}' - \mathbf{K}\mathbf{u}')$. Recalling that $\boldsymbol{\epsilon}$ is a diagonal matrix, we rewrite it the index notation as

$$\epsilon_k = \frac{D_{ki}(q'_i - K_{ij}u'_j)}{\delta_k} \qquad \text{(no summation over } k). \tag{2}$$

Equation (2) is the recipe for the adaptive penalty factor values. Unfortunately, the displacement correctors u'_j on the right hand side are not known the moment and we need to replace them by their available approximate. Since in the convergent iteration scheme the subsequent corrections tend to zero, it is proposed to set $u'_j = 0$, except for the nodes where active contact constraints apply – there u'_j are set to simple orthogonal projection vectors of the current node position onto the contact surface. Thus, the formula (2) does actually yield approximate rather than exact values of desired penalty factors, which does not guarantee fulfillment of the imposed accuracy condition of contact modelling, but appears to be sufficient to keep the inaccuracy at least at the order of magnitude of the allowed limits, $\mathbf{D}\mathbf{u} - \hat{\mathbf{u}} \sim \boldsymbol{\delta}$.

3. Numerical example: Deep drawing of a plastic sheet

The numerical example is a deep drawing of a sheet. The drawing parameters and geometry of tools are taken from the benchmark proposed by Woo in [3]. The geometry of the sheet and tools are presented in figure 2. The assumed inaccuracy of contact modelling at all nodes is $\delta = 0.001$ mm.

Figure 2 (right) presents the contact modeling inaccuracy, i.e. the penetration depth of sheet nodes into rigid tools. The picture presents the results for the step for which the worst inaccuracy was detected. As it can be seen, its magnitude is kept at the level of 10^{-3} mm.

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