Nonparametric identification of added masses in frequency domain: a numerical study

Grzegorz Suwała[1], Jan Biczyk[2], Łukasz Jankowski[1,*]

[1]Institute of Fundamental Technological Research (IPPT PAN)
Department of Intelligent Technologies
ul. Pawińskiego 5b
02-106, Warsaw, Poland
[2]Adaptronica sp.z o.o.
ul. Szpitalna 32
05-092 Łomianki k. Warszawy
[*]ljank@ippt.pan.pl

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Abstract

This paper presents a theoretical derivation and reports on a numerical verification of a model-free method for identification of added masses in truss structures. No parametric numerical model of the monitored structure is required, so that there is no need for initial model updating and fine tuning. This is a continuation and an improvement of a previous research that resulted in a time-domain identification method, which was tested to be accurate but very time-consuming. A general methodology is briefly introduced, including the inverse problem, and a numerical verification is reported. The aim of the numerical study is to test the accuracy of the proposed method and its sensitivity to various parameters (such as simulated measurement noise and decay rate of the exponential FFT window) in a numerically controlled environment. The verification uses a finite element model of the same real structure that was tested with the time-domain version of the approach. A natural further step is a lab verification based on experimental data.

Introduction

This paper presents a derivation and reports on a numerical verification of a frequency-domain version of a nonparametric approach to identification of added masses in truss structures. The general approach has been recently developed in IPPT PAN [1–3], and it is based on the essentially nonparametric methodology of the virtual distortion method (VDM) [4]. The monitored structure is characterized in a purely experimental way, by means of its impulse response functions. As a result, no parametric numerical modelling is required, which obviates the need for model updating and fine-tuning that is typical for other model-based methods.

Most of the low-frequency identification methods used in global structural health monitoring (SHM) can be classified into two general groups: (1) *Model-based methods* that rely on a parametric numerical model of the monitored structure [5,6]. Their appealing feature is the physicality of the model and its identified modifications. However, an accurate parametric model is not easy to obtain and update. (2) *Pattern recognition methods*, which rely on a database of numerical fingerprints that are extracted from the experimentally measured responses [7]. No parametric modeling is required. The identification rarely goes beyond detection or approximate localization of the modification.

The developed approach exploits the advantages of both groups: it makes use of a nonparametric model of the monitored structure based on experimentally measured data, but it allows parametrically expressed modifications to be identified. In [1], a time-domain version of the approach has been proposed and experimentally verified. It proved to be accurate, and thanks to the iterative CGLS solution scheme, provided a good control over numerical regularization of the computed time-domain response [8]. However, the fundamental equation is a system of linear integral equations of the Volterra type, whose solution is significantly time-consuming. This paper develops and verifies a frequency-domain approach, which uses the fast Fourier transform (FFT) to solve the equations. A significant reduction in computation time is attained (up to four orders of magnitude). However, this is at the cost of losing the control of the process of numerical regularization: regularization in frequency domain seems to be relatively weakly researched and understood. The aim of this study is to test the accuracy of the proposed method, as well as its sensitivity to various parameters, such as the decay rate of the exponential FFT window (which seems to play the role of the regularization parameter) and the simulated measurement noise. The verification uses a finite element model of the same real structure that was tested with the time-domain version of the approach.

Nonparametric modeling and identification

The direct problem

Mass modifications are modeled with the equivalent pseudo-loads that act in the involved degrees of freedom (DOFs) of the original unmodified structure. The influence of the pseudo-loads on the response is computed using a convolution with the experimentally obtained local impulse-responses. The original, time-domain solution presented in [1] is transferred here to frequency domain, which converts the original Volterra integral equation into a series of simple decoupled linear equations.

The problem is formulated in terms of the finite element method. The unmodified structure is assumed to be linear and to satisfy the equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}}^{\mathrm{L}}(t) + \mathbf{C}\dot{\mathbf{x}}^{\mathrm{L}}(t) + \mathbf{K}\mathbf{x}^{\mathrm{L}}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ denote respectively the structural matrices of mass, damping and stiffness, $\mathbf{f}(t)$ is the external testing excitation and $\mathbf{x}^{\mathrm{L}}(t)$ denotes the corresponding response of the unmodified structure (reference response). In frequency domain, (1) takes the following quasi-static form:

$$\mathbf{D}(\omega)\mathbf{x}^{\mathrm{L}}(\omega) = \mathbf{f}(\omega), \tag{2}$$

where ω is the angular frequency, $\mathbf{x}^{\mathrm{L}}(\omega)$ and $\mathbf{f}(\omega)$ denote respectively the complex amplitudes of the reference response and the excitation, and $\mathbf{D}(\omega)$ is the complex dynamic stiffness matrix,

$$\mathbf{D}(\omega) = -\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C} + \omega\mathbf{K}, \tag{3}$$

whose inverse $\mathbf{H}(\omega) = \mathbf{D}^{-\mathbf{1}}(\omega)$ is called the dynamic compliance matrix and allows (2) to be represented in the direct form:

$$\mathbf{x}^{\mathrm{L}}(\omega) = \mathbf{D}^{-\mathbf{1}}(\omega)\mathbf{f}(\omega) = \mathbf{H}(\omega)\mathbf{f}(\omega). \tag{4}$$

The added masses, $\mathbf{m} = (m_1, m_2, \dots, m_N)$, are represented in terms of the modification $\Delta\mathbf{M}(\mathbf{m})$ to the original mass matrix $\mathbf{M}$. The mass matrix $\widetilde{\mathbf{M}}$ of the modified structure is thus given by

$$\widetilde{\mathbf{M}} = \mathbf{M} + \Delta\mathbf{M}(\mathbf{m}), \tag{5}$$

while its dynamic stiffness matrix can be expressed as

$$\widetilde{\mathbf{D}}(\omega, \mathbf{m}) = \mathbf{D}(\omega) - \omega^2\Delta\mathbf{M}(\mathbf{m}). \tag{6}$$

As a result, the response $\mathbf{x}(\omega)$ of the modified structure (the original structure with added masses) satisfies the following counterpart of (2):

$$\mathbf{D}(\omega)\mathbf{x}(\omega) = \mathbf{f}(\omega) + \mathbf{p}(\omega, \mathbf{m}), \tag{7}$$

where the vector $\mathbf{p}(\omega, \mathbf{m})$ denotes the pseudo-loads that act in the involved DOFs of the unmodified structure to model the inertial effects of the added masses,

$$\mathbf{p}(\omega, \mathbf{m}) = \omega^2\Delta\mathbf{M}(\mathbf{m})\mathbf{x}(\omega). \tag{8}$$

Equations (7), (4) and (8) yield:

$$\mathbf{x}(\omega) = \mathbf{x}^{\mathrm{L}}(\omega) + \mathbf{H}(\omega)\mathbf{p}(\omega, \mathbf{m}). \tag{9}$$

Equation (9) can be substituted into (8) to yield the following linear equation:

$$[\mathbf{I} - \omega^2 \Delta\mathbf{M}(\mathbf{m})\mathbf{H}(\omega)]\mathbf{p}(\omega,\mathbf{m}) = \omega^2 \Delta\mathbf{M}(\mathbf{m})\mathbf{x}^{\mathrm{L}}(\omega), \qquad (10)$$

where $\mathbf{I}$ is the identity matrix of the appropriate dimensions. Notice that $\Delta\mathbf{M}(\mathbf{m})$ is a diagonal matrix with non-vanishing entries only in the DOFs that correspond to the added masses. As a result, the pseudo-loads $\mathbf{p}(\omega,\mathbf{m})$ vanish in all other DOFs and (10) is reduced to a small system with dimensions $3N$ x $3N$, where N is the number of the added masses. Similarly, only the corresponding small submatrix of the full dynamic compliance matrix $\mathbf{H}(\omega)$. Based on (9) and (10), the direct problem is easily solved: for each angular frequency ω and vector $\mathbf{m}$ of the added masses, (10) is solved and the resulting pseudo-loads are substituted into (9) to compute the corresponding response of the modified structure.

The inverse problem

The inverse problem is formulated in the form of an optimization problem of minimization of a certain objective function with respect to the vector $\mathbf{m}$. In general, any objective function that expresses a discrepancy between the modeled response $\mathbf{x}(\omega)$ of the modified structure and its actually measured response $\mathbf{x}^{\mathrm{L}}(\omega)$ is plausible. Here, the following form is used:

$$F(\mathbf{m}) = \frac{\int I(\omega)\|\mathbf{x}(\omega)-\mathbf{x}^{\mathrm{L}}(\omega)\|d\omega}{\int I(\omega)\|\mathbf{x}^{\mathrm{L}}(\omega)\|d\omega}, \qquad (10)$$

where $I(\omega) \in \{0,1\}$ is an indicator function of the considered frequency range, which should coincide with the frequency range of the testing excitation $\mathbf{f}(t)$.

Numerical verification

The verification uses a model of the setup tested experimentally in [1], see Fig. 1. Each element cross-sectional area is 66 mm^2, the density is 7800 kg/m^3 and the Young modulus is 210 GPa. The sampling frequency is 65.5 kHz, all the sensors are accelerometers and the time interval of 230 ms is sampled. Four different masses (1.36 kg, 2.86 kg, 3.86 kg, 5.36 kg) are added in turn to one of the nodes M_1, M_2 or M_3. The location of the modification is assumed to be known. The identification is performed separately for each of the 12 considered modification cases, and the identification range is 0 kg to 10 kg with the step of 0.01 kg. As the testing excitation an impact by a modal hammer is used. Its placement is selected so that it excites at least two bending vibration modes. The objective function is based on the frequency range up to 485 Hz, which has been determined by including the spectral fringes (of the impact) with the energy above 10% of the maximum fringe energy.

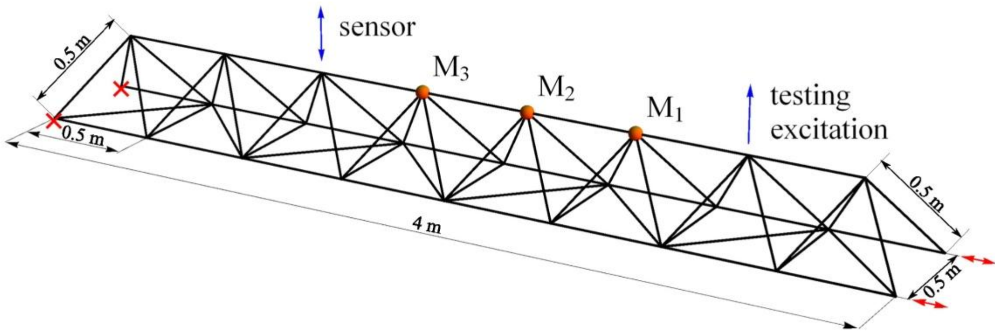
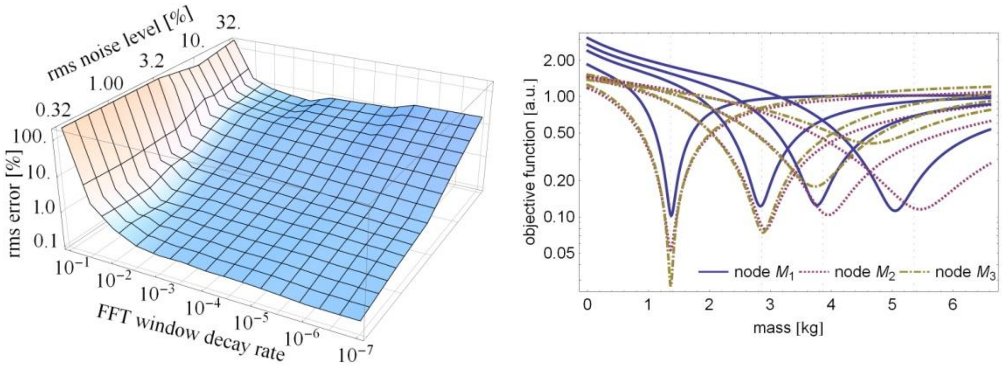


Figure 1. Truss structure used in the numerical example.

Figure 2(a) shows the dependence of the root mean square relative identification error on the decay rate of the FFT exponential window (defined by the value of the window at the end of the time interval) and on the simulated measurement noise (per cent of the signal rms, applied to the accelerations and excitation). Figure 2(b) plots typical objective functions (decay rate 10^{-5}, simulated measurement error 10% rms).



(a) (b)

Figure 2. (a) Root mean square relative identification errors in dependence on the decay rate of the FFT exponential window and the simulated measurement noise. (b) Typical objective functions (FFT window decay rate 10^{-5}, measurement noise 10% rms).

Conclusions

The numerical study confirms that the developed frequency-domain version of the identification method is accurate and relatively insensitive to (simulated) measurement noise. The decay rate of the exponential window used with FFT seems to play the role of the regularization parameter. The method performs up to four orders of magnitude faster than its previously developed time-domain counterpart. A lab verification based on experimental data is the natural further step.

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Bibliography

1. G. Suwała and Ł. Jankowski. A model-free method for identification of mass modifications. *Structural Control & Health Monitoring*, 19(2):216–230, 2012.
2. G. Suwała and Ł. Jankowski. *Experimental study of a model-free method for identification of stiffness-related structural damages.* Proc. of the 6th European Workshop on Structural Health Monitoring, Dresden, Germany, July 3-6, 2012, pp. 714–721.
3. G. Suwała and Ł. Jankowski. A model-less method impact mass identification. Proc. of the 4th European Workshop on Structural Health Monitoring, Kraków, Poland, July 2-4, 2008, pp. 374–381.
4. P. Kołakowski, M. Wikło and J. Holnicki-Szulc. The virtual distortion method–a versatile reanalysis tool for structures and systems. *Structural and Multidisciplinary Optimization*, 36(3):217–234, 2008.
5. C.-P. Fritzen and P. Kraemer. Self-diagnosis of smart structures based on dynamical properties. *Mechanical Systems and Signal Processing*, 23(6):1830–1845, 2009.
6. S. W. Doebling, C. R. Farrar and M. B. Prime. A summary review of vibration-based damage identification methods. *The Shock and Vibration Digest*, 30:91–105, 1998.
7. L. E. Mujica Delgado. *A hybrid approach of knowledge-based reasoning for structural assessment*. Ph.D. thesis. University of Girona, 2006.
8. P. C. Hansen. *Discrete inverse problems: insights and algorithms*. SIAM, Philadelphia, 2010.