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# Numerical analysis of the cup deep drawing process with use of new yield condition for plane stress states

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## Abstract

The elastoplastic constitutive equations for materials under plane stress condition with new yield criterion have been proposed. This yield condition accounts for the effect of strength differential effect. The system of equations of sheet metal forming process is solved by algorithm using the return mapping procedure. Plane stress constrain is incorporated into the Newton-Raphson iteration loop. The proposed algorithm is verified by performing numerical tests using shell elements in commercial FEM software ABAQUS/EXPLICIT with developed VUMAT subroutine. It is shown that the proposed approach provides the satisfactory prediction of material behaviour, at least in the cases when the anisotropy effects are not so advanced.

*Keywords: anisotropic behaviour of metal sheets, strength differential effect, explicit finite element analysis, plane stress*

## 1. Introduction

Finite element method is an efficient numerical tool to analyse such problems of shell deformation as for instance the sheet metal forming processes including cup drawing and stamping. Proper description of material properties is crucial for accurate analysis. In particular, the anisotropy and asymmetry of elastic range, related with strength differential effect (SDE), of considered materials play an important role in finite element simulation. For metal forming analysis many experimental tests are needed to obtain the proper description of anisotropic behaviour of metal sheets. There are some attempts to account for both, anisotropy and the SDE, e.g. [6]. However, according to our opinion, there is still lack of workable description of these effects, which could allow analysing effectively practical problems. In our disposal we usually have the yield stress in uniaxial tension, uniaxial compression and biaxial compression.

For integration of plane stress state problem one needs to satisfy the condition that the out-of-plane components of stress are zero. Because of this condition, which is called the plane stress constraint, particular schemes have been developed for plane stress elastoplastic finite element analysis [3]. The simple scheme is based on plane stress projected constitutive models with the in-plane stress  $\sigma_x, \sigma_y, \sigma_{xy}$  and strain  $\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$  components. For complex constitutive models when it is not so easy to derive plane stress-projected models, one can use other ideas [3].

## 2. Constitutive equations

The considered material model is based on modified  $J_2$  plasticity theory with an account for the influence of SDE. The strain rate is additively decomposed into the elastic part obeying isotropic Hooke's law and the plastic part governed by the associated flow law:

$$\begin{aligned} \dot{\boldsymbol{\varepsilon}} &= \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p \\ \dot{\boldsymbol{\sigma}} &= \mathbf{C} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p) \\ \mathbf{C} &= 2G\mathbf{I}' + K\mathbf{1} \otimes \mathbf{1} \end{aligned} \quad (1)$$

where  $G$  is shear modulus and  $K$  is the bulk modulus, while the flow law has the form:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda} \frac{\partial \Phi}{\partial \boldsymbol{\sigma}} \quad (2)$$

Finally, loading/unloading may be simply formulated in the Kuhn-Tucker form, that is:

$$\dot{\lambda} \geq 0, \Phi \leq 0, \dot{\lambda} \Phi = 0$$

## 3. Yield condition

The proposed yield condition is based on the analysis of limit criterion for transversally isotropic solids presented in [7]. In case of plane stress state the yield condition takes the following form:

$$f = \frac{1}{2k_1} \left\{ 3(k_1 - 1)p + \sqrt{9(k_1 - 1)^2 p^2 + 4k_1 q^2} \right\} - \sigma_Y^T(\bar{\varepsilon}_p) = 0 \quad (3)$$

where

$$\begin{aligned} p &= \frac{\sigma_x + \sigma_y}{3}, \\ q &= \sqrt{\sigma_x^2 + \sigma_y^2 + R_B \sigma_x \sigma_y + (2 - R_B) \sigma_{xy}^2}, \\ R_B &= 2 - \frac{1}{k_1 k_2^2} - \frac{2}{k_2} + \frac{2}{k_1 k_2}, \\ k_1 &= \sigma_Y^C / \sigma_Y^T, \\ k_2 &= \sigma_Y^{CC} / \sigma_Y^C, \end{aligned}$$

where  $\sigma_Y^C$  corresponds to the initial yield stress in uniaxial compression,  $\sigma_Y^T$  is initial yield stress in uniaxial tension and  $\sigma_Y^{CC}$  is initial yield stress in biaxial compression.

## 4. Integration of the elastoplasticity equations

After increment of time the stress is defined as:

$$\boldsymbol{\sigma}_{(n+1)} = \boldsymbol{\sigma}_{(n+1)}^{trial} - \mathbf{C} : \Delta \boldsymbol{\varepsilon}^p = \boldsymbol{\sigma}_{(n+1)}^{trial} - (2G\mathbf{I}' + K\mathbf{1} \otimes \mathbf{1}) : \Delta \boldsymbol{\varepsilon}^p$$

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where trial stress is:

$$\sigma_{(n+1)}^{trial} = \sigma_{(n)} + C : \Delta \epsilon$$

### 2.1. Plane stress state

Let us consider a thin plate element in plane stress states. We then have:

$$\sigma_z = 0, \sigma_{zx} = \sigma_{zy} = 0$$

$$\epsilon_z \neq 0, \gamma_{zx} = \gamma_{zy} = 0$$

where  $\gamma_{ij}$  indicates the shear strain.

## 6. Cup deep drawing process

Figure 1 shows a schematic description of the tools and blank discretization by the finite elements for the square cup deep drawing. The blank is modelled by four-nodes shell element (ABAQUS elements library type - S4R), whereas the die, punch and holder are modelled by rigid elements (ABAQUS elements library type - R3D4).

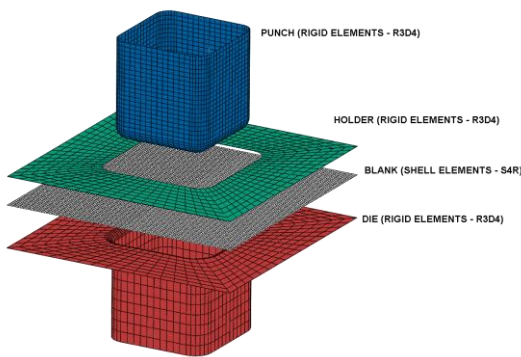


Figure 1: Initial configuration of the tools and the blank for the square cup deep drawing.

The material properties and process variables used in the simulation are as follows:

- size of the blank 150x150 mm;
- thickness of the blank 0.78 mm;
- blankholding force 19.6kN;
- coefficient of friction 0.1;
- material of the blank - mild steel;
- Young's modulus 210 GPa,
- Poisson ratio 0.3.

The uniaxial true stress–strain data measured in the tension test for steel were fit to the power law equation

$$\sigma_y^T(\bar{\epsilon}_p) = A + B(\bar{\epsilon}_p)^C$$

and the obtained coefficients are: A=200 MPa, B=232 MPa, C=0.3.

The blank is composed of 1849 elements and 1936 nodes.

Figure 2 illustrates an example for plasticity theory with Huber-Mises yield condition. In the presentation the results of the same problem are shown for the plasticity model with the proposed new yield condition.

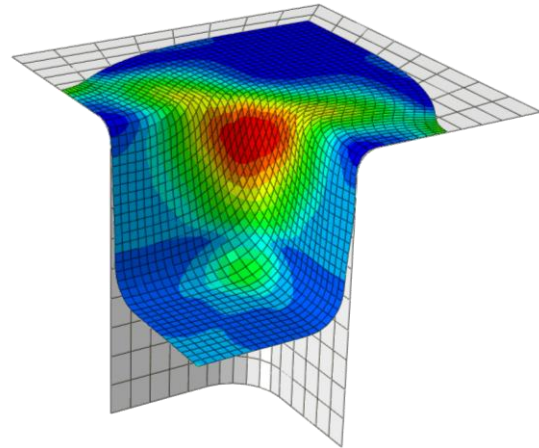


Figure 2: Deformation of the blank at the punch stroke 50mm for the square cup deep drawing with application of ABAQUS/EXPLICIT for Huber-Mises yield condition and isotropic hardening law.

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