39th Solid Mechanics Conference
Zakopane, Poland, September 1–5, 2014

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RECENT IMPROVEMENTS IN MIXED/ENHANCED SHELL ELEMENTS WITH DRILLING ROTATION

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1. Introduction

The purpose of the presentation is to summarize features of our recent formulation of mixed/enhanced four-node shell elements based on the Hu-Washizu (HW) functional *with* rotational degrees of freedom and illustrate their numerical performance [1–5].

The drilling rotation, defined as the rotation about the shell director, is not naturally present in the shell equations derived from the non-polar Cauchy continuum for the Reissner hypothesis and Green strain. To alleviate this restriction, we consider the *extended configuration* space, consisting of the deformation $\boldsymbol{\chi} \in R^3$ and rotations $\mathbf{Q} \in SO(3)$ subjected to the Rotation Constraint (RC) equation $skew(\mathbf{Q}^T\mathbf{F}) = \mathbf{0}$, where $\mathbf{F} = \nabla\boldsymbol{\chi}$. For shells, $\mathbf{Q}$ is reduced to the rotation at the reference surface $\mathbf{Q}_0$ and only the drilling component (12) of the RC equation, $[skew(\mathbf{Q}_0^T\mathbf{F})]_{12} = 0$, is used. This approach enables us to use standard constitutive equations for shells; in contrast to the Cosserat-type shells, which also use the drilling rotation but need specialized constitutive equations.

2. Shell pure HW functional with rotations

Mixed finite elements were pioneered by Pian in 1964 and a lot of work has been done since then to improve their theoretical foundations and performance. In the class of four-node elements, the best seem to be the ones based on the three-field Hu-Washizu (HW) functional; they exhibit a higher accuracy of displacements and stresses and significantly better convergence properties in non-linear problems than other elements, such as the enhanced strain (EAS or EADG) elements.

Our formulation is based on the 2nd Piola-Kirchhoff stress $\mathbf{S}$ and the Green strain $\mathbf{E}(\mathbf{u}) = \frac{1}{2}(\mathbf{F}^T\mathbf{F} - \mathbf{I})$. For 3D formulation, the Lagrange multiplier method is used to append the RC equation to the HW functional,

$$(1)\quad F_{\mathrm{HW}}(\mathbf{u}, \mathbf{Q}, \mathbf{S}^*, \mathbf{E}^*, \mathbf{T}^*) = \int_{\mathbf{V}} \{W(\mathbf{E}^*) + \mathbf{S}^* \cdot [\mathbf{E}(\mathbf{u}) - \mathbf{E}^*] + \mathbf{T}^* \cdot skew(\mathbf{Q}^T\mathbf{F})\}\, dV - F_{\mathrm{ext}}\,,$$

where $W(\mathbf{E}^*)$ is the strain energy expressed by the independent strain $\mathbf{E}^*$, and $\mathbf{S}^*$ is the independent stress which serves as the Lagrange multiplier for the difference of strains $\mathbf{E}^*$ and $\mathbf{E}(\mathbf{u})$. Besides, $\mathbf{T}^*$ is the skew-symmetric Lagrange multiplier for the RC equation $skew(\mathbf{Q}^T\mathbf{F}) = \mathbf{0}$.

To derive the HW functional for shells, we use the Reissner shell kinematics,

$$(2)\quad \mathbf{x}(\zeta) = \mathbf{x}_0 + \zeta\mathbf{Q}_0\mathbf{t}_3\,, \quad \zeta \in [-1, +1],$$

where $\mathbf{x}_0$ is a position of the reference surface and $\mathbf{t}_3$ is the shell director. The rotation of the reference surface $\mathbf{Q}_0$ is parameterized by the canonical rotation vector. For this kinematics, the Green strain linearized in ζ becomes $\mathbf{E}(\zeta) \approx \boldsymbol{\varepsilon} + \zeta\boldsymbol{\kappa}$. Let us also assume the independent strain in the analogous form, $\mathbf{E}^*(\zeta) \approx \boldsymbol{\varepsilon}^* + \zeta\boldsymbol{\kappa}^*$. Then, by integration of the HW functional of Eq. (1) over the shell thickness, we obtain its shell counterpart,

$$(3) \qquad F_{\mathrm{HW}}{}^{\mathrm{sh}} = \int_{\mathbf{A}} \{W^{\mathrm{sh}}(\boldsymbol{\varepsilon}^*, \boldsymbol{\kappa}^*) + \mathbf{N}^* \cdot [\boldsymbol{\varepsilon}(\mathbf{u}, \mathbf{Q}_0) - \boldsymbol{\varepsilon}^*] + \mathbf{M}^* \cdot [\boldsymbol{\kappa}(\mathbf{u}, \mathbf{Q}_0) - \boldsymbol{\kappa}^*] + F_{\mathrm{RC}}{}^{\mathrm{drill}}\}\, dA - F_{\mathrm{ext}}{}^{\mathrm{sh}},$$

where $\mathbf{N}^*$ and $\mathbf{M}^*$ are defined as the integrals of $\mathbf{S}^*$ and $\zeta\mathbf{S}^*$ over the shell thickness, and A is the area of the shell reference surface. The drilling RC part is used in the Perturbed Lagrange (PL) form,

$$(4) \qquad F_{\mathrm{RC}}{}^{\mathrm{drill}} = T^*[skew(\mathbf{Q}_0^{\mathrm{T}}\mathbf{F})]_{12} - (1/2\gamma)(T^*)^2$$

where T^* is the (scalar) Lagrange multiplier and $\gamma \in (0, \infty)$ is the regularization parameter. Note that seven fields $(\mathbf{u}, \mathbf{Q}_0, \mathbf{N}^*, \mathbf{M}^*, \boldsymbol{\varepsilon}^*, \boldsymbol{\kappa}^*, T^*)$ are used in $F_{\mathrm{HW}}{}^{\mathrm{sh}}$. This methodology yields pure shell HW functional, the shell element for which performs very well but requires a large number of parameters and so is not efficient.

3. Shell partial HW functionals with rotations

In an alternative approach, we derive the shell HW functional differently. First, the 3D potential energy functional is integrated, to obtain its shell counterpart

$$(5) \qquad F_{\mathrm{PE}}{}^{\mathrm{sh}} = \int_{\mathbf{A}} W^{\mathrm{sh}}(\boldsymbol{\varepsilon}, \boldsymbol{\kappa})\, dA - F_{\mathrm{ext}}{}^{\mathrm{sh}}.$$

Then, we construct the *partial* shell HW functional for selected strain components only while still using the potential energy functional for the other components. The so-derived functionals can be used to select the "optimal" formulation, which ensures the required numerical performance using a minimum number of parameters. For instance, in the HW29 shell element, we use the HW functional only for the 0-th order strain components $\varepsilon_{\alpha\beta}$ and $\varepsilon_{\alpha 3}$ $(\alpha,\, \beta = 1, 2)$,

$$(6) \qquad F_{\mathrm{HW}}{}^{\mathrm{sh}} = \int_{\mathbf{A}} \{W^{\mathrm{sh}}(\varepsilon^*_{\alpha\beta}, \varepsilon^*_{\alpha 3}, \kappa_{\alpha\beta}) + N^*_{\alpha\beta}[\varepsilon_{\alpha\beta} - \varepsilon^*_{\alpha\beta}] + N^*_{\alpha 3}[\varepsilon_{\alpha 3} - \varepsilon^*_{\alpha 3}] + F_{\mathrm{RC}}{}^{\mathrm{drill}}\}\, dA - F_{\mathrm{ext}}{}^{\mathrm{sh}}.$$

Using such a methodology, we have developed several mixed/enhanced shell elements based on the Hu-Washizu functional *with* rotational degrees of freedom, [1–4]. Numerical examples will illustrate various aspects of their performance, such as: accuracy, radius of convergence, required number of iterations of the Newton method or the arc-length method and time of computations. Additionally, examples enabling comparisons with our new HW elements using only translational degrees of freedom will be provided.

References

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