XIII KONFERENCJA NAUKOWO-TECHNICZNA
TKI 2014 TECHNIKI KOMPUTEROWE W INŻYNIERII
Licheń Stary • 6–9 maja 2014

Effective numerical techniques for identification of structural mass modifications

Grzegorz Suwała, Łukasz Jankowski
Department of Intelligent Technologies, Institute of Fundamental Technological Research (IPPT PAN)
email: gsuwala@ippt.pan.pl, ljank@ippt.pan.pl

ABSTRACT: This contribution focuses on effective numerical techniques used in a nonparametric method for identification of structural mass modifications. The approach utilizes the Virtual Distortion Method (VDM), which allows experimentally measured data to be directly used in the modeling process. As a result, experimentally obtained characteristics of the involved structure are used directly, so that no parametric modeling and time-consuming fine-tuning of the parameters are necessary. On the other hand, there are significant computational costs related to the need of direct processing of the measured time series, which require effective numerical techniques. Mass identification is formulated as an optimization problem of minimizing the mean square distance between the measured and the computed structural responses, where the optimization variables are mass-related parameters. Given the testing excitation (which can be unknown but should be reproducible) and the measured response of the original undamaged structure, the corresponding response of the structural mass modifications is computed by using certain mass-equivalent pseudo loads, which are convolved with experimentally obtained local impulse responses of the unaffected structure. The methodology is validated numerically and experimentally using a 4-meter-long, 70-element truss.

SŁOWA KLUCZOWE: mass identification, virtual distortion method (VDM), conjugate gradient least squares, FFT, nonparametric

1. Introduction

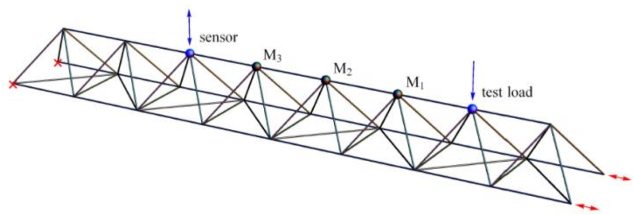
This paper presents and discusses numerical techniques used in a time-domain version of a nonparametric approach to identification of added masses in truss structures. The general approach has been developed in IPPT PAN [1–3], and it is based on the essentially nonparametric methodology of the virtual distortion method (VDM) [4]. The monitored structure is characterized in a purely experimental way, by means of its impulse response functions. As a result, no parametric numerical modeling is required, which obviates the need for model updating and fine-tuning that is typical for other model-based methods.

A 3D truss structure with 26 nodes and 70 elements was used in the experimental verification, see fig. 1. The structure was 4 m long, and the elements were circular steel tubes with the radius of 22 mm, the thickness of 1 mm and the lengths of 500 mm or 707 mm; the total weight was approximately 32 kg. The right-hand side nodes were free to move in the longitudinal direction only, whereas the two opposite left-hand side nodes were fixed. Only nodal mass modifications were considered. They were implemented by fixing concentrated masses at one or two of the nodes marked M1, M2 and M3 in fig. 1; the location of the modifications was assumed to be known. Two modification scenarios were investigated:

1) modification of a single nodal mass in M1, M2, or M3;
2) simultaneous modification of two nodal masses in the nodes M1 and M3.

2. Nonparametric modeling and identification

In agreement with the general approach of the VDM, modifications of structural mass are modeled with pseudo loads $\mathbf{p}(t)$, which are response-coupled and act in the unmodified structure to imitate the inertial effects of the modifications. For a given modification $\Delta\mathbf{M}$ and excitation $\mathbf{f}(t)$, the response of the modified structure is computed in two steps [1]:



Rys. 1. Truss structure

1) The pseudo-loads $\mathbf{p}(t)$ that model the mass modifications are found by solving the following linear integral equation of the Volterra type:

$$\mathbf{p}(t) + \Delta\mathbf{M}\int_0^t \ddot{\mathbf{B}}(t-\tau)\mathbf{p}(\tau)d\tau = -\Delta\mathbf{M}\ddot{\mathbf{u}}^L(t) \qquad (1)$$

2) The response is then computed by

$$\mathbf{u}(t) = \mathbf{u}^L(t) + \int_0^t \mathbf{B}(t-\tau)\mathbf{p}(\tau)d\tau \qquad (2)$$

Computations in both steps require, besides $\Delta\mathbf{M}$, solely the characteristics of the unmodified structure. These are:

1) the responses $\ddot{\mathbf{u}}^L(t)$ and $\mathbf{u}^L(t)$ to the same considered excitation $\mathbf{f}(t)$;
2) the matrices of the impulse responses $\ddot{\mathbf{B}}(t)$ and $\mathbf{B}(t)$.

All these characteristics can be measured experimentally prior to modeling of the modifications, so that there is no need to build and update a parametric numerical model of neither the unmodified nor the modified structure. The pseudo-loads vanish in the DOFs that are unrelated to the mass modification $\Delta\mathbf{M}$. As a result, the measured responses and the impulse responses can be restricted to the DOFs that are related to the mass modifications, which makes the experimental measurements more feasible

Identification is formulated as an optimization problem of minimizing (with respect to $\Delta\mathbf{M}$) the least square discrepancy between the actually measured response of the modified structure to $\mathbf{f}(t)$ and the response modeled by (2).

3. Numerical techniques

Theoretically, given the discretized versions of the direct (1) and the inverse problems, identification of mass modifications is straightforward: it amounts to an iterative minimization of the objective function. In each iteration, a sensitivity analysis can be also performed [1], which requires an additional adjoint linear system to be solved. If a second-order optimization method is used, then also all derivatives of the pseudo load $\mathbf{p}_i(t)$ have to be computed by solving a linear system of the size of (1) separately for each optimization variable. In all cases, the system matrix is denoted by $\mathbf{A}$ (obtained in discretization of (1)) or $\mathbf{A}^T$. However, all responses are stored and processed in time domain, which can result in large dimensions of $\mathbf{A}$. It is a dense $3n$ x $3n$ block matrix with T x T blocks, where T is the number of the time steps and n is the number of the nodes related to mass modifications; the total dimensions are thus $3nT$ x $3nT$, see fig. 2. In case of a longer time interval or a non-localized modification, the matrix can become huge and unmanageable by standard numerical techniques. Moreover, as can be expected from the Toeplitz structure of its blocks [5], the matrix is significantly ill-conditioned, and a regularization technique has to be used in order to obtain meaningful solutions. It is proposed to use the fast iterative algorithm of conjugate gradient least squares (CGLS) [6] to solve the involved systems, since

1) the CGLS method has good regularizing properties. The number of iterations plays the role of the regularization parameter: the more iterations, the more exact but less regularized (that is more influenced by the measurement error) is the solution;
2) the method uses the system matrix $\mathbf{A}$ only in the form of the matrix-vector products $\mathbf{Ax}$ and $\mathbf{A}^T\mathbf{y}$, so that no matrix decomposition and no direct access to its elements are necessary, only two black-box procedures implementing the respective multiplications.

Moreover, the block Toeplits structure of the system matrix $\mathbf{A}$ can be also exploited:

1) Each block of $\mathbf{A}$ is a T x T lower triangular Toeplitz block, hence it can be stored in computer memory in a reduced form using only T elements instead of T^2.
2) In the CGLS method, the system matrix $\mathbf{A}$ is present only implicitly in the form of matrix-vector products. For each of the T x T Toeplitz blocks, the product can be computed in frequency domain using the FFT in time $O(T \log T)$ instead of $O(T^2)$.

4. Conclusions

This contribution focuses on numerical techniques used in a nonparametric approach to identification of modifications of structural mass. The approach is based on the VDM and requires neither parametric numerical model of the monitored structure nor any topological information, besides the locations of the potential modifications. A 70-element 3D truss structure was used in the experimental validation. Modifications of one and two nodal masses were identified using a single impact test excitation and a single test sensor. The average relative errors of identification of single modifications were less than 5%. The errors were larger in the cases of two modifications, which was a result of the ill-conditioning of the problem.

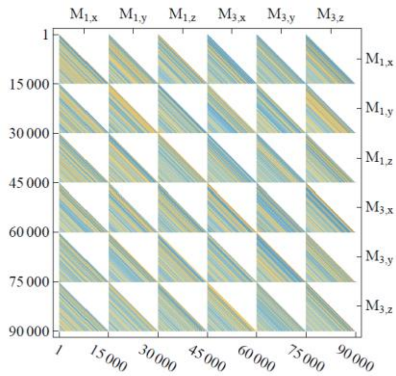


Fig. 2. Structure of the system matrix **A**

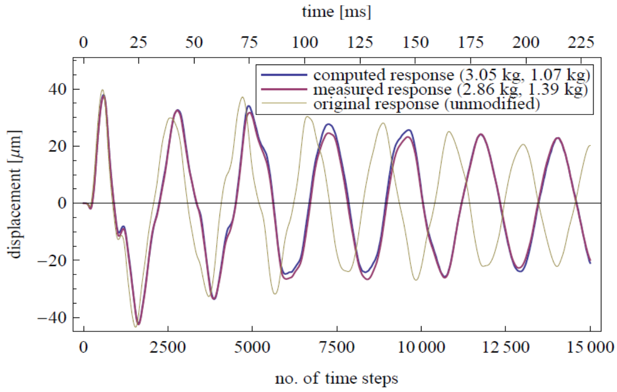


Fig. 3. Measured and computed responses

A significant computational cost of the proposed time-domain approach is addressed by selected effective numerical techniques, which exploit the block-Toeplitz structure of the discretized system. These techniques include the iterative regularization with the CGLS method, exact FFT-based matrix-vector multiplications, reduced-memory representation, etc.

Financial support of Structural Funds in the Operational Programme - Innovative Economy (IE OP) financed from the European Regional Development Fund - Project "Modern material technologies in aerospace industry", Nr POIG.01.01.02-00-015/08-00, and of the Polish National Science Centre (Project "AIA") is gratefully acknowledged.

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