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## Multiscale modelling of the acoustic waves in rigid porous and fibrous materials

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### Abstract

This paper presents the multiscale approach to the problem of acoustic waves propagating in a fluid (air) inside rigid fibrous or porous materials with open porosity. The approach essentially consists of the finite element analyses of three relevant problems defined on the representative fluid domain of a porous medium, the averaging and up-scaling techniques applied to calculate some necessary parameters of the porous microstructure which are used to model the effective properties of a homogenized fluid equivalent to the porous medium, and finally, the solution of a relevant Helmholtz problem on the macro-scale level in order to estimate, e.g., the acoustic absorption of the porous medium. This approach is illustrated by two examples: experimentally validated analyses of a fibrous material made up of a copper wire based on two Representative Volume Elements, and an analysis of a foam with spherical pores using a randomly generated periodic representative cell.

*Keywords: multiscale modelling, representative microstructures, porous materials, acoustic waves*

### 1. Introduction

The absorption and propagation of acoustic waves in porous (or fibrous) media with sufficiently rigid frame (skeleton) and open porosity can be predicted by the so-called Johnson-Champoux-Allard-Pride-Lafarge (JCAPL) model [1], where a porous medium is represented on the macro-scale level by an effective equivalent fluid so that the classic Helmholtz equation of linear acoustics can be used. The effective speed of sound and density of the equivalent fluid are frequency-dependent, they are calculated from the JCAPL formulas which involve parameters of the actual fluid (air) which fills the pores and the so-called transport parameters which solely depend on the micro-scale geometry of the porous medium.

A multiscale modelling of the problem of acoustic wave propagation and dissipation in porous media consists of three stages: first, the transport parameters are calculated from a microstructure of porous medium, then, the JCAPL formulas are used to estimate the effective speed of sound and density of the equivalent fluid, which finally serve to solve a relevant Helmholtz problem on the macro-scale. The transport parameters are computed by solving three independent Boundary Value Problems (BVPs) on the micro-scale level of porous medium [2, 3], namely:

- the viscous incompressible flow (i.e., the Stokes flow) through porous medium with no-slip boundary conditions on the solid skeleton walls;
- the steady heat transfer with isothermal boundary conditions on the solid skeleton walls;
- the Laplace problem.

The BVPs are defined on the fluid domain of a Representative Volume Element (RVE) of porous medium; they are usually solved using the Finite Element Method (FEM). After the finite element analyses are carried out, the averaging over the fluid domain is applied to the solutions and the up-scaling formulas are used to determine the transport parameters.

### 2. Examples of multiscale modelling

#### 2.1. Two RVEs for a fibrous material

Two fibrous samples were manufactured from a copper wire with diameter 0.5 mm: they were manually woven and fitted into an impedance tube with diameter 29. mm. The heights of samples are 30 mm and 60 mm, while the total length of wire used for the first sample was 10 m, and for the second one it was 20 m, so that both samples have the same porosity of 90%. The acoustic absorption coefficient was measured for both samples in the impedance tube: first, for each sample separately, and then, for two configurations of both sample, that is, with one sample on the top of the other, so that the total height of such fibrous layer was 90 mm (i.e., 30 mm + 60 mm or 60 mm + 30 mm).

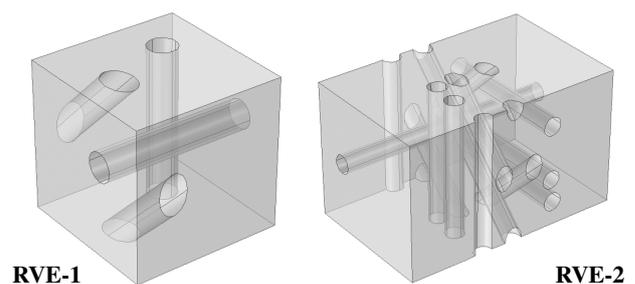


Figure 1: Simplified periodic representations of microstructure for the fibrous material of copper wire

Two Representative Volume Elements, RVE-1 and RVE-2 (see Fig. 1), were constructed for the fibrous material. Both are periodic and contain only straight fibres, which entails the assumption that the copper fibres in fibrous samples are locally straight. The RVE-1 has uniformly-spaced fibres, while in the RVE-2 the fibres are grouped in a layer normal to the direction of the propagation of acoustic waves; the porosity is always 90%

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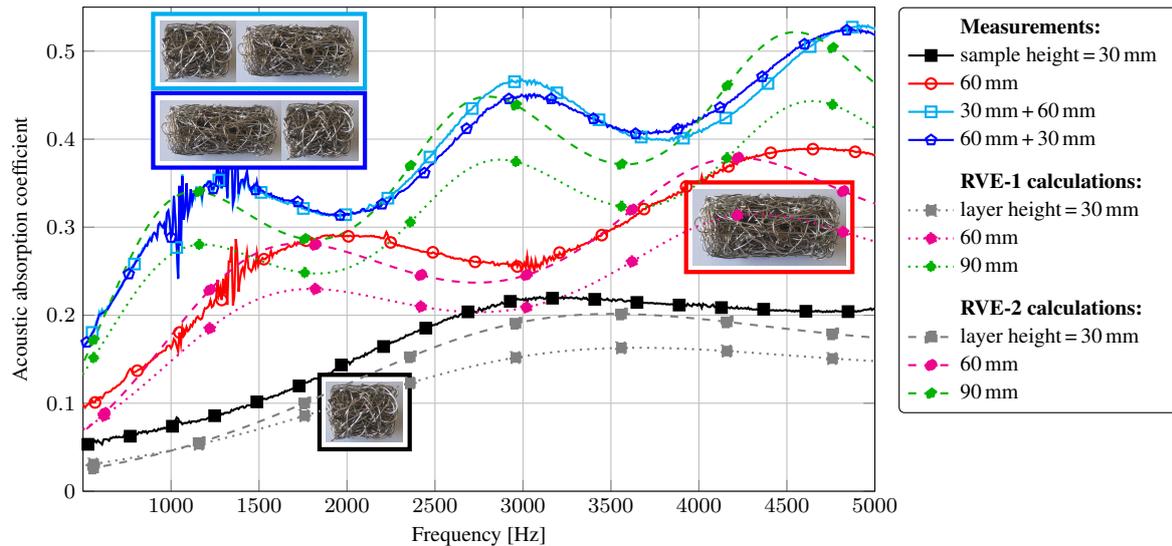


Figure 2: Sound absorption in fibrous layers: the results of the multiscale (microstructure-based) modelling and measurements

and the diameter of all fibres is 0.5 mm. For both RVEs the microstructure-based calculations were effectuated to estimate the effective speed of sound and density, so that the sound absorption could be computed for layers 30 mm, 60 mm, and 90 mm thick. Figure 2 compares these results with the corresponding measurements from the impedance tube, in the frequency range from 500 Hz to 5 kHz. It appears that the results obtained for the RVE-2 are very similar to the experimental curves, which means that the simplifying assumption of straight fibres is admissible.

2.2. A foam with spherical pores

A periodic representative microstructure for a foam with spherical pores was randomly generated using an algorithm which simulates the dynamic mixing of spheres (see Fig. 3). In the algorithm, the spheres may to some small extent penetrate each other, and eventually, they become pores linked with windows. The procedure was carried out for a case with five total pores in a cubic RVE (each pore represented by a family of eight identical spheres), and it was accomplished when the designed porosity of 70% was reached (see Fig. 4).

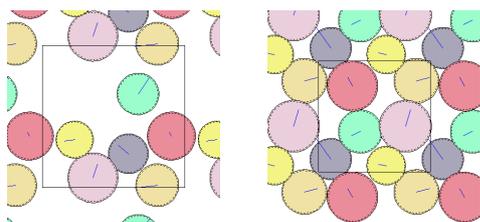


Figure 3: Random generation of periodic arrangements of overlapping spheres (pores)

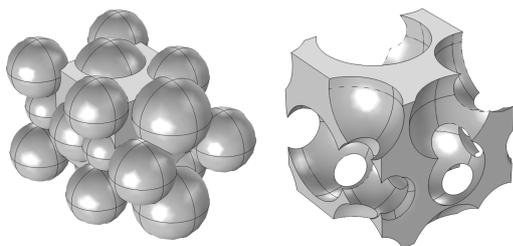


Figure 4: Final periodic arrangement of pores in a foam and the corresponding solid frame (skeleton)

The size of RVE was set so that the average diameter of pores was 0.33 mm. The micro-scale finite element analyses were carried out on the fluid domain of RVE in order to calculate the transport parameters, and then, to estimate the macro-scale effective celerity of acoustic waves propagating in such a foam (see Fig. 5).

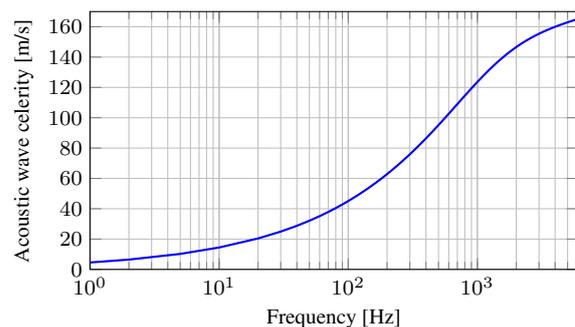


Figure 5: Celerity of acoustic waves in the designed foam with spherical pores

3. Conclusions

Usually, more than a few pores or fibres in a periodic cell (RVE) are necessary to well represent real porous or fibrous materials, however, this makes the cell larger, and the size of a large RVE may at higher frequencies become comparable with the wavelengths, which may decrease the accuracy and reliability of estimations, because of a weak separation of scales.

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