Constitutive and material instabilities in rate-independent single crystals deformed by multiple slip

H. Petryk* and M. Kursa

Institute of Fundamental Technological Research (IPPT), Polish Academy of Sciences, Pawinskiego 5B, 02-106 Warsaw, Poland hpetryk@ippt.pan.pl, mkursa@ippt.pan.pl

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A long-standing difficulty in rate-independent crystal plasticity is related to non-uniqueness in selection of active slip-systems. The common way to circumvent that problem in calculations is to apply a viscoplastic model where slip-rates are given functions of state and therefore uniquely defined by definition. However, uniqueness of solutions in crystal viscoplasticity is sometimes accompanied by strong sensitivity to initial imperfections, which replaces the explicit problem of non-uniqueness by a kind of hidden instability.

It is thus of interest to examine single crystal behaviour within the general framework of stability in materials with rate-independent dissipation [1]. In this paper, the attention is restricted to spatially discretized single crystals without strain-gradient effects. Instability is meant in the extended sense applicable to quasi-static deformation processes (paths), assuming that instability of equilibrium does not occur if the loading device is sufficiently stiff.

Path instabilities either of constitutive origin or with respect to formation of deformation patterns can be addressed in a unified manner by using the energy criterion of path instability [1]. This requires the incremental (or rate) problem to be of potential type, which is not true in general in rate-independent plasticity of single crystals deformed by multislip. Recently, the selective symmetrization of the slip-system interaction moduli matrix has been proposed [2], restricted to active slip-systems. As shown in [3], it admits an asymptotically exact formulation of the set of constitutive equations and inequalities for a crystal as a minimization problem for the incremental work. If there exist more than one solution to the original incremental problem then only the incremental work minimizer is regarded as stable while those corresponding to a higher value of the incremental work are eliminated.

The respective algorithm has been built up that enables numerical simulations of large plastic deformation of single crystals to be performed with the automatic crossing of constitutive branching points. An example is shown in Fig. 1 for uniform simple shear of an fcc crystal with latent-to-self hardening ratio of 1.4 for a fully prescribed deformation gradient $\mathbf{F}(\lambda) = \mathbf{1} + \lambda \mathbf{B} \otimes \mathbf{A}$, with λ increasing from 0 to 10 and $\mathbf{B} \leftrightarrow [100]$, $\mathbf{A} \leftrightarrow (001)$ initially. It can be seen that unstable solutions with more that 5 active slip-systems have been eliminated.

If the deformation gradient is only partially restricted then the number of locally active slipsystems can be much smaller, as in Fig. 2(a) for uniaxial tension in the high-symmetry direction with a symmetry-breaking response. In Fig. 2(b) the calculated energetically preferable pattern of deformation banding is visualized for another fcc crystal subjected to tension with two components of the macroscopic deformation gradient left free. The pattern depends on the loading direction, in qualitative agreement with experimental observations.



Figure 1: (a) Shear stress vs shear strain for uniform simple shear of an fcc single crystal, (b) changes in the orientation of crystallographic axes.



Figure 2: Simulations of uniaxial tension of fcc crystal: (a) slip-system selection, stress-strain diagram and changes of crystallographic orientation of the tensile axis; (b) deformation banding in tension in different directions [3].

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References

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