

24<sup>th</sup> International Congress of Theoretical and Applied Mechanics

> Palais des congrès, Montréal, Canada August 21 – 26, 2016

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# Preface

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ISBN: NR16-127/2016E-EPUB Catalogue Number: 978-0-660-05459-9

## THREE MODES OF THE DYNAMICS OF FLEXIBLE FIBERS IN SHEAR FLOW

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<u>Summary</u> Dynamics of a single non-Brownian flexible fiber in shear flow at low Reynolds number is investigated numerically. Initially, the fiber is straight and at the equilibrium. For different initial orientations and values of bending stiffness, three generic scenarios are observed: the fiber tends to: align along the vorticity direction, tumble within the plane perpendicular to vorticity, or perform a periodic motion superposed with translation along the flow.

#### SYSTEM

Dynamics of non-Brownian flexible microfibers in external flows have been recently intensively investigated experimentally and numerically, see e.g. [1, 2] and the references therein. One of the basic questions is how to classify the modes of their dynamics in shear flow, depending on the initial position [3, 4, 5] and flexibility [6]. In this work, we contribute towards solving this problem. We denote the fluid viscosity as  $\eta$  and the shear flow velocity as  $\mathbf{v} = \dot{\gamma} z \mathbf{e}_x$ , where  $\mathbf{e}_x$  is the unit vector along x. A flexible fiber is modeled as a chain of N beads of equal diameters d, with the consecutive beads connected by springs, and resisting bending [7]. At equilibrium, the fiber is straight with a very small distance  $l_0$  between the centers of the consecutive beads. As in [1, 6], the constraint forces are characterized by the dimensionless bending stiffness parameter A equal to the ratio of the Young modulus to  $\pi\eta\dot{\gamma}$ , and the dimensionless ratio k of the spring constant to the hydrodynamic force per unit length,  $\pi\eta d\dot{\gamma}$ . The value of k is assumed to be large, to provide almost constant fiber length.

The Reynolds number of the system is much smaller than unity, and the fluid motion satisfies the Stokes equations. Numerical simulations of the dynamics of each fiber bead are performed using the multipole expansion, corrected for lubrication [8], and implemented in the HYDROMULTIPOLE numerical code. We choose N = 40, k = 1000,  $l_0 = 1.02$ . We assume that initially the fiber is at equilibrium, and we perform a systematic study of how the evolution depends on the bending stiffness A and the initial orientation, which is determined by two angles:  $\Theta_0$  - the angle between the end-to-end vector of the fiber and the vorticity direction y, and  $\Phi_0$  - the angle between the projection of the end-to-end vector on the x - z plane and the x axis.

### RESULTS

### **Identification of modes**

In our simulations of the fiber motion, we have found three generic types of evolution. Basic features of these three modes of the fiber dynamics are illustrated in three columns of Figure 1, where we plot the time-dependent angle  $\Theta$  between the instantaneous end-to-end vector and the vorticity direction, and the time-dependent distance  $\Delta L$  between the centers of the first and last beads, normalized by its equilibrium value  $L_0$ . The time unit is  $1/\dot{\gamma}$ .

In the first mode, shown in the left column of Figure 1, colored blue and called spin-rotation [4], the fiber tends to stay straight,  $\Delta L/L_0 \rightarrow 1$ , and along the vorticity direction  $y, \Theta \rightarrow 0$ , rolling around it. In the second (red) mode, called the tumbling one, the fiber tends to the x-z plane,  $\Theta \rightarrow 90^{\circ}$ , and it regularly straightens out and becomes coiled while tumbling, with large-amplitude oscillations of  $\Delta L/L_0$ . In the third (green) mode, each bead of the fiber evolves towards a periodic orbit of a very complex three-dimensional shape, translating along the flow with a constant speed. This mode, called the periodic one, has not been observed before.

#### **Diagrams of the modes**

We have systematically investigated the dependence of the dynamical modes on the fiber initial orientation  $(\Phi_0, \Theta_0)$  with respect to the vorticity direction. The results for fibers of a different bending stiffness A = 4, 10 and 40 are shown in Figure 2. Very flexible fibers (A = 4, left) with all the initial orientations belong to the spin-rotation (blue) mode. For very stiff fibers (A = 40, right), the tumbling (red) mode dominates for most of the initial positions, excluding some of those which are close to the vorticity direction and lead to the spin-rotation (blue mode). For fibers of a moderate stiffness (A = 10, middle), there exists a range of the initial orientations which correspond to the periodic (green) mode. The boundaries of this range are not smooth, and a relatively small change of initial orientations can result in a different dynamical mode.



Figure 1: Typical examples of three modes of the fiber dynamics: blue, red and green. Top: the time dependent angle  $\Theta$ . Middle: the instantaneous relative length  $\Delta L/L_0$  of the fiber end-to-end vector. Bottom: examples of shapes when the end-to-end vector is perpendicular to the flow.



Figure 2: Dynamical modes of fibers at different initial orientations  $(\Phi_0, \Theta_0)$ , depending on their bending stiffness A.

## CONCLUSIONS

We have shown numerically that there exist three – rather than two as reported before [3, 4, 5] – essentially different modes of the flexible fiber dynamics in shear flow. The remarkable feature is that for moderate values of the bending stiffness, periodic solutions exist, with a very complex three-dimensional shape of the periodic trajectories. These orbits are essentially different than the classical Jeffery's orbits.

This work was supported in part by the National Science Centre under grant No. 2014/15/B/ST8/04359.

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