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On the semi-active control of carrying structures under a moving load

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Abstract

In this paper we address a group of recent research focused on the semi active control problems in carrying structures systems subjected to a travelling load. The magnitude of the moving force is assumed to be constant by neglecting inertial forces. The response of the system is solved in modal space. The optimal control problem is stated and it is solved by using of Pontryagin Maximum Principle. Switching control method is verified by numerical examples. The controlled system widely outperforms passive solutions. Due to its simplicity in practical design, the presented solution should be interesting to engineers.

Keywords: Semi-active control, structural control, optimization, moving load

1. Introduction

An increasing speed requirements in transport and technological processes forces engineers to apply new and unique solutions for the carrying structures design. From the last few decades the main role in such a design play an integrated systems of control. Semi-active methods superiority over active result from its reliability and low power consumption.

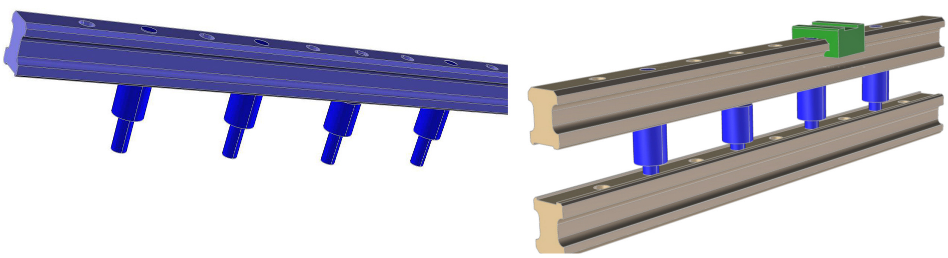

Figure 1. Semi-active controlled guideways.

In this paper we propose two fields of applications of semi-active controlled systems. The first one is dedicated to technological processes such as cutting or bonding, where the straight passage of a moving load is essential. The second one is directed to largescale engineering structures like bridges that span gaps or beams that must resist loads due to heavy and fast vehicles.

Technical difficulties with the rigid support of the bottom parts of the dampers require new, more practical solutions. One of them is presented in Fig 1.

A good number of semi-active control methods have spread widely and some of them have been put into practice recently. They are usually based on sky-hook and ground-hook ideas. These strategies are used for the active suspension of a moving oscillator in [3, 4]. The idea of a beam vibrations control by dampers and preliminary results were presented in [5]. The early papers deal with the problem of active control of a beam vibrations [6]. An active constrained layer is applied in [7].

In this paper we propose an open loop switching control method. The optimal solution is based on the Maximum Principle [8]. The form of cost integrand depends on the aim of control.

2. Mathematical background

In this section we present a control method and its optimal solution in a short. The aim of the proposed strategy is to provide a straight passage for the moving load. We consider the double-beam system as shown in Fig. 1. The solution scheme for a single-beam system is analogous.

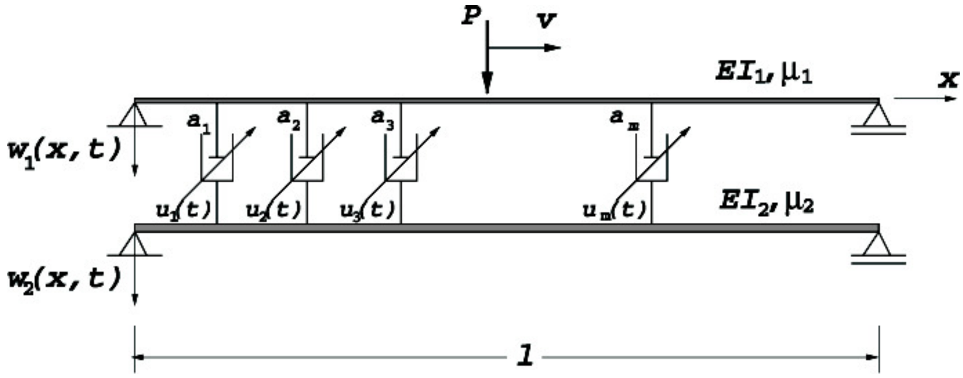

Figure 2. Double Euler-Bernoulli beam system coupled by a set of semi-active dampers.

We can write the governing equation for the considered system as follows:

$$EI_1 \frac{\partial^4 w_1(x,t)}{\partial x^4} + \mu_1 \frac{\partial^2 w_1(x,t)}{\partial t^2} = -\sum_{i=1}^{m} u_i \left[\frac{\partial w_1(a_i,t)}{\partial t} - \frac{\partial w_2(a_i,t)}{\partial t} \right] \delta(x-a_i) + $$
$$+ P\delta(x - vt), \tag{1}$$
$$EI_2 \frac{\partial^4 w_2(x,t)}{\partial x^4} + \mu_2 \frac{\partial^2 w_2(x,t)}{\partial t^2} = -\sum_{i=1}^{m} u_i \left[\frac{\partial w_2(a_i,t)}{\partial t} - \frac{\partial w_1(a_i,t)}{\partial t} \right] \delta(x-a_i),$$

together with the boundary and initial conditions:

$$w_1(0,t)=0, w_1(l,t)=0, w_1(x,0)=0, \dot{w}_1(x,0)=0,$$
$$w_2(0,t)=0, w_2(l,t)=0, w_2(x,0)=0, \dot{w}_2(x,0)=0. \tag{2}$$

Here, $w_1(x,t)$ and $w_2(x,t)$ are the transverse deflections of the beams at point (x,t), $u_i(t)$ is the i-th damping coefficient as a function of time, m is the number of viscous supports and P is the concentrated force passing the upper beam at constant velocity v and δ is the Dirac delta function. For the control design we use a representation of the system in modal space. Respecting the boundary conditions we look for the solution expressed upon the sine serie base.

$$V_{1(2)}(j,t)=\int_0^l w_{1(2)}(x,t)\sin\frac{j\pi x}{l}\mathrm{d}x\,, w_{1(2)}(x,t)=\frac{2}{l}\sum_{j=1}^{\infty}V_{1(2)}(j,t)\sin\frac{j\pi x}{l}. \tag{3}$$

Furthermore, we consider only approximate solutions of Eq. 1 by using a finite-dimensional modal space, i.e. j, k = 1,2,…, M. The transformation (3) yelds the following system of ODEs

$$\mu_1\ddot{V}_1(j,t)+\frac{2}{l}\sum_{i=1}^{m}\sum_{k=1}^{\infty}u_i(t)\left[\dot{V}_1(k,t)-\dot{V}_2(k,t)\right]\sin\frac{k\pi a_i}{l}\sin\frac{j\pi a_i}{l}+EI_1\frac{j^4\pi^4}{l^4}V_1(j,t)=P\sin\frac{j\pi vt}{l},$$
$$\mu_2\ddot{V}_2(j,t)+\frac{2}{l}\sum_{i=1}^{m}\sum_{k=1}^{\infty}u_i(t)\left[\dot{V}_2(k,t)-\dot{V}_1(k,t)\right]\sin\frac{k\pi a_i}{l}\sin\frac{j\pi a_i}{l}+EI_2\frac{j^4\pi^4}{l^4}V_2(j,t)=0. \tag{4}$$

Entering the generalized state vector $y(t)\in R^n$, where $y_{4k-3}(t)=V_1(k,t)$, $y_{4k-2}(t)=\dot{V}_1(k,t)$, $y_{4k-1}(t)=V_2(k,t)$, $y_{4k}(t)=\dot{V}_2(k,t)$, $k=1,2,...,n/4=M$, we can formulate the optimal control problem:

$$\text{Minimize}\quad J=\int_0^{t_f=(v/l)}\left[\sum_{k=1}^{n/4}y_{4k-3}(t)\sin\frac{k\pi vt}{l}\right]^2\mathrm{d}t, \tag{5}$$

$$\text{subject to}\quad \dot{\mathbf{y}}(t)=\mathbf{A}\mathbf{y}(t)+\sum_{i=1}^{m}\mathbf{B_i}\,\mathbf{y}(t)u_i(t)+\mathbf{f}(t), \tag{6}$$

$$y_{4k-3}(0)=V_1(k,0),\quad y_{4k-2}(0)=\dot{V}_1(k,0),$$
$$y_{4k-1}(0)=V_2(k,0),\quad y_{4k}(0)=\dot{V}_2(k,0),\quad k=1,2,...,n/4\,, \tag{7}$$

$$u_i(t)\in[0,u_{max}],\quad \forall t\in[0,t_f],\quad i=1,2,...,m\,. \tag{8}$$

Introducing a new state variable $\dot{y}_{n+1}(t)=1$, $y_{n+1}(0)=0$ and rebuilding $A\rightarrow\hat{A}$, $B_i\rightarrow\hat{B}_i$, $f(t)\rightarrow\hat{f}(y)$ in such a way they respect a new variable, we replace (5)-(8) with the autonomous optimal control problem so Maximum Principle can be applied directly. The Hamiltonian function is given by

$$H(\mathbf{y},\mathbf{u},\boldsymbol{\eta}) = \langle \boldsymbol{\eta}, \hat{\mathbf{A}}\mathbf{y} \rangle + \sum_{i=1}^{m} \langle \boldsymbol{\eta}, \hat{\mathbf{B}}_i \mathbf{y} \rangle u_i + \langle \boldsymbol{\eta}, \hat{\mathbf{f}} \rangle - \left[\sum_{k=1}^{n/4} y_{4k-3} \sin \frac{k\pi v t}{l} \right]^2 . \tag{9}$$

The adjoint differential equation and the transversality conditions are as follows:

$$\dot{\boldsymbol{\eta}}(t) = -\frac{\partial H}{\partial \mathbf{y}}, \qquad \boldsymbol{\eta}(t_f) = 0 \ . \tag{10}$$

The Hamiltonian (9) takes the maximum value when the controls equal:

$$u_i(t) = \begin{cases} u_{max}, & \langle \boldsymbol{\eta}(t), \hat{\mathbf{B}}_i \mathbf{y}(t) \rangle > 0 \\ 0, & \langle \boldsymbol{\eta}(t), \hat{\mathbf{B}}_i \mathbf{y}(t) \rangle < 0 \end{cases} . \tag{11}$$

However Maximum Principle is only a nessesary condition for the optimal solution. We suppose the most efficient control method is generated by switching controls.

The implicit solution of stated problem can be solved numerically by the shooting method for instance. However, it can be extremely difficult due to high dimensional problem. For the alternate method we assume a priori a number of switchings for every control and then transform the problem into mathematical programming.

3. Numerical examples

Here, we present a few numerical solutions of optimal control problem stated in the previous section. We use Hooke-Jeeves Direct Search Method, where we consider at least 3 different starting points with 3 reducing step size schemes for each case. The number of switchings was first assumed as 3, then 2, and finally 1 for every control. Reduced number of switching actions is a great advantage from the practical point of view while the cost (Eq. 5) is comparable.

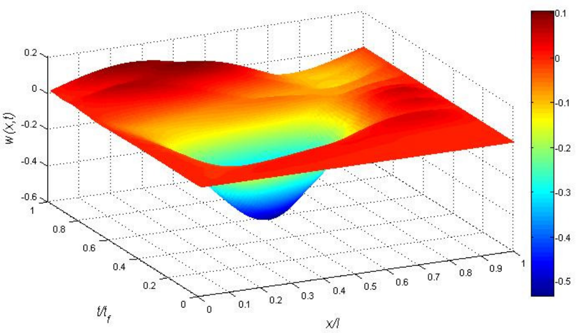

Figure 3. Transverse vibration of controlled beam in space-time domain.

The idea of straight-line passage is based on the principle of a two-sided lever. The first part of the beam which is subjected to a moving load is supported by an active damper placed on the rigid base. The first damper is active while the second one is passive. At this stage, a part of the beam is turned around its centre of gravity, levering the right hand part with a passive damper attached. The temporal increment of displacements on the right hand part of the beam enables us to exploit it during the second stage of the passage. This phenomenon can be observed in the space time-domain (Fig 3).

Below, we present the exemplary optimal deflection trajectories under a moving load in two different cases. In the first one (Fig. 4), we consider a single beam with two active dampers placed in the positions 0.25*l*, 0.75*l*. In the second one (Fig. 5), four dampers placed in positions 0.2*l*, 0.4*l*, 0.6*l*, 0.8*l* are attached to the double beam system. Trajectories for passive cases (all dampers are on) are added for comparison.

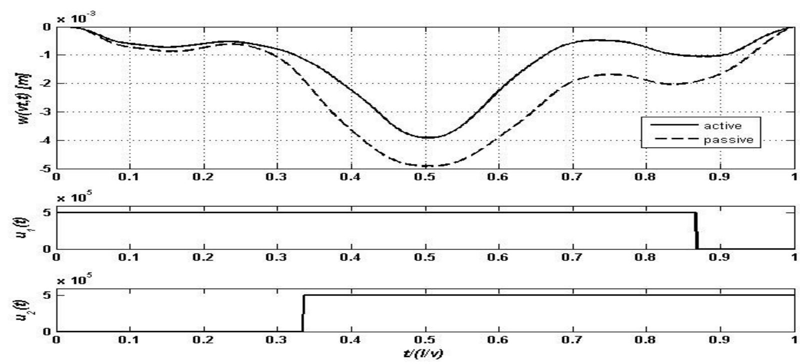

Figure 4. Optimal deflection trajectory and switching controls.

While the cost integrand is calculated with respect to velocities or accelerations of vibrations we do not observe a significant efficiency of the proposed method. High-frequency harmonics included in those trajectories can be reduced by high-frequency switching controls. This is the ongoing research topic of the authors.

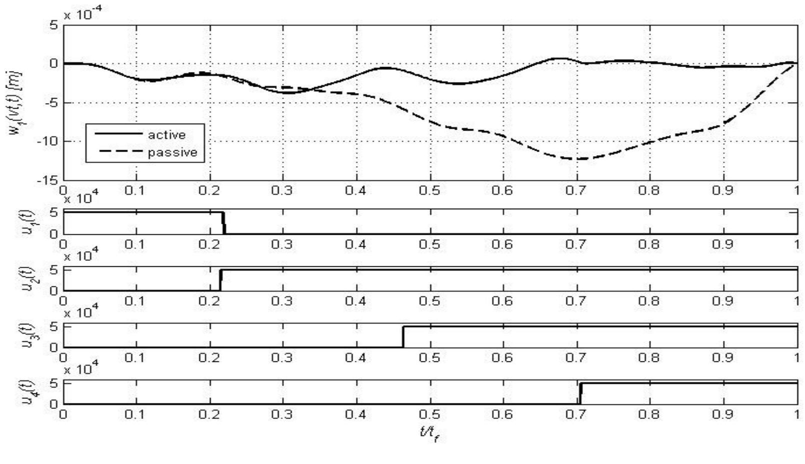

Figure 5. Optimal deflection trajectory and switching controls.

4. Conclusions

In this paper, a semi-active control method for linear carrying structures has been presented. A bang-bang control method has been proposed and its performance has been verified by numerical examples. The best efficiency is obtained at high travel speeds. The controlled system can efficiently decrease the mass of the guideway. The control strategy is simple for practical design. It can be implemented by creating an optimal control map in the memory of the controller. Integration of a neural network with the system will be addressed in future works.

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O semi-aktywnym sterowaniu układów nośnych pod ruchomym obciążeniem.

W pracy przedstawiono wyniki badań półaktywnego sterowania w układach nośnych poddanym obciążeniom ruchomym. Obciążenie zostało przedstawione jako bezinercyjne. Odpowiedź układu została wyznaczona w reprezentacji modalnej. Sformułowano zadanie sterowania optymalnego. Uzasadniono zastosowanie sterowań typu bang-bang opierając się na Twierdzeniu o Maksimum Pontryagina. Proponowana metoda sterowania została zweryfikowana na podstawie przykładów numerycznych. Wykazano przewagę układów sterowanych nad układami tłumienia pasywnego. Opracowana strategia sterowania jest prosta w implementacji i może być atrakcyjnym rozwiązaniem dla inżynierów.