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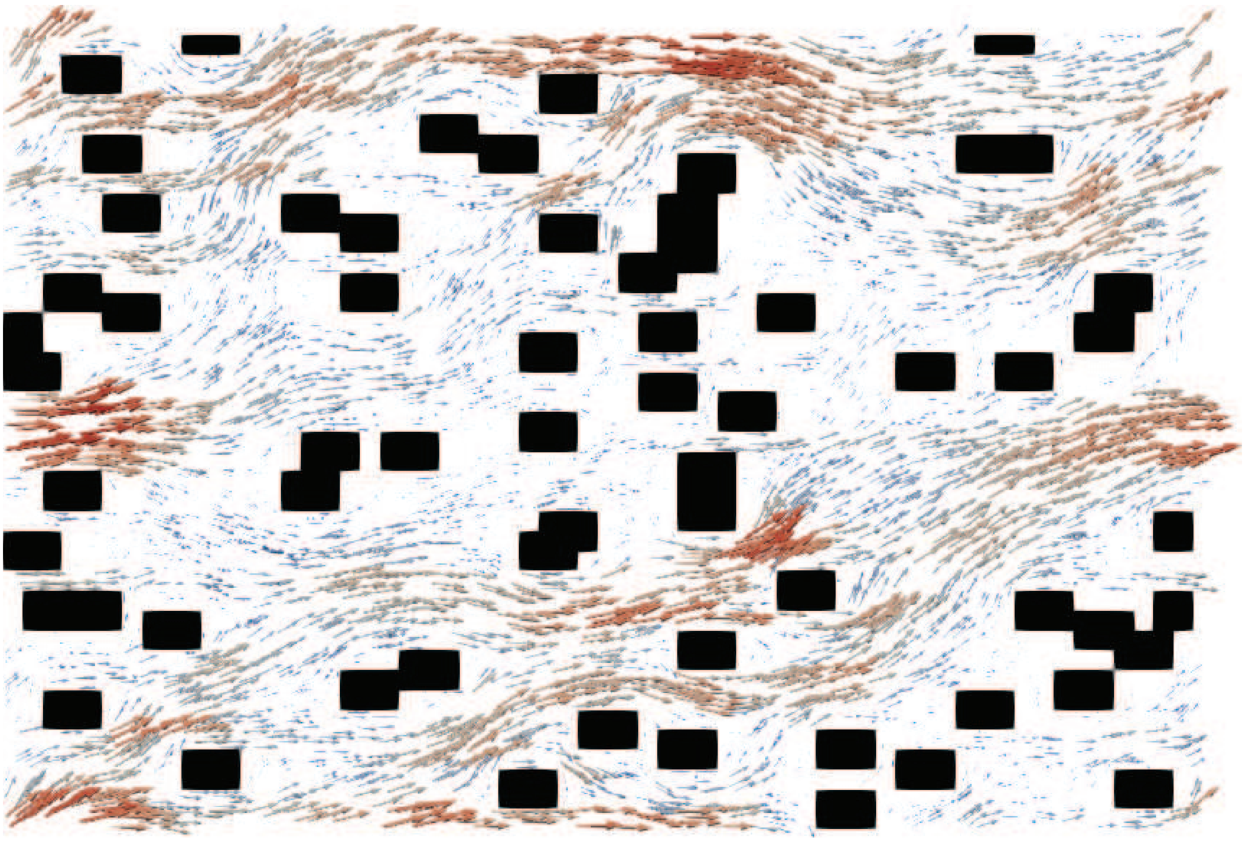

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Plane flow through the porous medium with chessboard-like distribution of permeability

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Theoretical trials to determine the effective properties of heterogeneous porous media are widely performed. In particular, the methods of asymptotical homogenisation are applied, cf. [1, 2]. In this contribution we study a special case of a chessboard-like distribution of permeability coefficients, in which simple algebraic arguments are applied.

First, Darcy's flow is considered, described by the equation.

$$\boldsymbol{v} = -\frac{K}{\eta}\nabla(p+U)$$

Here the vector $\boldsymbol{v}$ is the flow velocity, K – permeability, η - viscosity, p – the pressure, and U – the external potential. We assume the flow being incompressible, it is $\nabla\cdot\boldsymbol{v} = 0$. It is known, from the papers by Keller [3] and Dykhne [4] that the symmetry of equations describing stationary potential flows in the planar systems composed of two constituents, combined with geometrical symmetry of both constituents of the considered medium permits to find the effective conductivity of such a body. This elegant non-perturbative result is known as Keller - Dykhne's formula. There are three main assumptions needed:

(i) considered fields are two-dimensional;
(ii) the flow is stationary and has a potential;
(iii) statistical symmetry and isotropy of the composite is assumed; both components are equivalent in statistical meaning: they have the areas of the same dimension, and can be mutually changed without the change of the whole composite, cf. Figure 1.

If one constituent has the permeability K_1, and the second K_2, then the effective permeability is $\sqrt{K_1\,K_2}$, in agreement with experiments and simulations of Warren and Price [5].

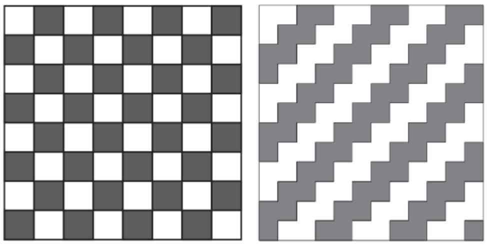

Fig.1. Chessboard-like distributions of the permeability. Examples of different tessellations of more dense (dark) and more rare (light) areas of a rock cross-section, which are in agreement with Keller-Dykhne's geometrical assumption.

Similar result we obtain for the problem in which the flow is described by Brinkman's equation, cf. [6],

$$\boldsymbol{v} = \frac{K}{\eta}\left[-\nabla(p+U) + \eta'\,\Delta\boldsymbol{v}\right]$$

Here the coefficient η' means Brinkman's effective viscosity. This equation permits to satisfy in full the boundary conditions at the interfaces of regions with different permeabilities, what, as it is known, is impossible when Darcy's equation is used. But now, the flow $\boldsymbol{v}$ is not potential, any more. Let us substitute $\boldsymbol{u} = \boldsymbol{v} - (K\,\eta'/\eta)\,\Delta\,\boldsymbol{v}$. Then, Brinkman's equation has the form of Darcy's equation $\boldsymbol{u} = -(K/\eta)\nabla(p+U)$. Such vector $\boldsymbol{u}$ is the potential and divergence-free one, as the vector $\boldsymbol{v}$ is divergence-free. Thus, it satisfies all Keller - Dykhne's formula assumptions. Hence, in the case Brinkman's flow the effective permeability is $\sqrt{K_1\,K_2}$ also.

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