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First-order approach for structural topology optimization with low-cycle fatigue constraints

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ABSTRACT: In the present study topology optimization with constraints on the probability of failure is presented. The proposed approach allows controlling the safety level during the optimization process. The number of cycles to structural failure in the case of low-cycle fatigue problem is chosen as a constraint. The low-cycle fatigue requires elastic-plastic formulation of the problem, because in the optimized structure the phenomenon related to accumulation of plastic deformation is observed. In order to estimate the probability of failure for this numerically complex problem, the First Order Reliability Method (FORM) is proposed. The above mentioned methodology is implemented in object-oriented code written in MATLAB programming language.

Keywords: Topology optimization, Reliability analysis, Low-cycle fatigue, Object oriented programming.

1. INTRODUCTION

In order to estimate a safety level of a structure, the uncertainty of selected parameters must be taken into account. This is particularly important in the topology optimization process involving iterative material removal. This process causes the deterioration of the mechanical properties of the structure and increases the probability of failure. Therefore, it is crucial to represent a quantities such as compliance, stresses, displacements or material fatigue in a probabilistic way. The approach taking into account uncertainty allows to control the probability of failure during the optimization process. In a deterministic approach, the optimal design corresponds to a chosen set of input parameters (loads, material constants), but most often variations of these parameters causes that the structure goes beyond the limit state surface to failure zone. In this work, we propose the methodology that allows engineers to control the probability of failure for selected constraints representing the safety limit of the structure's operation. The probability of that the number of cycles to fatigue failure will not be less than the assumed limit number is presented in this approach.

The numerical example shows optimal topology as well as dependence of fatigue resistance on the assumed volume fraction. All aspects of numerical analysis, including finite element formulation, topology optimization algorithm, reliability analysis, low-cycle fatigue as well as reliability analysis are implemented in our own object-oriented computational environment implemented in MATLAB.

2. METHODOLOGY

The proposed methodology consists of two parts: finite element analysis for determination of stress field and a First Order Reliability Method (FORM) for reliability assessment. Both methods operate in nested loop where at each iteration step update in the topology of the structure is performed and then reliability constraint based on low-cycle fatigue is checked.

2.1. Topology optimization algorithm

This work presents an iterative, nested (double loop) algorithm for topological optimization with regard to reliability constraints. The external topological optimization loop implements a heuristic algorithm of material removal in the least stressed areas. The internal loop controls whether the failure probability is not exceeded. The applied first-order approach (FORM) to the safety control in the topological optimization process is not a high time overhead. Flowchart is presented on the Fig. 1.

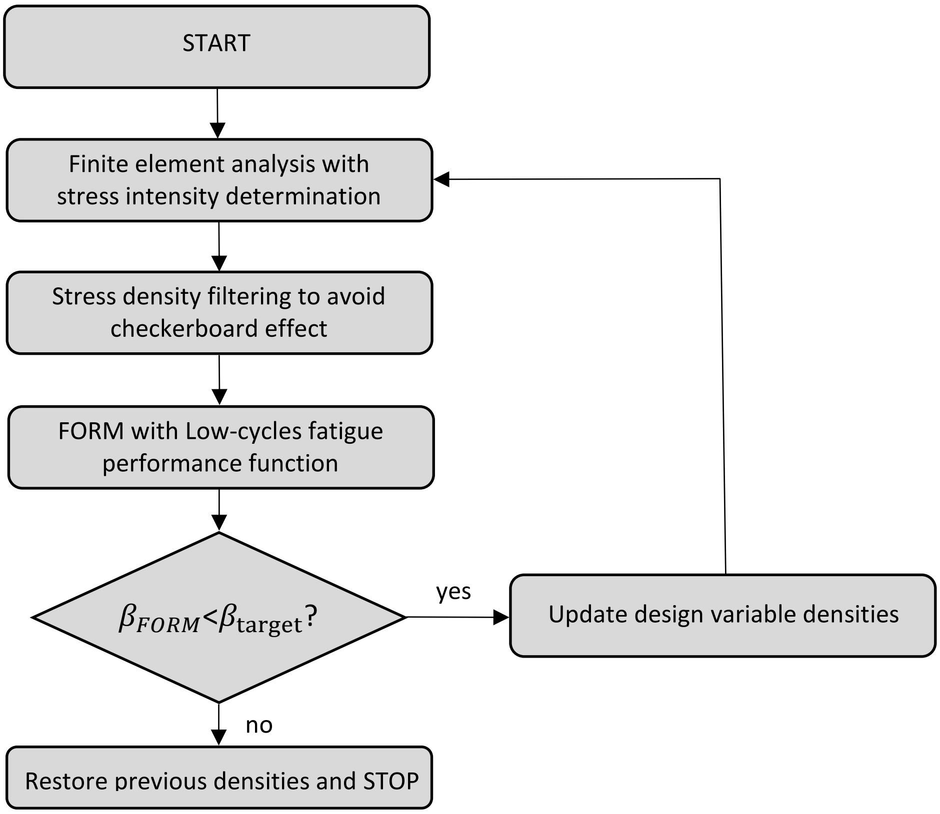

Fig. 1 Flowchart of the proposed method for topology optimization under reliability constraint.

2.2. First Order reliability assessment

The FORM is based on the assumption that the limit state surface is approximated by a line (Fig 2). This assumption works well for low probabilities of failure and this is expected of safe engineering structures. The probability of failure should be usually around 0.001. What makes the FORM, as a fastest method, a good choice for coupling with topological optimization.
The design point represents set of realization of random variable for most probable failure scenario. This is the point which lays on the of the limit state surface and is closest to the mean point. Therefore its determination is optimization task formulated in standardized space as follows:

$$\text{Find} \quad \min\|\mathbf{u^2}\| = \mathbf{u}\cdot\mathbf{u}^T, \qquad (1)$$
$$\text{constrains} \quad g(\mathbf{u}) = 0.$$

The Rackwitz-Fiesler iterative formula has the following form:

$$\mathbf{u}^{(n+1)} = \frac{1}{\|\nabla g(\mathbf{u}^{(n)})\|^2}\left(\nabla g\left(\mathbf{u}^{(n)}\right)^T \mathbf{u}^{(n)} - g\left(\mathbf{u}^{(n)}\right)\right)\nabla g(\mathbf{u}^{(n)}). \quad (2)$$

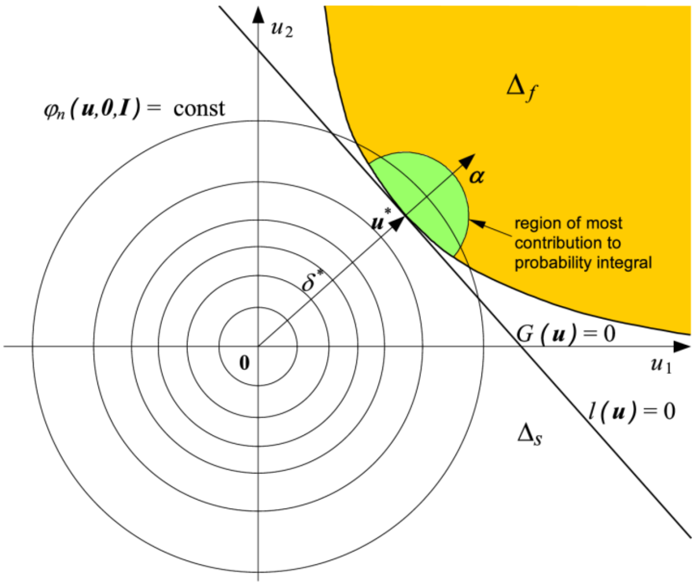

Fig 2. First Order Reliability Method. Design point concept.

Having a most probable point $\mathbf{u}^*$ reliability index can be estimated as follow:

$$\beta_{FORM} = \text{sign}\big(g(\mathbf{0})\big)\|\mathbf{u}^*\|. \quad (3)$$

where β is an Hasofer-Lind's reliability index. The linear estimation of the probability of failure has the form:

$$P_{FORM} = \Phi(-\beta_{FORM}), \quad (4)$$

2.3. Low cycle fatigue

Low-cycle fatigue is based on the accumulation of plastic failure in each load cycle. Multi-level load program denoting a different amplitude in each level of cycles is assumed. It consists of 13 levels of cycles. In the first, the longest one, 38,000 cycles is performed. Next, 11 levels have 100 cycles of loads in each. In the last 13th level the number of cycles to fatigue failure have to be determined. Having the number of cycles at each level and the value of the damage increment per cycle, the total value of the accumulated plastic damage D for 12 levels can be computed according the formula:

$$D = (38000 - N_{\text{D}})\frac{\delta D}{\delta N}^{(1)} + 100\frac{\delta D}{\delta N}^{(2)} + \cdots + 100\frac{\delta D}{\delta N}^{(12)} \quad (5)$$

Assuming the value of plastic damage representing fatigue damage D_{C} the number of cycles to failure can be determined:

$$N_r^{(13)} = \frac{D_{\text{C}} - D}{\frac{\delta D}{\delta N}^{(13)}} \quad (6)$$

3. NUMERICAL EXAMPLE

The objective function for the reliability problem is $g(x) = N_{\mathrm{r}} - N_{\mathrm{TARGET}}$. An unsafe state is understood as a structure for which the number of fatigue cycles is lower than $N_{\mathrm{TARGET}} = 20000$. The target reliability index is $\beta_{\mathrm{target}} = 2.0$. The probabilistic data of random variables are presented in the table below:

varable	*mean*	*St. dev*
E [GPa]	200	10%
σ_y [MPa]	180	10%
P [kN]	10	10%

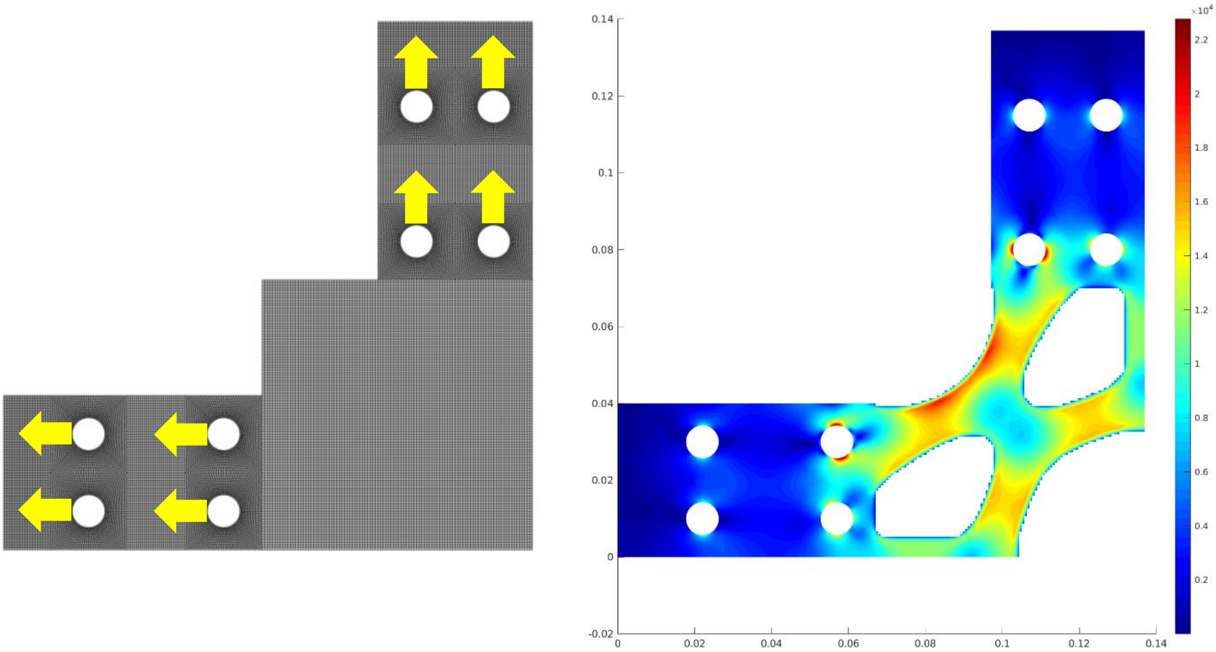

Fig 3. FEM model and topology of analyzed specimen (the specimen model is taken from:[2]).

4. CONCLUSIONS

Simple algorithm for density based topology optimization is efficient tool to solve advanced optimization problem with reliability constraints. The number of cycles of the low-cycle fatigue is well suited for the definition of the performance function and for the safety control of elastoplastic structures loaded with cyclic loads. FORM is able to efficiently and reasonably fast estimate the probability of failure in the complex engineering problem such as topological optimization combined with low-cycle fatigue.

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