POLSKA AKADEMIA NAUK
INSTYTUT PODSTAWOWYCH PROBLEMÓW TECHNIKI

ARCHIWUM MECHANIKI STOSOWANEJ

ARCHIVES DE MECANIQUE APPLIQUEE

6
XIII

VOL. XIII WARSZAWA 1961 No 6
PAŃSTWOWE WYDAWNICTWO NAUKOWE

ARCHIWUM
MECHANIKI
STOSOWANEJ
6, 13 (1961)

THE PROBLEM OF CONCENTRATION OF PERIODIC THERMAL STRESSES AT CYLINDRICAL HOLES AND SPHERICAL CAVITIES IN UNIFORM PLANE HEAT FLOW

JÓZEF IGNACZAK and WITOLD NOWACKI (WARSZAWA)

The concentration problem of stresses produced by a steady-state heat flow past a cylindrical hole and a spherical cavity has been solved by A. L. FLORENCE and J. N. GOODIER, [1]. The problem considered in the present paper is that of heat flow varying harmonically in time. Retaining the inertia terms in the fundamental equations, the problem is treated as dynamic, the coupling between the temperature and strain field not, however, being introduced.

Our assumptions are based on the classical linear theory of thermal stress, [2], [3]. The material constants both mechanical and thermal are assumed to be independent of the temperature. It is also assumed that the cylindrical hole and spherical cavity are impermeable to heat and free from stress.

The solution method consists in superposing the solutions of the equations of periodic heat flow, and those of periodic thermoelastic vibration expressed in displacements, [4], [5].

1. The Heat Flow in the Neighbourhood of a Cylindrical Hole

Let us consider the problem of heat flow past a cylindrical hole of radius a. It is assumed that the hole is parallel to the plane over which a periodic heat source is distributed in a uniform manner.

If a plane periodic heat source acts in the plane $x=0$ of a Cartesian system x, y, z in an infinite elastic solid without hole, the temperature field T_1 satisfies the equation

$$\Box_3^2 T_1^* = -\frac{q}{\lambda_0}\delta(x), \qquad T_1 = T_1^* e^{i\omega t}, \tag{1.1}$$

where

$$\Box_3^2 = \nabla^2 + h_3^2, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad h_3 = \frac{1}{i}\sqrt{\frac{\omega i}{\varkappa}}, \quad i^2 = -1;$$

and q in the Eq. (1.1) is the uniform intensity of the heat source per unit area; λ_0, $\varkappa$ — coefficients determining the heat conductivity, δ — the Dirac function and ω the frequency.

The solution (1.1) has the form:

$$T_1(h_3) = q\frac{e^{i\omega t}}{2\lambda_0}\frac{e^{-ih_3x}}{ih_3} \tag{1.2}$$

To determine the modification of the heat flow due to the presence of the cylindrical hole, impermeable to heat, let us consider the solution:

$$e^{-ih_3x} = \sum_{n=0}^{\infty}(-1)^n \delta_n I_n(ih_3r)\cos n\theta, \quad x = r\cos\theta, \tag{1.3}$$

$$\delta_n = \begin{Bmatrix} 1 & \text{for} & n=0 \\ 2 & \text{for} & n\geqslant 1 \end{Bmatrix},$$

where $I_n = I_n(\alpha)$, a modified Bessel function of the first kind, and (r, θ) are plane polar coordinates with the pole located on the axis of the cylinder. Therefore $x = r\cos\theta, y = r\sin\theta,\ z = z$.

To satisfy the condition of thermal impermeability of the cylinder, we must superpose on every term of the sum (1.3) a component containing a Bessel function of the second kind. The solution T_2 satisfying the equation

$$\Box_3^2 T_2^* = 0 \tag{1.4}$$

should be superposed over the solution (1.2).

The final form of the solution of the heat conduction problem, satisfying the condition $[\partial T/\partial r]_{r=0} = 0$ is, [6]:

$$T = T_1(h_3) - T_2(h_3), \tag{1.5}$$

where

$$\begin{cases} T_1(h_3) = \dfrac{qe^{i\omega t}}{2\lambda_0}\displaystyle\sum_{n=0}^{\infty}(-1)^n\delta_n\frac{I_n(ih_3r)}{ih_3}\cos n\theta, \\ T_2(h_3) = \dfrac{qe^{i\omega t}}{2\lambda_0}\displaystyle\sum_{n=0}^{\infty}(-1)^n\delta_n\frac{I_n'(ih_3a)}{K_n'(ih_3a)}\frac{K_n(ih_3r)}{ih_3}\cos n\theta. \end{cases} \tag{1.6}$$

If the frequency ω of the periodic heat flow decreases $(\omega \to 0)$, the form of the flow approaches that of a steady-state flow. This principle may be applied to every separate term of the series obtained.

In the case of plane heat source under consideration, this principle becomes somewhat less simple. For, taking into consideration the development (1.6) and development of the functions $I_n(x)$, $K_n(x)$ in the neighbourhood of $x = 0$, we have

$$I_n(x) = \left(\frac{x}{2}\right)^n\frac{1}{n!}+\ldots, \qquad K_n(x) = \frac{(n-1)!}{2}\left(\frac{2}{x}\right)^n+\ldots, \qquad n = 0, 1, 2,\ldots$$

It becomes evident that a steady-state flow furnishes the second term of the series ($n = 1$). For $n \geqslant 2$, the components vanish if $\omega \to 0$, and the first term becomes infinitely large.

The difference (1.5) gives for $\omega \to 0$ the steady-state flow considered in Ref. [1]:

$$T_{\omega=0} = -\frac{q}{2\lambda_0}\left(r + \frac{a^2}{r}\right)\cos\theta \,. \tag{1.7}$$

To find the solution of the thermoelastic problem, the method of potential of thermoelastic displacement Φ will be used. This potential satisfies the equation:

$$\begin{gathered} \Box_1^2 \Phi^* = \vartheta_0 T, \quad \Box_1^2 = \nabla^2 + h_1^2, \quad h_1 = \omega / c_1 \,; \\ c_1^{-2} = \frac{\varrho}{\lambda + 2\mu}, \quad \vartheta_0 = \frac{3\lambda + 2\mu}{\lambda + 2\mu}\, \alpha_t \,, \end{gathered} \tag{1.8}$$

where ϱ is the density of the medium, λ, μ — Lamé's constants, and α_t — the coefficient of thermal dilatation.

Making use of the Eq. (1.1), and performing the operation $\Box_3^2$ on the Eq. (1.8), we find:

$$\Box_1^2 \Box_3^2 \Phi^* = -\vartheta_0 \frac{q}{\lambda_0}\, \delta(x) . \tag{1.9}$$

Assuming the function Φ^* in a form analogous to (1.5)

$$\Phi^* = \Phi_1^* - \Phi_2^* , \tag{1.10}$$

$$\Box_1^2 \Box_3^2 \Phi_1^* = -\frac{\vartheta_0 q}{\lambda_0}\, \delta(x) , \qquad \Box_1^2 \Box_3^2 \Phi_2^* = 0 , \tag{1.11}$$

we obtain from (1.11):

$$\Phi_1^* = -\frac{\vartheta_0 q}{\lambda_0} \frac{1}{\Box_1^2 \Box_3^2}\, \delta(x) = -\frac{\vartheta_0 q}{\lambda_0} \frac{1}{h_1^2 - h_3^2} \left(\frac{1}{\Box_3^2} - \frac{1}{\Box_1^2}\right) \delta(x) . \tag{1.12}$$

Since $-\dfrac{q}{\lambda_0} \dfrac{1}{\Box_3^2}\, \delta(x) = T_1^*(h_3)$, by virtue of (1.1), therefore

$$\Phi_1^* = \frac{\vartheta_0}{h_1^2 - h_3^2} \left[T_1^*(h_3) - T_1^*(h_1)\right] . \tag{1.13}$$

The function Φ_2^* is chosen so that Φ^* satisfies (1.8), and the condition of vanishing of the normal derivative on the surface of the cylinder. We find:

$$\Phi^* = \frac{\vartheta_0}{h_1^2 - h_3^2} \{[T_1^*(h_3) - T_1^*(h_1)] - [T_2^*(h_3) - T_2^*(h_1)]\} . \tag{1.14}$$

It can easily be shown, by considering the equation

$$\Box_1^2 = \Box_3^2 + (h_1^2 - h_3^2), \tag{1.15}$$

and the Eqs. (1.1) and (1.4), that (1.14) satisfies the equation of the potential (1.8) in the region considered. The stress produced by the potential and denoted by $[\bar{S}]$, will be computed from the equation

$$\bar{\sigma}_{ij} = 2\mu(\Phi_{,ij} - \Phi_{,kk}\delta_{ij}) + \varrho\delta_{ij}\ddot{\Phi}, \quad i, j = 1, 2, 3. \tag{1.16}$$

The tensor notation for space and time differentiation in (1.16) concerns the Cartesian coordinates ($x_1 = x$, $x_2 = y$, $x_3 = z$). μ — is the shear modulus and δ_{ij} the Kronecker's delta. The dot denotes differentiation with respect to time. In the cylindrical coordinates, we obtain from (1.16):

$$\begin{cases} \bar{\sigma}_{rr} = -2\mu r^{-2}(r\Phi_{,r} + \Phi_{,\theta\theta}) + \varrho\ddot{\Phi}, \\ \bar{\sigma}_{r\theta} = -2\mu r^{-2}(\Phi - r\Phi_{,r})_{,\theta}. \end{cases} \tag{1.17}$$

Bearing in mind the relation

$$\Phi^*_{,r}|_{r=a} = 0, \tag{1.18}$$

we obtain

$$\begin{cases} \bar{\sigma}^*_{rr}|_{r=a} = [-2\mu a^{-2}\Phi^*_{,\theta\theta} - \varrho\omega^2\Phi^*]_{r=a}, \\ \bar{\sigma}^*_{r\theta}|_{r=a} = [-2\mu a^{-2}\Phi^*_{,\theta}]_{r=a}. \end{cases} \tag{1.19}$$

Thus, the load on the cylinder produced by the potential Φ^* can by virtue of (1.6) be expressed in the form of the following series

$$\begin{cases} \bar{\sigma}^*_{rr}|_{r=a} = \mu C \sum_0^\infty (-1)^n \delta_n \left(\frac{1}{2} a^2 h_2^2 - n^2\right) [\chi_n(ih_3 a) - \chi_n(ih_1 a)] \cos n\theta, \\ \bar{\sigma}^*_{r\theta}|_{r=a} = \mu C \sum_0^\infty (-1)^n \delta_n [\chi_n(ih_3 a) - \chi_n(ih_1 a)](\cos n\theta)_{,\theta}, \end{cases} \tag{1.20}$$

where

$$\chi_n(z) = 1/z^2 K_n'(z), \quad C = q\vartheta_0/a\lambda_0(h_1^2 - h_3^2), \quad h_2 = \omega/c_2, \quad c_2^{-2} = \varrho/\mu.$$

For the derivation of (1.20), the following relation was used

$$1 = [K_n(z) I_n'(z) - I_n(z) K_n'(z)] z. \tag{1.21}$$

To suppress the load $\bar{\sigma}^*_{rr}|_{r=a}$ and $\bar{\sigma}^*_{r}|_{r\theta=a}$, we add to every term of the series (1.20) an appropriate additional component produced by two vector fields satisfying the Lamé condition of periodic vibration without the temperature. The total solution $[S^c]$ for the additional load has the form

$$[S^c] = \sum_{n=0}^\infty \{x_n^c[S_n^{1c}] + y_n^c[S^{2c}]\}. \tag{1.22}$$

For the displacements u_r^c, u_θ^c and the stresses σ_{rr}^c, $\sigma_{r\theta}^c$, we have the equations:

$$(1.23)\qquad [S_n^{1c}]:\begin{cases} u_r^{1c}=-h_1^{-2}[H_n^{(2)}(h_1r)]_{,r}\cos n\theta,\\ u_\theta^{1c}=-h_1^{-2}r^{-1}H_n^{(2)}(h_1r)(\cos n\theta)_{,\theta},\\ \sigma_{rr}^{1c}=\mu A_n^c(\omega r)\cos n\theta,\\ \sigma_{r\theta}^{1c}=\mu C_n^c(\omega r)(\cos n\theta)_{,\theta};\end{cases}$$

$$(1.24)\qquad [S_n^{2c}]:\begin{cases} u_r^{2c}=-r_n^{-1}n^2H_n^{(2)}(h_2r)\cos n\theta,\\ u_\theta^{2c}=-[H_n^{(2)}(h_2r)]_{,r}(\cos n\theta)_{,\theta},\\ \sigma_{rr}^{2c}=\mu B_n^c(\omega r)\cos n\theta,\\ \sigma_{r\theta}^{2c}=\mu D_n^c(\omega r)(\cos n\theta)_{,\theta},\end{cases}$$

where

$$(1.25)\qquad \begin{cases} A_n^c(\omega r)=(h_1r)^{-2}\{[(h_2r)^2-2n^2]H_n^{(2)}(h_1r)+2h_1rH_n'^{(2)}(h_1r)\},\\ B_n^c(\omega r)=2r^{-2}n^2\{H_n^{(2)}(h_2r)-h_2rH_n'^{(2)}(h_2r)\},\\ C_n^c(\omega r)=2(h_1r)^{-2}[H_n^{(2)}(h_1r)-h_1rH_n'^{(2)}(h_1r)],\\ D_n^c(\omega r)=r^{-2}\{[(h_2r)^2-2n^2]H_n^{(2)}(h_2r)+2h_2rH_n'^{(2)}(h_2r)\}.\end{cases}$$

The additional displacement fields assumed in the Eqs. (1.23) and (1.24), in agreement with [4], are such that the plane conditions of radiation into infinity are satisfied. Thus, the factor depending on the radius r contains Hankel's function $H_n^{(2)}(hr)$ of the second kind, [7]. The condition of no load on the cylindrical surface determining two sequences of coefficients $\{x_n^c\}$, $\{y_n^c\}$ appearing in Eq. (1.22) has the form

$$(1.26)\qquad \begin{cases} x_n^cA_n^c(\omega a)+y_n^cB_n^c(\omega a)+C(-1)^n\delta_n\left(\frac{1}{2}a^2h_2^2-n^2\right)[\chi_n(ih_3a)-\chi_n(ih_1a)=0,\\ x_n^cC_n^c(\omega a)+y_n^cD_n^c(\omega a)+C(-1)^n\delta_n[\chi_n(ih_3a)-\chi_n(ih_1a)]=0.\end{cases}$$

Hence,

$$(1.27)\qquad \begin{cases} x_n^c=(\Delta_n^c)^{-1}C(-1)^n\delta_n[\chi_n'(ih_3a)-\chi_n(ih_1a)]\left[B_n^c(\omega a)-\left(\frac{1}{2}a^2h_2^2-n^2\right)D_n^c(\omega a)\right],\\ y_n^c=(\Delta_n^c)^{-1}C(-1)^n\delta_n[\chi_n(ih_3a)-\chi_n(ih_1a)]\left[A_n^c(\omega a)-\left(\frac{1}{2}a^2h^2-n^2\right)C_n^c(\omega a)\right],\end{cases}$$

where

$$(1.28)\qquad \Delta_n^c(\omega a)=A_n^c(\omega a)D_n^c(\omega a)-B_n^c(\omega a)C_n^c(\omega a).$$

The resultant stress due to homogeneous plane heat flow past the cylindrical hole free from load is obtained by superposition:

$$(1.29)\qquad [S]=[\bar{S}]+[S^c].$$

According to the conditions to be satisfied by the temperature field and concerning the tendency of a periodic heat flow to the relevant steady-state-flow for $\omega \to 0$, and the decrease of the terms of the series for $n \geqslant 2$, it suffices, for sufficiently small ω, to take a few terms of the series (1.14) and (1.22).

2. Homogeneous Periodic Plane Heat Flow past a Spherical Cavity

The origin of the spherical coordinate system (R, θ, φ) is assumed at the centre of the sphere, the axis $z = R\cos\theta$ being normal to the plane of the heat flow. Proceeding in the same manner as in the case of a cylinder, and making use of the expansion

$$e^{-ih_3 R\cos\theta} = \sqrt{2\pi} \sum_{n=0}^{\infty} (-1)^n \left(n+\frac{1}{2}\right) \frac{I_{n+1/2}(ih_3R)}{(ih_3R)^{1/2}} \mathcal{L}_n(\cos\theta), \tag{2.1}$$

where $\mathcal{L}_n = \mathcal{L}_n(\cos\theta)$ is Legendre's polynomial, we find the following expression for the temperature field:

$$T = T_1(h_3) - T_2(h_3), \tag{2.2}$$

where

$$\begin{cases} T_1(h_3) = \dfrac{q\sqrt{2\pi}}{2\lambda_0} e^{i\omega t} \displaystyle\sum_{n=0}^{\infty} (-1)^n \left(n+\frac{1}{2}\right) \frac{1}{ih_3} \frac{I_{n+1/2}(ih_3R)}{(ih_3R)^{1/2}} \mathcal{L}_n(\cos\theta), \\ T_2(h_3) = \dfrac{q\sqrt{2\pi}}{2\lambda_0} e^{i\omega t} \displaystyle\sum_{n=0}^{\infty} (-1)^n \left(n+\frac{1}{2}\right) \frac{[I]'_{n+1/2}(ih_3a)}{[K]'_{n+1/2}(ih_3a)} \frac{K_{n+1/2}(ih_3R)}{ih_3(ih_3R)^{1/2}} \mathcal{L}_n(\cos\theta), \end{cases} \tag{2.3}$$

and

$$\begin{cases} [I]'_n(z) = I'_n(z)z - \dfrac{1}{2} I_n(z), \\ [K]'_n(z) = K'_n(z)z - \dfrac{1}{2} K_n(z). \end{cases} \tag{2.4}$$

The function (2.2) satisfies the thermal impermeability condition of the surface $R = a$ of the cavity

$$[\partial T/\partial R]_{R=a} = 0, \tag{2.5}$$

and the functions T_1^*, T_1^* satisfy the respective equations (1.1) and (1.4), where the variable x in (1.1) should be replaced by $z = R\cos\theta$.

Passing to the limit for $\omega \to 0$, we find that, similarly to the case of the cylinder, the steady-state heat flow is furnished by the second term of the series (2.3)

$(n = 1)$. The relevant equation for the steady-state flow in the neighbourhood of the cylinder has the form, [1]:

$$T_{\omega=0} = -\frac{q}{2\lambda_0}\left(R+\frac{1}{2}\frac{a^3}{R^2}\right)\cos\theta . \tag{2.6}$$

The potential function for Φ is taken, for the sphere, from (1.14) and the stresses from (1.16). We find, in spherical coordinates:

$$\begin{cases} \bar{\sigma}_{RR} = 2\mu[-2R^{-1}\Phi_{,R} - R^{-2}\operatorname{cosec}\theta(\sin\theta\Phi_{,\theta})_{,\theta}] + \varrho\ddot{\Phi}, \\ \bar{\sigma}_{R\theta} = 2\mu R^{-1}(\Phi_{,R} - R^{-1}\Phi_{,\theta}). \end{cases} \tag{2.7}$$

Taking into consideration the condition (2.5), we obtain on the spherical surface:

$$\begin{cases} \bar{\sigma}^*_{RR}\,|_{R=a} = [-2\mu a^{-2}\operatorname{cosec}\theta(\sin\theta\Phi^*_{,\theta})_{,\theta} - \varrho\omega^2\Phi^*]_{R=a}, \\ \bar{\sigma}^*_{R\theta}\,|_{R=a} = [-2\mu a^{-2}\Phi^*_{,\theta}]_{R=a}. \end{cases} \tag{2.8}$$

Making use of the relation

$$\begin{cases} \operatorname{cosec} e\theta\,\{\sin\theta[\mathcal{L}_n(\cos\theta)]_{,\theta}\}_{,\theta} = -n(n+1)\,\mathcal{L}_n(\cos\theta), \\ 1 = K_n(z)\,[I]'_n(z) - I_n(z)\,[K]'_n(z), \end{cases} \tag{2.9}$$

and of the Eqs. (1.14) and (2.3), we can represent the load on the sphere, according to (2.8), by the series:

$$\begin{cases} \bar{\sigma}^*_{RR}|_{R=a} = \mu S\sum_{n=0}^{\infty}(-1)^n\left(n+\frac{1}{2}\right)\left[\frac{1}{2}a^2h_2^2 - n(n+1)\right]\times \\ \qquad\qquad \times[\Psi_{n+1/2}(ih_3a) - \Psi_{n+1/2}(ih_1a)]\,\mathcal{L}_n(\cos\theta), \\ \bar{\sigma}^*_{R\theta}|_{R=a} = \mu S\sum_{n=0}^{\infty}(-1)^n\left(n+\frac{1}{2}\right)[\Psi_{n+1/2}(ih_3a) - \Psi_{n+1/2}(ih_1a)]\,[\mathcal{L}_n(\cos\theta)]_{,\theta}, \end{cases} \tag{2.10}$$

where

$$\Psi_n(z) = 1/z^{3/2}[K]'_n(z); \qquad S = q\vartheta_0\sqrt{2\pi}/a\lambda_0(h_1^2 - h_3^2).$$

To satisfy the condition of zero load on the surface $R = a$, an additional periodic state should be superposed over the state produced by the potential Φ. The additional solution $[S^s]$ has the form, [5],

$$[S^s] = \sum_{n=0}^{\infty}\{x_n^s[S_n^{1s}] + y_n^s[S_n^{1s}]\}, \tag{2.11}$$

where the corresponding additional components are described by the equations:

$$[S_n^{1s}]: \begin{cases} u_R^{1s} = -h_1^{-2}[R^{-1/2}H^{(2)}_{n+1/2}(h_1R)]_{,R}\,\mathcal{L}_n(\cos\theta), \\ u_\theta^{1s} = -h_1^{-2}R^{-1}[R^{1/2}H^{(2)}_{n+1/2}(h_1R)]\,[\mathcal{L}_n(\cos\theta)]_{,\theta}, \\ \sigma_{RR}^{1s} = \mu A_n^s(\omega R)\,\mathcal{L}_n(\cos\theta), \\ \sigma_{R\theta}^{1s} = \mu C_n^s(\omega R)\,[\mathcal{L}_n(\cos\theta)]_{,\theta}; \end{cases} \tag{2.12}$$

$$(2.13)\qquad [S_n^{2s}]:\quad \begin{cases} u_R^{2s} = R^{-1}[R^{-1/2}H^{(2)}_{n+1/2}(h_2R)]\,n(n+1)\,\mathcal{L}_n(\cos\theta)\,, \\ u_\theta^{2s} = R^{-1}[R^{1/2}H^{(2)}_{n+1/2}(h_2R)]_{,R}\,[\mathcal{L}_n(\cos\theta)]_{,\theta}\,, \\ \sigma_{RR}^{2s} = \mu(n+1)nB_n^s(\omega R)\mathcal{L}_n(\cos\theta)\,, \\ \sigma_{R\theta}^{2s} = \mu D_n^s(\omega R)[\mathcal{L}_n(\cos\theta)]_{,\theta}\,, \end{cases}$$

where

$$(2.14)\quad \begin{cases} A_n^s(\omega R) = R^{-1/2}(h_1R)^{-2}\{[(h_2R)^2-2(n-1)(n+2)]H^{(2)}_{n+1/2}(h_1R)+ \\ \qquad\qquad +4h_1RH'^{(2)}_{n+1/2}(h_1R)-6H^{(2)}_{n+1/2}(h_1R)\}\,, \\ B_n^s(\omega R) = R^{-5/2}[2(h_2R)H'^{(2)}_{n+1/2}(h_2R)-3H^{(2)}_{n+1/2}(h_2R)]\,, \\ C_n^s(\omega R) = -R^{-1/2}(h_1R)^{-2}[2h_1RH'^{(2)}_{n+1/2}(h_1R)-3H^{(2)}_{n+1/2}(h_1R)], \\ D_n^s(\omega R) = -R^{-5/2}\{[(h_2R)^2-2(n-1)(n+2)]H^{(2)}_{n+1/2}(h_2R)+ \\ \qquad\qquad +2(h_2R)H'^{(2)}_{n+1/2}(h_2R)-3H^{(2)}_{n+1/2}(h_2R)\}. \end{cases}$$

The condition of no load on the spherical surface $R = a$ has the form:

$$(2.15)\quad \begin{cases} x_n^s = (\Delta_n^s)^{-1}S(-1)^n\left(n+\dfrac{1}{2}\right)[\Psi_{n+1/2}(ih_3a)-\Psi_{n+1/2}(ih_1a)]\Big\{n(n+1)B_n^s(\omega a)- \\ \qquad\qquad -\left[\dfrac{1}{2}a^2h_2^2-n(n+1)\right]D_n^s(\omega a)\Big\}, \\ y_n^s = (\Delta_n^s)^{-1}S(-1)^n\left(n+\dfrac{1}{2}\right)[\Psi_{n+1/2}(ih_3a)-\Psi_{n+1/2}(ih_1a)]\Big\{A_n^s(\omega a)- \\ \qquad\qquad -\left[\dfrac{1}{2}a^2h_2^2-n(n+1)\right]C_n^s(\omega a)\Big\}, \end{cases}$$

where $\Delta_n^s(\omega a) = A_n^s(\omega a)D_n^s(\omega a)-n(n+1)B_n^s(\omega a)C_n^s(\omega a)$. In Eqs. (2.12), (2.13) the additional displacement field $[S^s]$ is assumed so that the space condition of radiation [7] is satisfied, therefore the factor depending on the radius R involves Hankel's function $H^{(2)}_{n+1/2}(hR)$.

References

[1] A.L. Florence, J.N. Goodier, *Thermal Stresses at Cylindrical Cavities and Circular Holes in Uniform Heat Flow*, J. Appl. Mech., **26** (1959).

[2] H. Parkus, *Instationäre Wärmespannungen*, Springer, Wien 1959.

[3] W. Nowacki, *Problems of Thermoelasticity* (in Polish), Warsaw 1960.

[4] J. Ignaczak, *A Plane Dynamic Problem of Thermoelasticity Concerning a Circular Hole*, Bull. Acad. Pol. Sci., Cl.IV, 7-8, **7** (1959).

[5] J. IGNACZAK, *Dynamic Thermoelastic Problem of a Spherical Cavity*, Arc. Mech. Stos., 4, **11** (1959).

[6] G. GREEN, *Corresponding Problems in Periodic and Steady Flow* (Part II), Phil. Mag. S. 7., Vol. 33, No 218, (1942).

[7] В. Д. КУПРАДЗЕ, *Граничные задачи теории колебаний и интегральные уравнения*, Москва - Ленинград 1950.

Streszczenie

PROBLEM PERIODYCZNEJ KONCENTRACJI NAPRĘŻEŃ CIEPLNYCH PRZY CYLINDRYCZNYCH I KULISTYCH OTWORACH POZOSTAJĄCYCH W JEDNORODNYM PŁASKIM STRUMIENIU CIEPŁA

W pracy rozpatrzono działanie periodycznego w czasie płaskiego źródła ciepła, które w nieograniczonym ciele sprężystym napotyka na cylindryczny lub też kulisty otwór, nieprzenikliwy dla ciepła oraz swobodny od obciążeń.

W podstawowych równaniach termosprężystości (bez sprzężenia pola temperatury i deformacji) uwzględniamy człony inercyjne traktując zagadnienie jako dynamiczne. Metoda rozwiązania polega na superpozycji rozwiązań równań przewodnictwa cieplnego i rozwiązań równań przemieszczeniowych termosprężystości.

Резюме

ЗАДАЧА О ПЕРИОДИЧЕСКОЙ КОНЦЕНТРАЦИИ ТЕРМИЧЕСКИХ НАПРЯЖЕНИЙ ПРИ ЦИЛИНДРИЧЕСКИХ И ШАРООБРАЗНЫХ ОТВЕРСТИЯХ ПОД ВЛИЯНИЕМ ОДНОРОДНОГО ПЛОСКОГО ПОТОКА ТЕПЛА

Рассматривается действие периодического во времени плоского источника тепла, который в бесконечном упругом теле встречает цилиндрическое или шарообразное отверстие непроницаемое для тепла и свободное от нагрузок.

В основных уравнениях термоупругости (без сопряжения температурного поля и деформации) учитываются инерционные члены, рассматривая задачу как динамическую. Метод решения состоит в суперпозиции решений уравнений теплопроводности и решений уравнений термоупругости в перемещениях.

DEPARTMENT OF MECHANICS OF CONTINUOUS MEDIA
IBTP POLISH ACADEMY OF SCIENCES

Received Juny 26, 1961.