BULLETIN
DE
L'ACADÉMIE POLONAISE DES SCIENCES

SÉRIE DES SCIENCES TECHNIQUES

Volume XVII Numéro 2

VARSOVIE 1969

BULLETIN DE L'ACADÉMIE
4. POLONAISE DES SCIENCES
Série des sciences techniques
Volume XVII, No. 2 — 1969

APPLIED MECHANICS
(ELASTICITY)

Generation of Waves in an Infinite Micropolar Elastic Solid Body I.

by

W. NOWACKI and W. K. NOWACKI

Presented by W. NOWACKI on November 18, 1968

1. Introduction

The aim of the present paper is to find a method for determining the displacement $\mathbf{u}(\mathbf{x}, t)$ and rotation $\boldsymbol{\omega}(\mathbf{x}, t)$ field formed in an infinite micropolar elastic medium under the effect of action of body forces and body couples.

We shall consider an isotropic, homogeneous, centrosymmetric elastic medium wherein the displacement and rotation fields are described by the linearized equations of asymmetric elasticity

$$(1.1)\qquad (\varkappa+\alpha)\,\nabla^2\mathbf{u}+(\lambda+\varkappa-\alpha)\,\text{grad div}\,\mathbf{u}+2\alpha\,\text{rot}\,\boldsymbol{\omega}+\mathbf{X}=\varrho\ddot{\mathbf{u}},$$

$$(1.2)\qquad (\gamma+\varepsilon)\,\nabla^2\boldsymbol{\omega}+(\gamma+\beta-\varepsilon)\,\text{grad div}\,\boldsymbol{\omega}-4\alpha\boldsymbol{\omega}+2\alpha\,\text{rot}\,\mathbf{u}+\mathbf{Y}=J\ddot{\boldsymbol{\omega}}.$$

In the above formulae the following notations are adopted: the symbol $\mathbf{u}$ denotes the displacement vector, while $\boldsymbol{\omega}$ stands for the rotation vector; $\mathbf{X}$ represents the body forces vector, while $\mathbf{Y}$ denotes the body couple vector, $\varkappa$, λ, α, β, γ, ε denote material constants, ϱ stands for the density and J for rotational inertia. The quantities $\mathbf{u}$, $\boldsymbol{\omega}$, $\mathbf{X}$, $\mathbf{Y}$ are functions of the position $\mathbf{x}$ and time t.

The very complicated system of six differential equations (1.1) and (1.2) may be reduced — by decomposing the vectors $\mathbf{u}$, $\boldsymbol{\omega}$ as well as $\mathbf{X}$, $\mathbf{Y}$ into their potential and solenoidal parts — to a system of more simple wave equations. Substituting for $\mathbf{u}$ and $\boldsymbol{\omega}$ into Eqs. (1.1) and (1.2), respectively, the following expressions

$$(1.3)\qquad \mathbf{u}=\text{grad}\,\Phi+\text{rot}\,\boldsymbol{\Psi},\quad \text{div}\,\boldsymbol{\Psi}=0,$$

$$(1.4)\qquad \boldsymbol{\omega}=\text{grad}\,\varphi+\text{rot}\,\boldsymbol{\Omega},\quad \text{div}\,\boldsymbol{\Omega}=0,$$

and also for $\mathbf{X}$ and $\mathbf{Y}$:

$$(1.5)\qquad \mathbf{X}=\varrho\,(\text{grad}\,\vartheta+\text{rot}\,\boldsymbol{\chi}),\quad \text{div}\,\boldsymbol{\chi}=0,$$

$$(1.6)\qquad \mathbf{Y}=J\,(\text{grad}\,\sigma+\text{rot}\,\boldsymbol{\eta}),\quad \text{div}\,\boldsymbol{\eta}=0$$

we arrive at the following system of equations

$$(1.7)\qquad \left(\nabla^2-\frac{1}{c_1^2}\partial_t^2\right)\Phi+\frac{1}{c_1^2}\vartheta=0,$$

$$(1.8)\qquad \left(\nabla^2-\tau^2-\frac{1}{c_3^2}\partial_t^2\right)\varphi+\frac{1}{c_3^2}\sigma=0,$$

$$(1.9)\qquad \left[\left(\nabla^2-\frac{1}{c_2^2}\partial_t^2\right)\left(\nabla^2-\nu^2-\frac{1}{c_4^2}\partial_t^2\right)+\eta_0^2\nabla^2\right]\mathbf{\Psi}= =\frac{p}{c_4^2}\operatorname{rot}\boldsymbol{\eta}-\frac{1}{c_2^2}\left(\nabla^2-\nu^2-\frac{1}{c_4^2}\partial_t^2\right)\boldsymbol{\chi},$$

$$(1.10)\qquad \left[\left(\nabla^2-\frac{1}{c_2^2}\partial_t^2\right)\left(\nabla^2-\nu^2-\frac{1}{c_4^2}\partial_t^2\right)+\eta_0^2\nabla^2\right]\mathbf{\Omega}= =\frac{s}{c_2^2}\operatorname{rot}\boldsymbol{\chi}-\frac{1}{c_4^2}\left(\nabla^2-\frac{1}{c_2^2}\partial_t^2\right)\boldsymbol{\eta}.$$

The following notations have been introduced in addition to those indicated above

$$c_1=\left(\frac{\lambda+2\varkappa}{\varrho}\right)^{1/2},\quad c_2=\left(\frac{\varkappa+\alpha}{\varrho}\right)^{1/2},\quad c_3=\left(\frac{\beta+2\gamma}{J}\right)^{1/2},\quad c_4=\left(\frac{\gamma+\varepsilon}{J}\right)^{1/2},$$

$$\tau^2=\frac{4\alpha}{\beta+2\gamma},\quad \nu^2=\frac{4\alpha}{\gamma+\varepsilon},\quad \eta_0^2=\frac{4\alpha^2}{(\gamma+\varepsilon)(\varkappa+\alpha)},\quad p=\frac{2\alpha}{\varkappa+\alpha},\quad s=\frac{2\alpha}{\gamma+\varepsilon}.$$

Eq. (1.7) describes the longitudinal wave, Eq. (1.8) represents the rotational wave, while Eqs. (1.9) and (1.10) are the formulae for modified transverse waves.

In the next section we derive — recurring to the four-dimensional Fourier transformation — a general solution of the equation of motion valid for any distribution of body forces and body couples. Taking advantage of the solution obtained in this way, we derive — in Sec. 3 — the general solution for the two-dimensional problem. Finally, in section 4, the particular case of the statical problem is discussed.

2. General solution of the equations of motion

To solve the system of wave equations (1.7)—(1.10) we make use of the four-dimensional Fourier transformation defined by the formulae

$$(2.1)\qquad \widetilde{\Phi}(\xi_1,\xi_2,\xi_3,\mu)=\frac{1}{4\pi^2}\int\limits_{E_4}\Phi(x_1,x_2,x_3,t)\exp\left[i(x_k\xi_k+\mu t)\right]dV,$$

$$(2.2)\qquad \Phi(x_1,x_2,x_3,t)=\frac{1}{4\pi^2}\int\limits_{W_4}\widetilde{\Phi}(\xi_1,\xi_2,\xi_3,\mu)\exp\left[-i(x_k\xi_k+\mu t)\right]dW,$$

where $dV=dx_1\,dx_2\,dx_3\,dt$ while E_3 denotes the whole $x_1\,x_2\,x_3\,t$ — space; $dW=$ $=d\xi_1\,d\xi_2\,d\xi_3\,d\mu$, while W_4 stands for the whole $\xi_1\,\xi_2\,\xi_3\,\mu$ — space.

Now, exploiting the results

$$\frac{1}{4\pi^2}\int\limits_{E_4}\left(\frac{\partial\Phi}{\partial x_j},\ \frac{\partial^2\Phi}{\partial t^2}\right)\exp\left[i(x_k\,\xi_k+\mu t)\right]dV=-(i\xi_j,\mu^2)\,\tilde{\Phi} \tag{2.3}$$

we obtain, in virtue of Eqs. (1.7)–(1.10) the following Fourier transforms:

$$\tilde{\Phi}=\frac{1}{c_1^2}\,\frac{\tilde{\vartheta}}{\xi^2-\sigma_1^2}, \tag{2.4}$$

$$\tilde{\varphi}=\frac{1}{c_3^2}\,\frac{\tilde{\sigma}}{\xi^2+\tau^2-\sigma_3^2}, \tag{2.5}$$

$$\tilde{\Psi}_j=\frac{1}{\Delta}\left[\frac{1}{c_2^2}(\xi^2+\nu^2-\sigma_4^2)\,\tilde{\chi}_j-\frac{ip\xi_k}{c_4^2}\,\epsilon_{jkl}\,\tilde{\eta}_l\right], \tag{2.6}$$

$$\tilde{\Omega}_j=\frac{1}{\Delta}\left[\frac{1}{c_4^2}(\xi^2-\sigma_2^2)\,\tilde{\eta}_j-\frac{is\xi_k}{c_2^2}\,\epsilon_{jkl}\,\tilde{\chi}_l\right]. \tag{2.7}$$

The following notations have been introduced in the above formulae.

$$\sigma_1=\frac{\mu}{c_1},\quad \sigma_2=\frac{\mu}{c_2},\quad \sigma_3=\frac{\mu}{c_3},\quad \sigma_4=\frac{\mu}{c_4},\quad \Delta=(\xi^2-\lambda_1^2)(\xi^2-\lambda_2^2),$$

$$\lambda_{1,2}^2=\frac{1}{2}\left[\sigma_2^2+\sigma_4^2+s(p-2)\pm\sqrt{[\sigma_4^2-\sigma_2^2+s(p-2)]^2+4ps\sigma_2^2}\right],\quad \xi^2=\xi_1^2+\xi_2^2+\xi_3^2.$$

We perform now the four-dimensional Fourier transformation on the expressions (1.3) and (1.4).

$$\tilde{u}_j=-i\xi_j\,\tilde{\Phi}-i\xi_k\,\epsilon_{jkl}\,\tilde{\Psi}_l, \tag{2.8}$$

$$\tilde{\omega}_j=-i\xi_j\,\tilde{\varphi}-i\xi_k\,\epsilon_{jkl}\,\tilde{\Omega}_l. \tag{2.9}$$

Introducing into these relations $\tilde{\Phi}$, $\tilde{\varphi}$, $\tilde{\Psi}_j$, $\tilde{\Omega}_j$, taking into account the relation

$$\epsilon_{ljk}\,\epsilon_{lmn}=\delta_{jm}\,\delta_{kn}-\delta_{jn}\,\delta_{km},$$

and bearing in mind that $\operatorname{div}\boldsymbol{\chi}=0$, $\operatorname{div}\boldsymbol{\eta}=0$, we obtain the following formulae

$$\tilde{u}_j=-\frac{i\xi_j\,\tilde{\vartheta}}{c_1^2(\xi^2-\sigma_1^2)}+\frac{1}{\Delta}\left[\frac{p}{c_4^2}\,\xi^2\,\tilde{\eta}_j-\frac{i}{c_2^2}(\xi^2+\nu^2-\sigma_4^2)\,\epsilon_{jkl}\,\xi_k\,\tilde{\chi}_l)\right], \tag{2.10}$$

$$\tilde{\omega}_j=-\frac{i\xi_j\,\tilde{\sigma}}{c_3^2(\xi^2+\tau^2-\sigma_3^2)}+\frac{1}{\Delta}\left[\frac{s}{c_3^2}\,\xi^2\,\tilde{\chi}_j-\frac{i}{c_3^2}(\xi^2-\sigma_2^2)\,\epsilon_{jkl}\,\xi_k\,\tilde{\eta}_l\right]. \tag{2.11}$$

Let us now apply a similar procedure to the formulae (1.5) and (1.6). We get

$$\tilde{X}_j=-\varrho\,(i\xi_j\,\tilde{\vartheta}+i\xi_k\,\epsilon_{jkl}\,\tilde{\chi}_l), \tag{2.12}$$

$$\tilde{Y}_j=-J(i\xi_j\,\tilde{\sigma}+i\xi_k\,\epsilon_{jkl}\,\tilde{\eta}_l). \tag{2.13}$$

By solving the above system of algebraic equations we arrive at

$$(2.14)\qquad \begin{aligned} \tilde{\vartheta} &= \frac{i\xi_k \tilde{X}_k}{\varrho \xi^2}, \qquad \tilde{\sigma} = \frac{i\xi_k \tilde{Y}_k}{J\xi^2},\\ \tilde{\chi}_j &= -\frac{i}{\varrho\xi^2}\epsilon_{jkl}\,\xi_k \tilde{X}_l, \qquad \tilde{\eta}_j = -\frac{i}{J\xi^2}\epsilon_{jkl}\,\xi_k \tilde{Y}_l. \end{aligned}$$

introducing the values thus obtained into the relations (2.10) and (2.11) and performing the inverse Fourier transformation according to Eq. (2.2) we obtain the general solution of the system of Eqs. (1.1) and (1.2) in the form of a quadruple integral, namely

$$(2.15)\qquad u_j(x_1, x_2, x_3, t) = \frac{1}{4\pi^2}\int\limits_{W_4}\left\{\frac{\xi_j\,\xi_k\,\tilde{X}_k}{\varrho c_1^2 \xi^2(\xi^2-\sigma_1^2)}+\right.$$

$$\left.-\frac{1}{\Delta}\left[\frac{\xi^2+\nu^2-\sigma_4^2}{c_2^2\,\varrho\xi^2}(\xi_j\,\xi_k\,\tilde{X}_k-\xi^2\,\tilde{X}_j)+\frac{ip}{Jc_4^2}\epsilon_{jkl}\,\xi_k\,\tilde{Y}_l\right]\right\}\exp[-i(\xi_k x_k+\mu t)]\,dW,$$

$$(2.16)\qquad \omega_j(x_1, x_2, x_3, t) = \frac{1}{4\pi^2}\int\limits_{W_4}\left\{\frac{\xi_j\,\xi_k\,\tilde{Y}_k}{Jc_3^2\xi^2(\xi^2+\tau^2-\sigma_3^2)}+\right.$$

$$\left.-\frac{1}{\Delta}\left[\frac{\xi^2-\sigma_2^2}{Jc_4^2\xi^2}(\xi_j\,\xi_k\,\tilde{Y}_k-\xi^2\,\tilde{Y}_j)+\frac{is}{\varrho c_2^2}\epsilon_{jkl}\,\xi_k\,\tilde{X}_l\right]\right\}\exp[-i(\xi_k x_k+\mu t)]\,dW.$$

Thus, the displacement and the rotations being known, we are able to determine the strain tensor γ_{ji} and the curvature-twist tensor $\varkappa_{ji}$.

$$(2.17)\qquad \gamma_{ji} = u_{i,j}-\epsilon_{kji}\,\omega_k, \qquad \varkappa_{ji} = \omega_{i,j}$$

and also the stress tensor σ_{ji} and couple tensor μ_{ji}

$$(2.18)\qquad \begin{aligned} \sigma_{ji} &= (\varkappa+\alpha)\,\gamma_{ji}+(\varkappa-\alpha)\,\gamma_{ij}+\lambda\gamma_{kk}\,\delta_{ij},\\ \mu_{ji} &= (\gamma+\varepsilon)\,\varkappa_{ji}+(\gamma-\varepsilon)\varkappa_{ij}+\beta\varkappa_{kk}\,\delta_{ij}. \end{aligned}$$

Let us now consider the particular case when $\alpha\to 0$. Eqs. (1.1) and (1.2) become then independent from each other

$$(2.19)\qquad \varkappa\nabla^2\mathbf{u}+(\lambda+\varkappa)\,\mathrm{grad\,div}\,\mathbf{u}+\mathbf{X} = \varrho\ddot{\mathbf{u}},$$

$$(2.20)\qquad (\gamma+\varepsilon)\,\nabla^2\boldsymbol{\omega}+(\gamma+\beta-\varepsilon)\,\mathrm{grad\,div}\,\boldsymbol{\omega}+\mathbf{Y} = J\ddot{\boldsymbol{\omega}}.$$

Eqs. (2.19) are equations of classical elastokinetics; Eqs. (2.20) refer to a hypothetical elastic medium wherein only rotations are possible. In such a limit case we have in virtue of (2.15) and (2.16) the following formulae

$$(2.21)\qquad u_j = \frac{1}{4\pi^2\varkappa}\int\limits_{W_4}\frac{\tilde{X}_j(\delta^2\xi^2-u^2/c_2^2)-(\delta^2-1)\,\xi_j\,\xi_k\,\tilde{X}_k}{(\xi^2-\mu^2/c_2^2)(\xi^2\,\delta^2-\mu^2/c_2^2)}\exp[-i(\xi_k x_k+\mu t)]\,dW,$$

$$(2.22)\qquad \omega_j = \frac{1}{4\pi^2(\gamma+\varepsilon)}\int\limits_{W_4}\frac{\tilde{Y}_j(\varrho_0^2\,\xi^2-\mu^2/c_4^2)-(\varrho_0^2-1)\,\xi_j\,\xi_k\,\tilde{Y}_k}{(\xi^2-\mu^2/c_4^2)(\xi^2\,\varrho_0^2-\mu^2/c_4^2)}\times$$

$$\times\exp[-i(\xi_k x_k+\mu t)]\,dW.$$

Here

$$\delta^2 = \frac{\lambda+2\varkappa}{\varkappa}, \quad \varrho_0^2 = \frac{\beta+2\gamma}{\gamma+\varepsilon}.$$

Formula (2.21) was derived in [4].

3. Solution of two-dimensional problem

Let us assume a plane state of strain where all sources of changes $(\mathbf{X}, \mathbf{Y})$ as also all effects $(\mathbf{u}, \boldsymbol{\omega})$ depend only on the variables x_1, x_2, t.

Then the system of Eqs. (1.1) and (1.2) splits into two idependent of each other systems of equations. In the first X_1, X_2, X_3 appear as sources while u_1, u_2, ω_3 as effects; in the second system the role of sources is played by Y_1, Y_2, Y_3 while ω_1, ω_2, u_3 represent the effects.

Denoting the body forces and body couples — considered as functions of x_1, x_2, t — by X_j^* and Y_j^*, respectively — we determine the quantities $\tilde{X}_j$ and $\tilde{Y}_j$ appearing in Eqs. (2.15) and (2.16) in the following may

$$(3.1) \qquad \tilde{X}_j(\xi_1, \xi_2, \xi_3, \mu) = \frac{1}{4\pi^2} \int_{E_3} X_j^*(x_1, x_2, t) \times \\ \times \exp[i(\xi_1, x_1 + \xi_2 x_2 + \mu t)]\, dS \int_{-\infty}^{\infty} c^{i\xi_3 x_3}\, dx_3.$$

Here $dS = dx_1\, dx_2\, dt$ while E_3 stands for the whole $x_1\, x_2\, t$-space

$$(3.2) \qquad \int_{-\infty}^{\infty} e^{i\xi_3 x_3} dx_3 = 2\pi\delta(\xi_3);$$

there is

$$(3.3) \qquad \tilde{X}_j(\xi_1, \xi_2, \xi_3, \mu) = \sqrt{2\pi}\, \delta(\xi_3)\, X_j^*(\xi_1, \xi_2, \mu),$$

where

$$\tilde{X}_j^*(\xi_1, \xi_2, \mu) = \frac{1}{(2\pi)^{3/2}} \int_{E_3} X_j^*(x_1, x_2, t) \exp[i(\xi_1 x_1 + \xi_2 x_2 + \mu t)]\, dS.$$

Introducing now Eq. (3.3) into Eq. (1.15) and a similar expression for $\tilde{Y}_j$ into Eq. (2.16) we get

$$(3.4) \qquad u_j = \frac{1}{(2\pi)^{3/2}} \int_{W_3} \left\{ \frac{\xi_j \xi_k \tilde{X}_k^*}{\varrho c_1^2 \boldsymbol{\xi}^2 (\boldsymbol{\xi}^2 - \sigma_1^2)} - \frac{1}{\Delta} \left[\frac{\boldsymbol{\xi}^2 + \nu^2 - \sigma_4^2}{\varrho c_2^2 \boldsymbol{\xi}^2} (\xi_j \xi_k \tilde{X}_k^* - \boldsymbol{\xi}^2 \tilde{X}_j^*) + \right.\right. \\ \left.\left. + \frac{ip}{J c_4^2} \epsilon_{jkl} \xi_k \tilde{Y}_l^* \right]\right\} \exp[-i(\xi_k x_k + \mu t)]\, dT,$$

$$(3.5) \qquad \omega_j = \frac{1}{(2\pi)^{3/2}} \int_{W_3} \left\{ \frac{\xi_j \xi_k \tilde{Y}_k^*}{J c_3^2 \boldsymbol{\xi}^2 (\boldsymbol{\xi}^2 + \tau^2 - \sigma_3^2)} + \right. \\ \left. - \frac{1}{\Delta} \left[\frac{\boldsymbol{\xi}^2 - \sigma_2^2}{J c_4^2 \boldsymbol{\xi}^2} (\xi_j \xi_k \tilde{Y}_k^* - \boldsymbol{\xi}^2 \tilde{Y}_j^*) + \frac{is}{\varrho c_2^2} \epsilon_{jkl} \tilde{X}_l^* \right]\right\} \exp[-i(\xi_k x_k + \mu t)]\, dT$$

$$j, k = 1, 2, \quad \boldsymbol{\xi}_1^2 = \xi_1^2 + \xi_2^2.$$

where $dT = dx_1\, dx_2\, dt$, while W_3 represents the whole $\xi_1\, \xi_2\, t$-space.

It is easily seen from the above formulae that $u_1\, u_2\, \omega_3$ may be brought about by the action of body forces X_1^*, X_2^* and body couples Y_3^*. The functions ω_1, ω_2, u_3 are connected with the action of body forces X_3^* and body couples Y_1^*, Y_2^*.

4. Solution of static problem

Let us first consider the three-dimensional problem. In the static problem the body forces and the body couples do not depend on time. Denoting by $G_j(x_1, x_2, x_3)$ the components of body forces and by $M_j(x_1, x_2, x_3)$ the components of body couples we may express the transform $\tilde{X}_j$ as

$$(4.1)\qquad \tilde{X}_j(\xi_1, \xi_2, \xi_3, \mu) = \frac{1}{4\pi^2} \int_{B_3} G_j(x_1, x_2, x_3) \exp[ix_k\, \xi_k]\, dA \int_{-\infty}^{\infty} e^{i\mu t}\, dt.$$

It results therefrom — bearing in mind (3.2) — that

$$(4.2)\qquad \tilde{X}_j(\xi_1, \xi_2, \xi_3, \mu) = \sqrt{2\pi}\, \delta(\mu)\, \tilde{G}_j(\xi_2, \xi_2, \xi_3),$$

where

$$\tilde{G}_j(\xi_1, \xi_2, \xi_3) = \frac{1}{(2\pi)^{3/2}} \int_{B_3} G_j(x_1, x_2, x_3)\, e^{i\xi_k x_k}\, dA.$$

Here $dA = dx_1\, dx_2\, dx_3$ while B_3 represents the whole $x_1\, x_2\, x_3$-space.

Introducing (4.2) into Eqs. (2.15) and (2.16) and performing the appropriate integration we obtain

$$(4.3)\qquad u_j = \frac{1}{(2\pi)^{3/2}} \int_{D_3} \left\{ \frac{\xi_j\, \xi_k\, \tilde{G}_k}{\varrho c_1^2 \xi^2 \xi^2} + \right.$$
$$\left. - \frac{1}{\Delta_0} \left[\frac{\xi^2 + \nu^2}{c_2^2 \varrho \xi^2} (\xi_j\, \xi_k\, \tilde{G}_k - \xi^2\, \tilde{G}_j) + \frac{ip}{Jc_4^2} \epsilon_{jkl}\, \xi_k\, \tilde{M}_l \right] \right\} \exp[-i\xi_k\, x_k]\, dD,$$

$$(4.4)\qquad \omega_j = \frac{1}{(2\pi)^{3/2}} \int_{D_3} \left\{ \frac{\xi_j\, \xi_k\, \tilde{M}_k}{Jc_3^2\, \xi^2 (\xi^2 + \tau^2)} + \right.$$
$$\left. - \frac{1}{\Delta_0} \left[\frac{1}{Jc_4^2} (\xi_j\, \xi_k\, \tilde{M}_k - \xi^2\, \tilde{M}_j) + \frac{is}{\varrho c_2^2} \epsilon_{jkl}\, \xi_k\, \tilde{G}_l \right] \right\} \exp(-i\xi_k\, x_k)\, dD,$$

$$j, k, l = 1, 2, 3, \quad \xi^2 = \xi_1^2 + \xi_2^2 + \xi_3^2, \quad \Delta_0 = \xi^2 (\xi^2 + \nu^2 - \eta_0^2).$$

Here $dD = d\xi_1\, d\xi_2\, d\xi_3$, while D_3 stands for the whole $\xi_1\, \xi_2\, \xi_3$-space.

Passing to the classical elastokinetics ($\alpha \to 0$) we obtain from (4.3) the following formula

$$(4.5)\qquad u_j = \frac{1}{\varkappa (2\pi)^{3/2}} \overline{\underline{\iiint_{-\infty}^{\infty}}} \left(\frac{\tilde{G}_j}{\xi^2} - \frac{(\delta^2 - 1)\, \xi_j\, \xi_k\, \tilde{G}_k}{\delta^2\, \xi^2\, \xi^2} \right) \exp(-i\xi_k\, x_k)\, dD,$$

which is conform with the results arrived at in [4]. Substituting $\alpha \to 0$ into Eq. (4.4) we get

$$\omega_j = \frac{1}{(\gamma+\varepsilon)(2\pi)^{3/2}} \int\int\int_{-\infty}^{\infty} \left(\frac{\tilde{M}_j}{\xi^2} - \frac{(\varrho_0^2 - 1)\,\xi_j\,\xi_k\,\tilde{M}_k}{\varrho_0^2\,\xi^2\,\xi^2} \right) \exp(-i\xi_k x_k)\, dD. \tag{4.6}$$

The last formula refers to a hypothetical medium wherein only rotations may occur.

Applying the *modus procedendi* indicated in Sec **2.** we now pass to the two-dimensional static problém. We get (cf. [4])

$$u_j = \frac{1}{2\pi} \int\int_{-\infty}^{\infty} \left\{ \frac{\xi_j\,\xi_k\,\tilde{G}_k}{\varrho c_1^2 \xi^2 \xi^2} - \frac{1}{\Delta_1} \left[\frac{\xi^2+\nu^2}{c_2^2\,\varrho\xi^2} (\xi_j\,\xi_k\,\tilde{G}_k - \xi^2\,\tilde{G}_j) + \right.\right.$$
$$\left.\left. + \frac{ip}{Jc_4^2} \epsilon_{jkl}\,\xi_k\,\tilde{M}_l \right]\right\} \exp[-i(\xi_1 x_1 + \xi_2 x_2)]\, d\xi_1\, d\xi_2, \tag{4.7}$$

$$\omega_j = \frac{1}{2\pi} \int\int_{-\infty}^{\infty} \left\{ \frac{\xi_j\,\xi_k\,\tilde{M}_k}{Jc_3^2 \xi^2 \xi^2} - \frac{1}{\Delta_1} \left[\frac{1}{c_4^2 J} (\xi_j\,\xi_k\,\tilde{M}_k - \xi^2\,\tilde{M}_j) + \right.\right.$$
$$\left.\left. + \frac{is}{\varrho c_2^2} \epsilon_{jkl}\,\xi_k\,\tilde{G}_l \right]\right\} \exp[-i(x_1\,\xi_1 + x_2\,\xi_2)]\, d\xi_1\, d\xi_2, \qquad j,k = 1,2. \tag{4.8}$$

where

$$\xi^2 = \xi_1^2 + \xi_2^2, \quad \Delta_1 = (\xi_1^2+\xi_2^2)(\xi_1^2+\xi_2^2+\nu^2-\eta^2),$$

$$\tilde{M}_j = \frac{1}{2\pi} \int\int_{-\infty}^{\infty} M_j(x_1, x_2) \exp[i(x^1\,\xi_1 + x_2\,\xi_2)]\, dx_1, d_2x.$$

A more ample discussion of the problems we are concerned with in this paper — particularly of those referring to the action of body forces and body couples, static as well as changing periodically in time — will appear in the journal "Proceedings of Vibration Problems".

DEPARTMENT OF MECHANICS OF CONTINUOUS MEDIA, INSTITUTE OF BASIC TECHNICAL PROBLEMS, POLISH ACADEMY OF SCIENCES

(ZAKŁAD MECHANIKI OŚRODKÓW CIĄGŁYCH, INSTYTUT PODSTAWOWYCH PROBLEMÓW TECHNIKI, PAN)

REFERENCES

[1] V. A. Palmov, Prikl. Mat. Mech., **28** (1964), 401.

[2] A. C. Eringen, E. S. Suhubi, Int. J. Engin., Sci. **2** (1964), 189.

[3] — , ibid., **2** (1964), 389.

[4] G. Eason, J. Fulton, I. N. Sneddon, Phil. Trans. R. S , **A955** (1956), 575.

В. НОВАЦКИЙ и В. К. НОВАЦКИЙ, **ГЕНЕРИРОВАНИЕ ВОЛН В БЕСКОНЕЧНОЙ МИКРОПОЛЯРНОЙ УПРУГОЙ СРЕДЕ. I.**

В настоящей работе предлагается метод общего решения уравнений движения для бесконечной микрополярной упругой среды. Источниками деформаций, образующихся в твердом теле, являются массовые моменты и массовые силы. Предлагаемое авторами решение получается, прибегая к интегральному экспоненциальному преобразованию Фурье. Дается общее решение для массовых сил и моментов изменяющихся во времени, а равно и таких, которые не зависят от временной переменной.

Рассматриваются трех- и двухмерные проблемы.