

## MATERIAL RESPONSES TO THE LABORATORY SIMULATION OF COMPLEX CYCLIC LOADINGS

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### **Abstract**

*The paper presents investigations identifying an influence of complex cyclic loading controlled by the strain signals on the selected mechanical properties of the 2024 aluminium alloy.*

*Investigations of proportional and non-proportional loading paths in the form of square and circle were carried out at room temperature using thin-walled tubular specimens enabling realisation of complex stress states due to acting of axial force and twisting moment.*

*The hardening effect of the material due to cyclic loading was observed on the basis of: stress responses into the strain controlled loading programme; hysteresis loop variations and amounts of stress amplitude. In the case of the non-proportional cyclic loading an additional hardening effect was identified.*

*Second-order effects, which allow better understanding of the influence of proportional and non-proportional loading paths on mechanical behaviour of engineering materials, were identified. In the case of the circular loading path a delay of the maximum stress signals with respect to strain ones is demonstrated. For the square loading path a softening effect was observed. It is reflected by the rapid stress drop on this direction which is perpendicular to that where the stress level begins to turn back.*

*The yield surface approach is also applied in order to assess the mechanical properties variations due to cyclic loading of the material.*

**Keywords:** *cyclic proportional and non-proportional loading, yield surface, complex stress state, hardening, softening*

## 1. Introduction

Complex cyclic loading is treated as a typical form of exploitation loadings met in many machinery elements used in different branches of industry. A representative example is shown in Fig. 1. It presents variations of the stress state components in the gas turbine and reflects a proportional and non-proportional character of loadings applied, especially in the first and third period. Moreover, the figure identifies the starting period of the gas turbine work as that, which has the essential influence on the further gas turbine rotor exploitation.

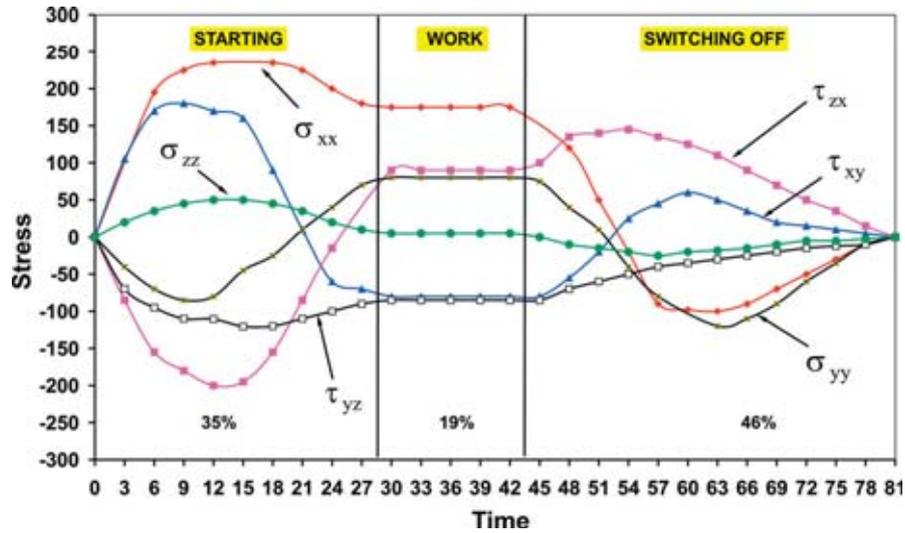


Fig. 1. Illustration of stress state variations of a gas turbine rotor [3]

In order to carry out a numerical simulation of proportional and non-proportional loadings due to a strain-controlled signals being tension-compression and torsion-reverse-torsion combination, the following equation describing an effective strain can be used

$$\varepsilon_e = \sqrt{\varepsilon_{xx}^2 + \frac{3}{(1+\nu)^2} \varepsilon_{xy}^2}, \quad (1)$$

where:

$\varepsilon_{xx}$  - axial strain,

$\varepsilon_{xy} = \gamma_{xy}/2$  - shear strain,

$\gamma_{xy}$  - shear strain angle,

$\nu$  - Poisson's ratio.

Possible variations of the principal directions of strain signals are calculated applying the following expression

$$\phi = \frac{1}{2} \cdot \tan^{-1} \left( \frac{\gamma_{xy}(t)}{\varepsilon_{xx}(t)} \right), \quad (2)$$

where t denotes time parameter.

Equations (1) and (2) can be used to identify effects associated with proportional (Fig. 2a) and non-proportional loading paths (Fig. 2b, 2c). They enable to reflect not only differences in the effective strain (Fig. 3), but also in the principal strain directions (Fig. 4) for non-proportional loading. Unfortunately, a change of principal strain directions does not identify the most preferred direction in which defects may nucleate and grow in the real machine elements under exploitation conditions. Therefore, a determination of the weakest points of the element is relatively difficult.

Taking into account these facts and knowing additionally that the phase shift between the maximum stress and strain signals takes place during non-proportional cyclic loading, many research centres [1-6] are focused on the analysis of effects associated with that type of loading. In this paper the problem of an influence of proportional and non-proportional loadings on the basic mechanical parameters variation is also investigated and discussed in detail for the 2024 aluminium alloy.

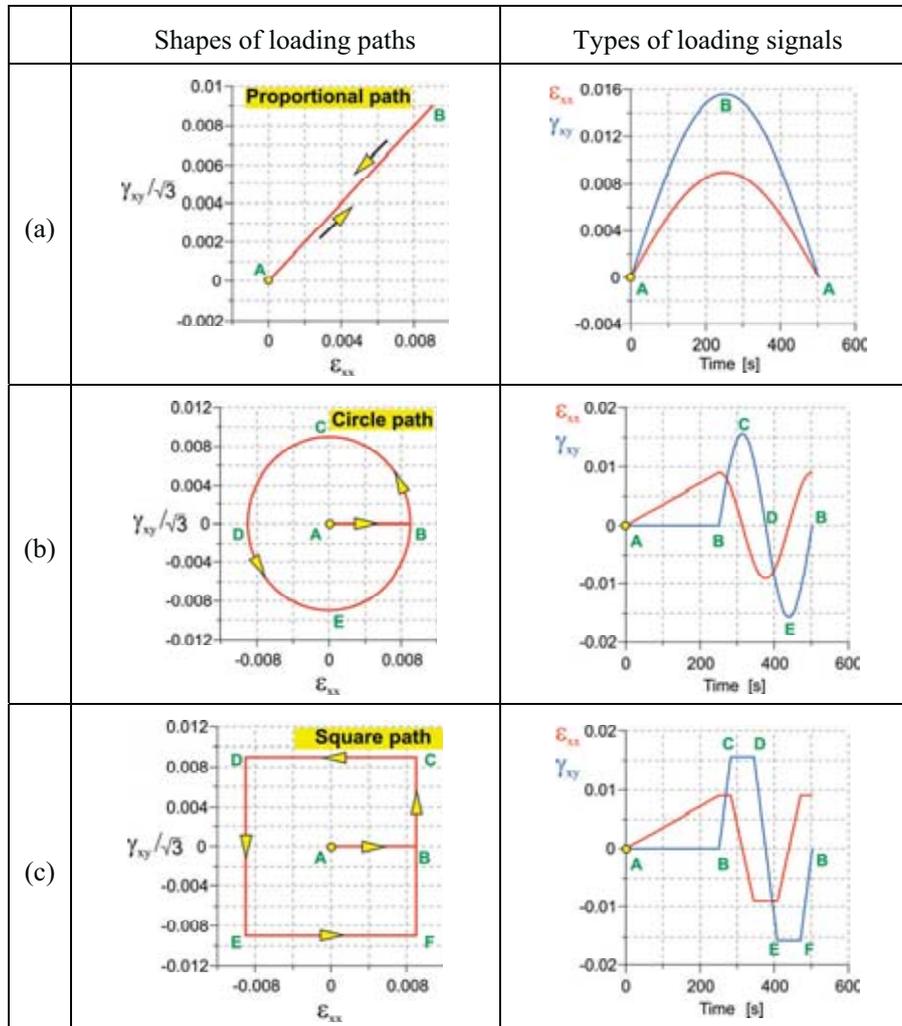


Fig. 2. Examples of cyclic loading paths designing: (a) proportional, (b) circle, (c) square

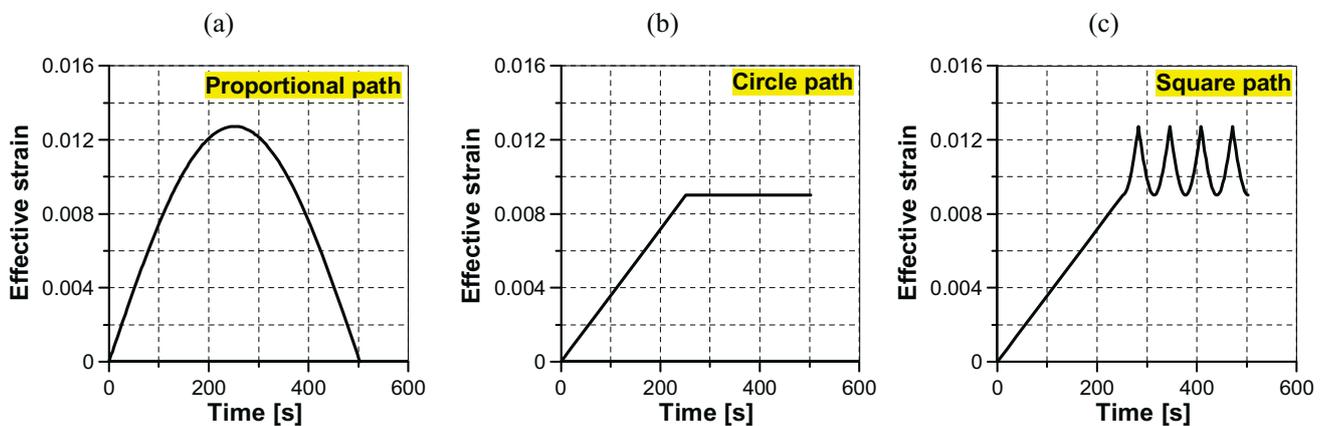


Fig. 3. Variations of the effective strain for the same values of deformation in the case of proportional and non-proportional loading paths: (a) proportional, (b) circle, (c) square

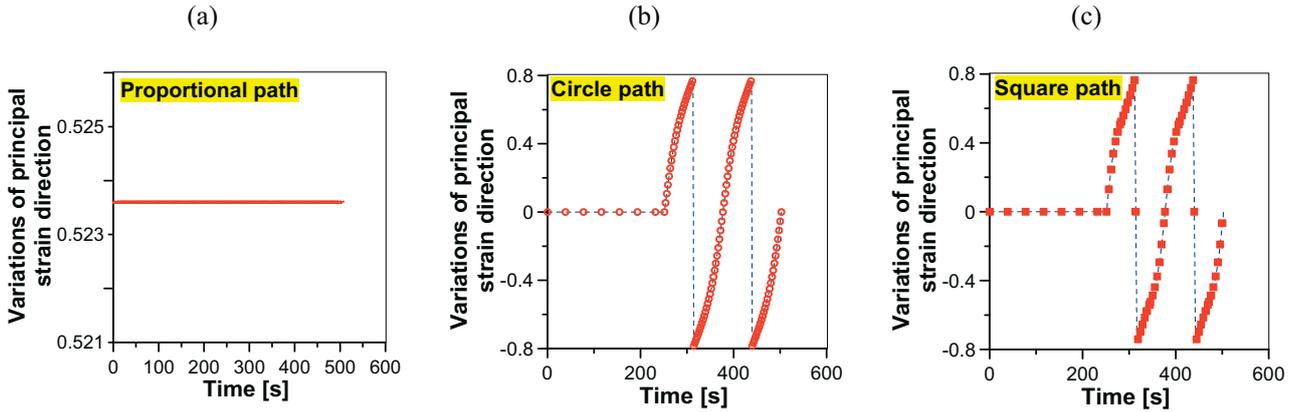


Fig. 4. Principal direction variations of loading signals for the same strain amplitudes of proportional and non-proportional loadings: (a) proportional path, (b) circle path, (c) square path

## 2. An influence of complex cyclic loading on hardening of the 2024 aluminium alloy

An influence of complex cyclic loadings on the mechanical behaviour of the 2024 aluminium alloy was investigated for proportional and non-proportional strain paths. The non-proportional loading paths were designed in such a way that the square loading path was circumscribed on the circle having a diameter equal of the length of the proportional loading path. The selected experimental results are presented in Fig. 5. Comparison of the results from proportional loading test, Fig. 5a, and non-proportional cycles, Fig. 5b, c, exhibits an additional hardening effect of the material. It is expressed by a gradual increase of stress amplitude and hysteresis loops with respect to those for the proportional cycles achieved. Moreover, the material does not reach the saturation state in the case of deformation realised using the square path (Fig. 5c) while for the circle path this feature can be observed (Fig. 5b). A shape of the stress responses into the strain controlled non-proportional cycles in the form of square identifies their distortion, Fig. 5c. Such phenomenon indicates the additional effects, which may be associated with the additional hardening obtained during non-proportional deformations in comparison to the hardening usually observed under loading along proportional paths.

A comparison of the courses of strain signals enables an identification of the second-order effects, which are connected with cyclic loading, Fig. 6. In the case of the proportional cyclic loading the maximum values of strain and stress signals are in phase, Fig. 6a. Contrary to that case, for the non-proportional cyclic loading along circular strain path the phase shift can be observed between these signals, Fig. 6b.

An interesting feature can be easily noticed looking on the courses of stress and strain signals in the case of cyclic deformation along square strain path. The softening effect takes place, which is expressed by a rapid drop of the stress component signals, Fig. 6c. The softening effect due to non-proportional cyclic loading along the square path can be observed for both stress components. It is visible when the one of the loading signals changes a direction (i.e. turns back).

The second-order effects mentioned above are considerably more visible when the magnitude of deformation increases. The biggest values of them were obtained for the deformation amounts equal to those required to attain yield limits of the material. For example, the retardation angle calculated on the basis of phase shift measured during cycles along circular strain path was equal to  $25^\circ$ , whereas in the case of square loading path a drop of the axial stress was equal around 300 MPa.

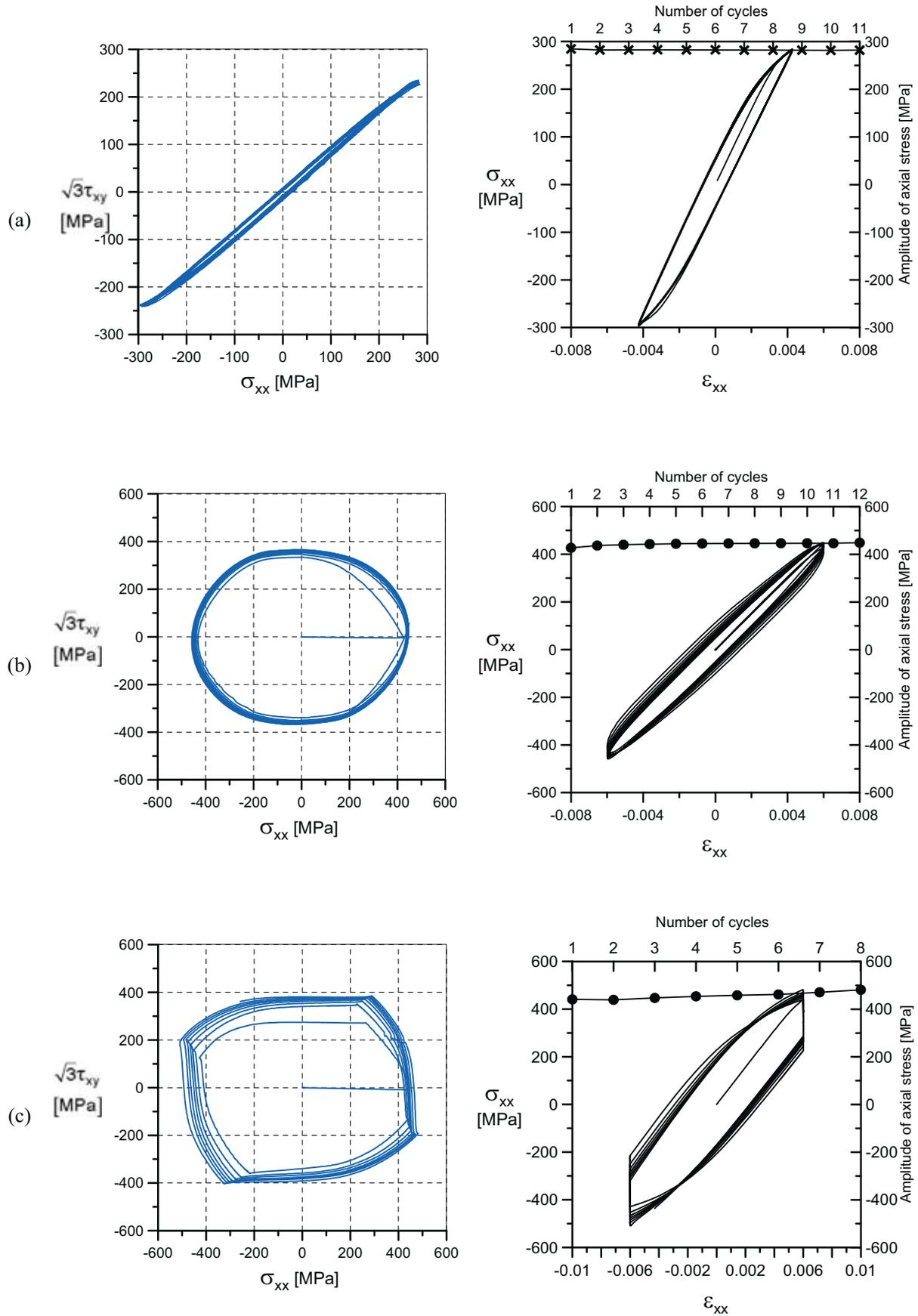


Fig. 5. Stress responses into cyclic loadings and illustration of hysteresis loops obtained for the same effective strain amplitude equal to 0.6%: (a) proportional path, (b) circle path, (c) square path

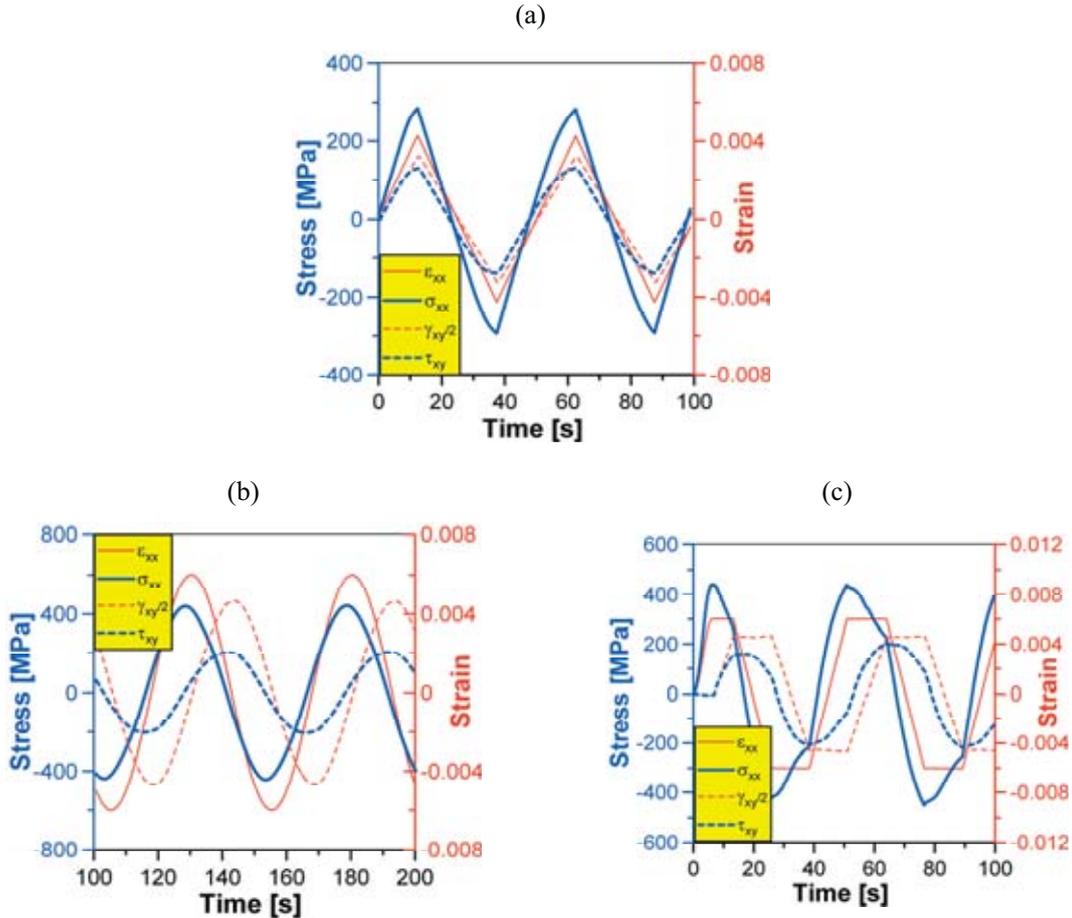


Fig. 6. Variations of the stress and strain signals due to proportional and non-proportional cyclic loadings for the same effective strain amplitude equal to 0.6%: (a) proportional path, (b) circle path, (c) square path

### 3. Yield surface approach in analysis of mechanical parameters variations due to prior cyclic loadings

An influence of the proportional and non-proportional loadings on the mechanical parameters of the 2024 aluminium alloy was evaluated on the basis of the initial yield surface evolution.

For the proportional loading path along tension-compression and torsion-reverse-torsion strain paths the hardening effect exhibits the isotropic-kinematic character, Fig. 7a. Moreover, the figure illustrates a tendency to the initial anisotropy reduction due to cyclic loading. It is confirmed by the significant decrease of the ratio of major to minor yield surface axes with the increase of strain amplitude. It varies from 2.21 for the initial yield surface to 1.96 for subsequent yield locus determined for the highest strain amplitude, Fig. 7b. A different behaviour was observed for the material tested after the circular loading path, Fig. 8. In this case a hardening of the material is also visible, however, the mechanical parameters changes have rather isotropic character.

It is easy to notice the hardening of the material which for the strain amplitude equal to  $\pm 0.8\%$  can be expressed by the significant increase of the yield point for tension and compression. Both yield points increased about 100 MPa, what means about 25% increase with respect to the initial values. Comparison of the initial yield surface with subsequent yield loci determined for the material prestrained due to different types of cyclic loading for the same strain amplitude (0.8%) indicates the non-proportional cycles as those having a dominant role in mechanical properties modifications, Fig. 9. As it is clearly seen, the biggest dimensions of subsequent yield surfaces were achieved for the non-proportional cycles along circular strain path.

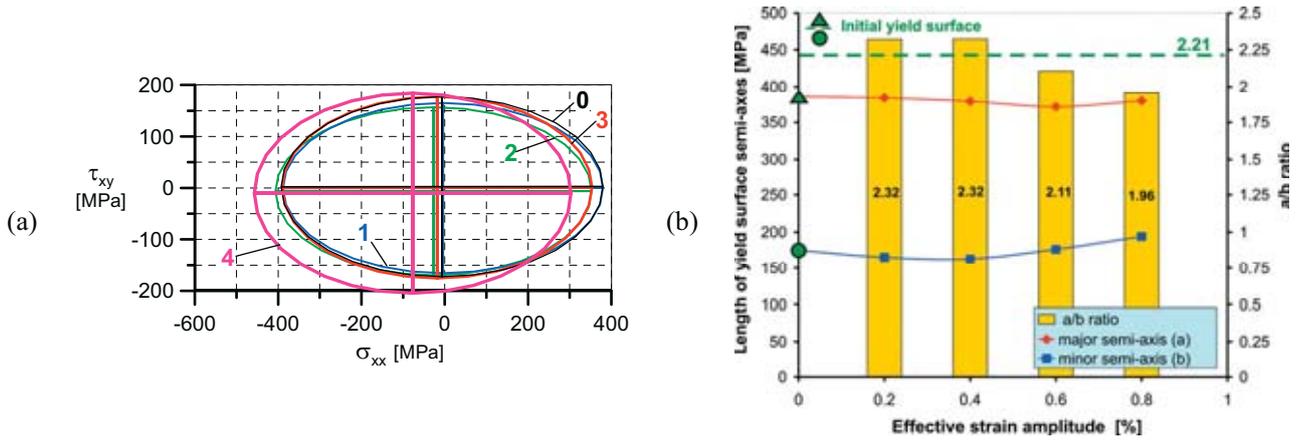


Fig. 7. Experimental results from proportional cyclic loading along rectilinear strain path being combination of axial force and twisting moment: (a) - evolution of the initial yield surface (0) due to proportional cyclic loading with strain amplitude equal to  $\pm 0.2\%$ ,  $\pm 0.4\%$ ,  $\pm 0.6\%$ ,  $\pm 0.8\%$ , yield loci denoted as 1, 2, 3, 4, respectively; (b) - selected parameters characterising yield surface evolution

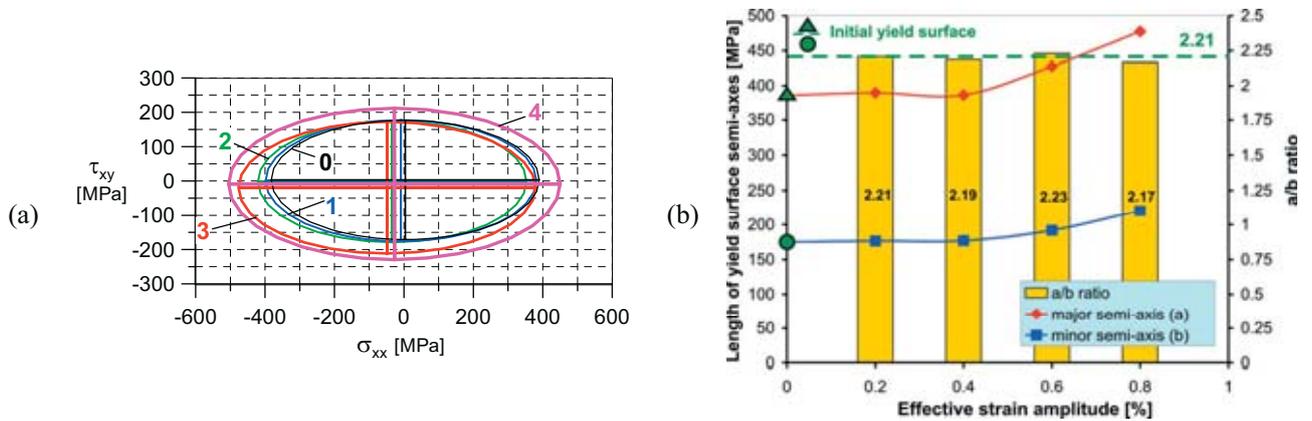


Fig. 8. Experimental results from non-proportional cyclic loading along circular strain path: (a) - evolution of the initial yield surface (0) due to circular cyclic loading with strain amplitude equal to  $\pm 0.2\%$ ,  $\pm 0.4\%$ ,  $\pm 0.6\%$ ,  $\pm 0.8\%$ , yield loci denoted as 1, 2, 3, 4, respectively; (b) - selected parameters characterising yield surface evolution

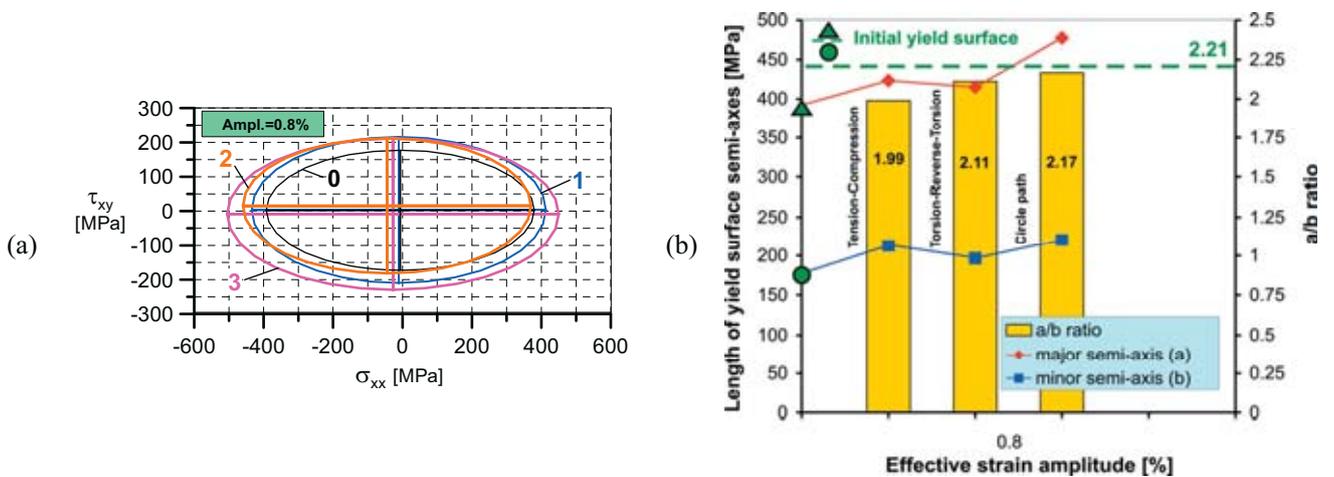


Fig. 9. Comparison of the results achieved from different types of cyclic loading: (a) - evolution of the initial yield surface (0) due to (1) proportional cyclic loading along rectilinear tension-compression strain path; (2) proportional cyclic loading along rectilinear torsion-reverse torsion strain path; (3) non-proportional cyclic loading along circular strain path; (b) - selected parameters characterising yield surface evolution

#### 4. Summary

The paper presents experimental results which indicate substantial material effects associated with complex proportional and non-proportional cyclic loadings. They can be used to assess a damage degree due to exploitation loadings of either machine elements or adequate materials.

A phase shift of the stress signals with respect to corresponding strain ones was observed in the case of non-proportional cyclic loading along circular strain path. Such behaviour suggests visco-elastic character of deformation.

For the non-proportional cyclic loading along square strain path a softening effect was identified. It was reflected by the rapid stress drop observed during each turn back of cyclic loading components. Such effect may find a significant application during technological processes designing for semi-finished products manufacturing.

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