New Ultrasonic Torsional Waves for Sensing Applications

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***Abstract*—In this paper (inspired by a newly developed class of materials, i.e., elastic metamaterials) we will show that torsional surface elastic waves can propagate on the surface of a metamaterial elastic rod embedded in a conventional elastic space. In this work, we assume that the elastic compliance $s_{44}(\omega)$ of a metamaterial cylindrical rod varies with frequency ω according to Drude model. The proposed torsional ultrasonic surface waves can be considered as an elastic analogue of Surface Plasmon Polariton (SPP) electromagnetic waves propagating on the surface of a metallic rod embedded in dielectric space. An analytical expression for the dispersion equation the new torsional elastic surface wave was developed. The newly discovered torsional elastic surface wave inherits many of fascinating properties of electromagnetic counterpart such as: strong subwavelength concentration of the wave field in the vicinity of the cylindrical surface, low phase and group velocity etc. Therefore, the proposed new torsional ultrasonic surface waves can be used in: a) near-field subwavelength acoustic imaging (superresolution), b) amplification of the evanescent waves c) acoustic wave trapping (zero group and phase velocity)**

Keywords—Torsional surface acoustic waves, Surface Plasmon Polaritons (SPP) electromagnetic waves, Dispersion equation, Phase velocity, Group velocity

I. Introduction

We are currently witnessing a fascinating development of the theory of surface and bulk acoustic waves. New extraordinary properties in the domain of acoustic waves appeared with the invention of a new class of materials, i.e., metamaterials. The use of elastic metamaterials for the construction of ultrasonic waveguides has created a fertile ground for the discovery of a series of new ultrasonic waves. In this paper, the author applied elastic metamaterials to develop new cylindrical liquid viscosity sensors.

Ultrasonic elastic waves propagating in rectangular and circular waveguide structures have found applications in sensors of physical quantities, such as viscosity sensors, to investigate the elastic parameters of surface layers, to investigate the physicochemical parameters of liquids, etc. [1-9].

Torsional waves propagating in cylindrical rods have been mainly used in viscosity sensors since the 1950s. Classical ultrasonic cylindrical liquid viscosity sensors are made of conventional elastic materials. These sensors are usually used to determine the viscosity of liquids in biosensors and chemosensors. However, sensors of this type are not free from disadvantages. Namely, the ultrasonic field of this classical torsional wave is distributed in a large volume of the waveguide, which results in a moderate mass sensitivity of the cylindrical sensor. Therefore, the need to solve this problem arose. The aim of the author's work was to overcome this drawbacks. To solve this problem, the author used the extraordinary properties of elastic metamaterials. The waveguide proposed by the author consists of a metamaterial core immersed in a three-dimensional (3D) elastic external medium (see figure 1). The mechanical susceptibility of the metamaterial cylindrical core follows the Drude model. Consequently, the mechanical compliance of the core cylinder can take negative and positive values. In this work, the author presented the theory of a new torsional ultrasonic wave propagating in metamaterial cylindrical waveguide structures. The new discovered cylindrical surface waves have only one angular mechanical displacement component (see figure 1).

The equations of motion written in a cylindrical coordinate system were formulated and solved. An analytical form of the dispersion equation was evaluated for the torsional wave propagating in layered cylindrical metamaterial structures.

The key property of the newly discovered torsional ultrasonic waves is that their mechanical displacement is concentrated close to the surface of the cylinder, which greatly increases the mass sensitivity of the sensor. The subject of this study will be the remarkable discovery of a new type of elastic torsional surface waves propagating at the interface of an elastic metamaterial rod embedded in an elastic outer conventional space.

The new developed torsional elastic wave is an elastic analogue of the electromagnetic wave of the Surface Plasmon Polariton (SPP) type that propagates at the interface between the metallic inner cylinder and the dielectric outer space [10-12].

This new torsional elastic ultrasonic wave has unusual properties such as:

1) ability to amplify the evanescent waves
2) the ability to concentrate the acoustic wave energy in a spot smaller than the wavelength (super lensing)
3) the possibility to achieve a resolution below the wavelength (super resolution)

According to the authors knowledge, a new torsional ultrasonic wave in cylindrical layered metamaterial waveguides has not been described in the world literature so far.

II. Geometry of the Problem

The geometry of the waveguide supporting new torsional elastic surface waves is presented in Fig.1. The waveguide consists of a metamaterial elastic rod ($0 < r < a$) embedded in a conventional elastic space ($r > a$).

The elastic compliance of a metamaterial ($0 < r < a$) can exhibit negative values $s_{44}^{(1)}(\omega) < 0$. The densities $(\rho_1, \rho_2) > 0$ in a metamaterial rod and in a conventional elastic outer space as well as the elastic compliance $s_{44}^{(2)} > 0$ are positive.

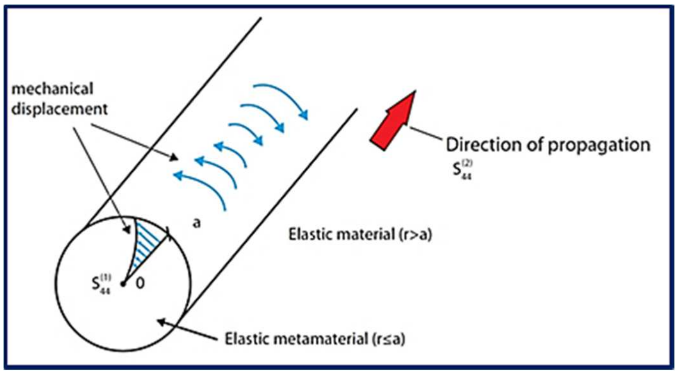

Fig. 1. Cross-section of the waveguide supporting the newly discovered torsional elastic surface waves, propagating in the direction along the cylindrical metamaterial rod ($r < a$) embedded in a conventional elastic half-space ($r > a$) Mechanical displacement u_θ of the new torsional elastic surface waves is polarized along the angular coordinate θ. The radius of the inner metamaterial rod is assumed to be 1 cm.

A. Elastic compliance $s_{44}^{(1)}(\omega)$ in the metamaterial elastic rod ($r < a$)

It is assumed that the elastic compliance of a metamaterial rod $s_{44}^{(1)}(\omega)$, as a function of angular frequency ω, changes obeying Drude's model.

$$s_{44}^{(1)}(\omega) = s_0\left(1 - \frac{\omega_p^2}{\omega^2}\right) \quad (1)$$

where: ω_p is the angular frequency of the local mechanical resonators in the metamaterial and s_0 is its reference elastic compliance for $\omega \to \infty$.

It can be noticed that the elastic compliance $s_{44}^{(1)}(\omega)$ given by Eq.1, is similar to the dielectric function $\varepsilon(\omega)$ in Drude's model of metals [13]. The angular frequency ω_p is called the angular frequency of bulk plasmon resonance [14].

The outer elastic space ($r > a$) is a conventional elastic material with a positive compliance $s_{44}^{(2)} > 0$ and density ρ_2 that are both frequency independent.

III. Theoretical Background

The new torsional surface waves have only one component of the mechanical displacement u_θ that is polarized along the angular coordinate θ and is tangential to the cylinder circumference, see Fig.1.

A. Mechanical Displacement and Shear Stresses

The new torsional wave propagates along the axis of the cylinder, i.e., along the z axis. The mechanical displacement of the new torsional wave in the inner metamaterial elastic cylinder can be expressed as follows:

$$u_\theta^{(1)}(r,\theta,z) = A \cdot f(r) \cdot \exp[j(Kz - \omega t)] \quad (2)$$

Consequently, the shear stress accompanying the new torsional wave in the inner metamaterial elastic rod (cylinder) can be written:

$$\sigma_{r\theta}^{(1)}(r,\theta,z) = \frac{1}{s_{44}^{(1)}} r \frac{\partial}{\partial r}\left(\frac{u_\theta^{(1)}(r,\theta,z)}{r}\right) \quad (3)$$

Similar expressions can be written for the outer conventional elastic space:

$$u_\theta^{(2)}(r,\theta,z) = B \cdot g(r) \cdot \exp[j(Kz - \omega t)] \quad (4)$$

and

$$\sigma_{r\theta}^{(2)}(r,\theta,z) = \frac{1}{s_{44}^{(2)}} r \frac{\partial}{\partial r}\left(\frac{u_\theta^{(2)}(r,\theta,z)}{r}\right) \quad (5)$$

where: K is the wave vector (wave number) of the new torsional elastic surface wave, ω is the angular frequency. A and B are constants. $f(r)$ and $g(r)$ are function of the radius r describing the change in the amplitude of the torsional wave inside the rod and outside the rod, respectively.

B. Equations of Motion

The mechanical displacements of the torsional surface wave: $u_\theta^{(1)}$ in the inner metamaterial elastic rod and $u_\theta^{(2)}$ in the outer conventional elastic space satisfy the following equations of motion written in the cylindrical system of coordinates:

$$\rho_1 s_{44}^{(1)} \frac{\partial^2 u_\theta^{(1)}}{\partial t^2} = \frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^3 \frac{1}{s_{44}^{(1)}} \frac{\partial}{\partial r}\left(\frac{u_\theta^{(1)}}{r}\right)\right) + \frac{\partial}{\partial z}\left(\frac{1}{s_{44}^{(1)}} \cdot \frac{\partial u_\theta^{(1)}}{\partial z}\right) \quad (6)$$

and

$$\rho_2 s_{44}^{(2)} \frac{\partial^2 u_\theta^{(2)}}{\partial t^2} = \frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^3 \frac{1}{s_{44}^{(2)}} \frac{\partial}{\partial r}\left(\frac{u_\theta^{(2)}}{r}\right)\right) + \frac{\partial}{\partial z}\left(\frac{1}{s_{44}^{(2)}} \cdot \frac{\partial u_\theta^{(2)}}{\partial z}\right) \quad (7)$$

C. Boundary Conditions

The continuity of a) mechanical displacement u_θ and b) shear stress $\sigma_{r\theta}$ on the cylinder surface ($r = a$) should be provided, namely:

$$u_\theta^{(1)}\big|_{r=a} = u_\theta^{(2)}\big|_{r=a} \quad (8)$$

$$\sigma_{r\theta}^{(1)}\big|_{r=a} = \sigma_{r\theta}^{(2)}\big|_{r=a} \quad (9)$$

D Dispersion Equation

By substituting Eq.2 into Eq.6 and Eq.4 into Eq.7, we obtain two differential equations for two unknown functions $f(r)$ and $g(r)$. Solution of these two equations gives:

$$f(r) = I_1(\gamma_1 r) \tag{10}$$

$$g(r) = K_1(\gamma_2 r) \tag{11}$$

where: $\gamma_1^2 = \left(K^2 - \omega^2 \rho_1 s_{44}^{(1)}\right)$ and $\gamma_2^2 = \left(K^2 - \omega^2 \rho_2 s_{44}^{(2)}\right)$. I_1 is the modified Bessel function, and K_1 is the Macdonald's function.

Finally, we obtain:

$$u_\theta^{(1)}(r,\theta,z) = A \cdot I_1(\gamma_1 r) \cdot \exp[j(Kz - \omega t)] \tag{12}$$

$$u_\theta^{(2)}(r,\theta,z) = B \cdot K_1(\gamma_2 r) \cdot \exp[j(Kz - \omega t)] \tag{13}$$

Subsequently, we substitute equations (12 and 13) along with corresponding stresses (Eqs. 3 and 5) into the boundary conditions given by Eqs. 8 and 9.

In this way, a homogeneous system of 2 linear equations for the coefficients A and B is produced. Equating the determinant of this system of equations to zero, we obtain the following dispersion equation of the new elastic torsional wave:

$$\frac{K_2(\gamma_2 a)}{K_2(\gamma_2 a)} - \frac{s_{44}^{(1)}(\omega)}{s_{44}^{(2)}} \frac{\gamma_1}{\gamma_2} \frac{I_2(\gamma_1 a)}{I_1(\gamma_1 a)} = F(\omega, K) = 0 \tag{14}$$

IV. RESULTS OF NUMERICAL CALCULATIONS

Numerical calculations were carried out on the example of the waveguide structure in which the outer space is made of PMMA polymer and the inner metamaterial rod is based on ST Quartz with embedded local oscillators. We assume that, the frequency of the local elementary oscillators equals 1 MHz. Losses in the cylindrical waveguide structure are neglected. The exact values of the material parameters used in the calculations are given in [15].

A. Dispersion Curve

The dispersion equation (10), for constant angular frequency ω, can be regarded as a nonlinear algebraic equation for the wave vector K. Solving this nonlinear algebraic equation we obtain a set of pairs (ω, K). This set of pairs determines the dispersion curve of the new torsional wave, see Fig. 2.

$v_2 = \left(\rho_2 s_{44}^{(2)}\right)^{1/2}$ is the phase velocity of the bulk shear acoustic waves in the surrounding conventional elastic space.

B. Phase Velocity

Using the solution of dispersion equation (10), the graph of the phase velocity $v_p(\omega) = \omega/K$, as a function of frequency f was evaluated and presented in Fig. 3. In our calculations the surface resonant frequency $f_{sp} = f_p/\sqrt{1 + s_{44}^{(2)}/s_0}$ is equal to 143.569 kHz.

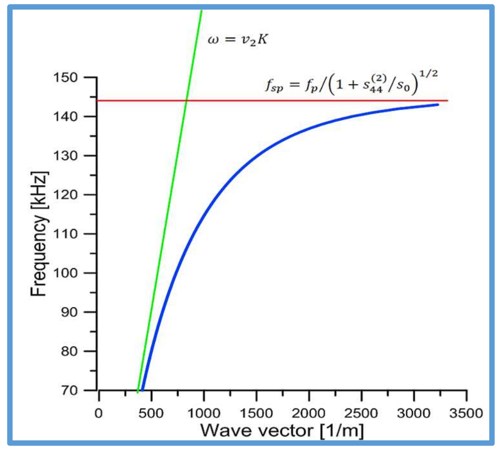

Fig. 2. Dispersion curve (f *versus* K) of new torsional elastic surface waves.

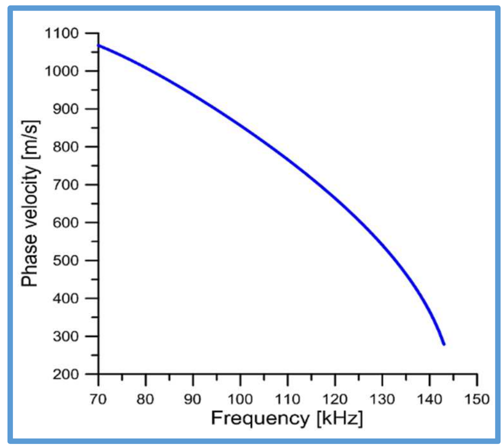

Fig. 3. Phase velocity of the new torsional elastic wave versus frequency.

C. Group Velocity

The group velocity $v_{gr}(\omega)$ of the new torsional elastic wave was calculated analytically using the following formula: $v_{gr}(\omega) = -\frac{\partial F/\partial K}{\partial F/\partial \omega}$. Figure (4) shows the graph of the group velocity $v_{gr}(\omega)$ of the new torsional elastic surface wave as a function of the wave frequency f.

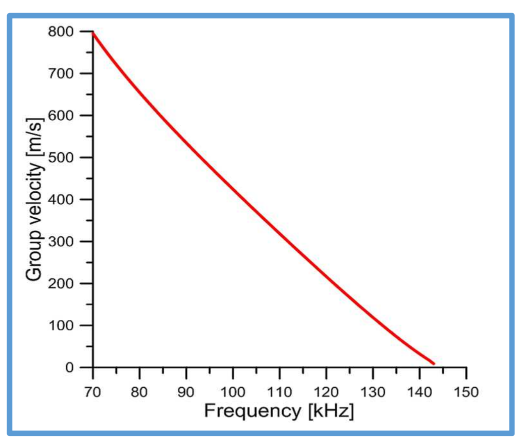

Fig. 4. Group velocity of the new torsional elastic wave versus frequency.

V. Conclusions

1. In this work, we discovered and presented a new ultrasonic torsional wave that propagates in elastic metamaterial rods embedded in a conventional elastic space.

2. The mechanical displacement field of new ultrasonic torsional waves is tangential to the circumference of the elastic cylinder.

3. These new torsional waves are characterized by a strong confinement of the wave energy near the surface of the cylinder.

4. For this reason, new torsional waves are ideally suited for investigating the viscosity of liquids. The new torsion waves are similar to surface polariton plasmon (SPP) electromagnetic waves propagating in complex dielectric-metal rods.

5. This property can result in much larger mass sensitivity compared to the sensitivity of conventional torsional waves.

6. A very fascinating property of the new elastic surface torsional wave is that its phase and group velocities slow down and simultaneously tend to zero as the wave frequency approaches the surface resonant frequency f_{sp}.

7. The new torsional elastic surface wave has a cut-off frequency. Below this cut-off frequency, the torsional wave cannot propagate.

8. The new torsional elastic waves are similar to Surface Plasmon Polariton (SPP) electromagnetic waves propagating in layered dielectric-metal rods.

9. Therefore, the new torsional ultrasonic waves inherit such fascinating properties of SPP as:

a) super-lensing
b) super resolution
c) the ability to amplify evanescent waves

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