BEM FOR INTERFACE PROBLEM OF BI-MATERIAL STRUCTURE UNDER STATIC LOADING*

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ABSTRACT. The behaviour of the interface of a pre-cracked bi-material ceramic-metal structure under static axial loading is an object of interest in the present paper. To solve the problem for an interface delamination of the structure and to determine the debond length along the interface, own 2D BEM code is created and applied. The interface plate is assumed as a very thin plate comparing with others two. The parametric (geometric and elastic) analysis of the debond length and interface shear stress is done. The obtained numerical results are compared with analytical one from 1D Shear-lag analysis of considered structure. The respective comparison is illustrated in figures and shows a good coincidence.

KEY WORDS: BEM, Shear-lag analysis, Layered structure

1. Introduction

The boundary element method (BEM) has been demonstrated to be a viable alternative to the FEM for many engineering problems, due to its futures of boundary-only discretization and high accuracy for stress analysis, especially in fracture mechanics [1, 2]. The meshing for the BEM is also much more efficient than those in other domain-based methods, especially for problems with changing boundaries such as crack propagation problems. Recently, it was shown in [3], both analytically and numerically, that the conventional boundary integral equation can be

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applied successfully to thin structures, such as layered structures, thin films and coatings. It has been shown in [4] that very accurate numerical solutions can be obtained for thin structures with the thickness to length ratio in the micro- and even nano-scales, using the newly developed BEM approach, without seeking refinement of the BEM mesh as the thickness decreases.

The interface strength, toughness and stiffness are important factors affecting mechanical response of the multi-material layered structures. A weak interface induces loss of structure stiffness and strength. On the other hand, a brittle and strong interface may induce excessive cracking of the bonded elements. Interfacial fracture of layered composite materials under mechanical loading was analysed in numerous papers (see, for example [5]).

The main idea of the shear-lag analysis is such an assumption which involves a simplification of in-plane shear stress and decouples the 2D problem into two 1D ones. Hedgepeth [6] was the first who applied shear-lag model to unidirectional composites. In the shear-lag models the hypothesis that the load is transferred from broken fibres to adjacent ones by the matrix shear force is stated. Hence, the matrix shear force is independent of the transverse displacements. In [7-9] the shear-lag approach are applied to the bi-material layered structure with pre-cracked first thin layer.

The present paper is aimed at the behaviour of the interface of bi-material ceramic-metal plates under static axial load. The interface plate is assumed as a very thin plate comparing with the second one and working only in shear stress. To validate the application domain of the shear-lag analysis the problem for a delamination of the interface of a bi-material structure own BEM code is created and used. The numerical model of the structure is considered in 2D plane-strain state. The delamination starts at assumed restrict condition for the value of shear stress of interface. The obtained numerical results are compared with analytical one from 1D shear lag analysis which can give a clear picture for application of 1D shear-lag analysis.

2. Shear-lag and BEM formulations

Consider two elastic plates *A* and *B* with finite lengths $2L$, thickness $2h_A$, $2h_B$, bonded by an interface *I* and loaded in tension by a strain ε_0 and the zero thickness interface in pure shear (Fig. 1). The modified shear-lag model is applied [7], taking into account plasticity and damage of the interface. In the shear-lag model the neglecting the bending effect results on the qualitative values of the stress-strain behavior. The main purpose of this study is to

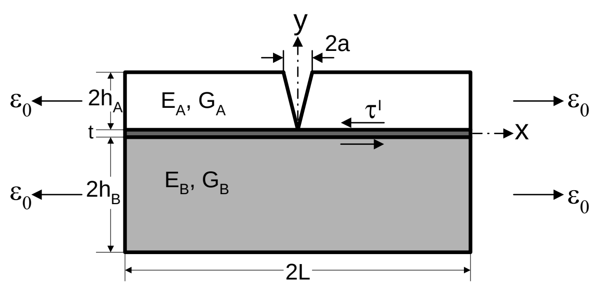

Fig. 1

compare the simple analytical solutions, helping the design of graded materials with the numerical results, obtained using BEM.

The obtained in [7] analytical expressions for the interfacial displacement and shear stress in dimensionless parameters have the form:

$$\text{(2.1)} \qquad \overline{u}_I^e(\overline{x}) = \frac{(1+\xi\eta)}{\overline{\lambda}} \frac{\text{ch}[\overline{\lambda}(\overline{L}-\overline{x})]}{\text{sh}(\overline{\lambda}\overline{L})} \quad , \quad \overline{\tau}^I(\overline{x}) = \overline{G}\,\frac{(1+\xi\eta)}{\overline{\lambda}} \frac{\text{ch}[\overline{\lambda}(\overline{L}-\overline{x})]}{\text{sh}(\overline{\lambda}\overline{L})} ,$$

where $\overline{\lambda}^2 = \dfrac{\overline{G}(1+\xi)(1+\xi\eta)}{2\xi\eta}$ and the following non-dimensional parameters are introduced:

$$\text{(2.2)} \qquad \begin{aligned} &(h_A + h_B)\overline{x} = x\,, \quad (h_A + h_B)\overline{u}_i^e = u_i^e, \quad E_B\varepsilon_0\overline{\sigma}_i^e = \sigma_i^e\,, \\ &E_B\varepsilon_0\overline{\tau}^I = \tau^I\,, \quad E_B\varepsilon_0\overline{G} = G, \quad \xi = \frac{h_A}{h_B}, \quad \eta = \frac{E_A}{E_B}, \quad (i = A, B, I) \end{aligned}$$

The debond length $\bar{l}_e$, which gives the magnitude of the brittle cracking along the interface is given as:

$$\text{(2.3)} \qquad \bar{l}_e = \overline{L} - \frac{1}{\overline{\lambda}} \ln\left[A + \sqrt{A^2 - 1}\right], \qquad A = \frac{\overline{\lambda}\,\overline{\tau}^{cr}\,\text{sh}(\overline{\lambda}\overline{L})}{\overline{G}(1+\xi\eta)}$$

The following boundary integral equations for two-dimensional elasticity problems can be applied in each material domain (index notation is used, where repeated subscripts imply summation) [1]:

$$\text{(2.4)} \qquad C_{ij}(P_0)u_j^{(\beta)}(P_0) = \int_\Gamma [U_{ij}^{(\beta)}(P,P_0)t_j^{(\beta)}(P) - T_{ij}^{(\beta)}(P,P_0)u_j^{(\beta)}(P)]d\Gamma(P)$$

in which $u_j^{(\beta)}$ and $t_j^{(\beta)}$ are the displacement and traction fields, respectively; $U_{ij}^{(\beta)}(P,P_0)$ and $T_{ij}^{(\beta)}(P,P_0)$ the displacement and traction kernels (Kelvin's solution), respectively; P the field point and P_0 the source point; and Γ the boundary of the single domain. $C_{ij}(P_0)$ is a constant coefficient matrix depending on the smoothness of the boundary Γ at the source point P_0. The superscript β on the variables in Eq. (2.4) signifies the dependence of these variables on the individual domains $\beta = A, B, I$.

In Eq. (2.4) the integral containing the $U_{ij}^{(\beta)}(P,P_0)$ kernel is weakly singular, while the one containing $T_{ij}^{(\beta)}(P,P_0)$ is strongly singular and must be interpreted in the Cauchy principal value sense. However, when the structure becomes thin in shape, such as the interphase, both integrals are difficult to deal with when the source point is on the one side and the integration is carried out on the other side of the thin structure. These types of integrals are called nearly singular integrals since the distance r is very small in this case but is still not zero. Recently, several techniques, including singularity subtractions, analytical integration, and nonlinear coordinate transformations have been developed to calculate the nearly singular integrals [4]. The combination of these techniques is found to be extremely effective and efficient in computing the nearly singular integrals in two-dimensional boundary integral equations, no matter how close the source point is to the element of integration.

The discretization of the BIE (2.4) using boundary elements follows the standard BEM procedures, except that the nearly-singular integrals. For multi-domain (material) problems, the resulting BEM equations for each material domains are coupled together by the interface conditions (continuity of both displacements and equilibrium of both tractions) and then solved to obtain the displacement and traction vectors at each node on the boundary and interfaces.

3. Numerical example

On the base of the obtained analytical formuli for an assumed interface shear stress lows, the stress behavior (especially debond length on the interface) of two plates structure with different mechanical and geometric properties under tension load ε_0 will be studied. The bending of the structure is avoided by imposed boundary condition $u_B(x, -(2h_B + t)) = 0$, where t is the thickness of the interface.

The following geometric and mechanical properties (Table 1) are used:

$$2\mathrm{L} = 24\ mm, \quad \left| \begin{array}{l} 2h_A = 2\ mm, \quad 2h_B = 6\ mm \quad (\xi = 1/3) \\ 2h_A = 1\ mm, \quad 2h_B = 6\ mm \quad (\xi = 1/6) \end{array} \right.$$

$$\tau^{cr} = 18\,[\mathrm{MPa}]\ , \quad a = 1\ mm\ , \quad t = 0.1\ mm\ , \quad \varepsilon_0 = 0.001 \div 0.008$$

Table 1. Characteristics of the materials [9]

	Material	E [GPa]	ν
Layer A	**C84** [Al_2O_3/Al composites, (C84=84 vol% Al_2O_3 + 16 vol% Al)]	285	0.28
Layer A	**Alumina** [DEGUSSIT Al 23, Friatec.]	380	0.24
Layer B	**100Cr6** [AISI 52100]	210	0.29
Interphase	**Polyacrylate thermoplastic glue**	2.5	0.50

In Fig.2 the comparison between 1D shear-lag and BEM 2D interface debond length predictions is shown. The values of debond length l_e versus the applied load ε_0 are obtained for two different ratio η of elastic moduli and for two different values of the thickness ratio ξ.

It can be seen, that geometric characteristics influence much more on the debond length than the material characteristics. Considering numerical and analytical results (BEM, shear-lag), bigger the thickness ratio ξ, smaller the value of applied load ε_0^{cr} needed for full delamination of the interface. The critical load ε_0^{cr} calculated using shear-lag at different thickness ratio ξ is much smaller comparing with ε_0^{cr} obtained by BEM. This difference for ε_0^{cr} can be explained with a presence of a normal crack which strongly reflects on the stress-strain behavior (BEM) of the first plate. On the other hand, the very thin first pre-cracked plate plays a significant role for full degradation of bi-material structure, allowing bigger critical load.

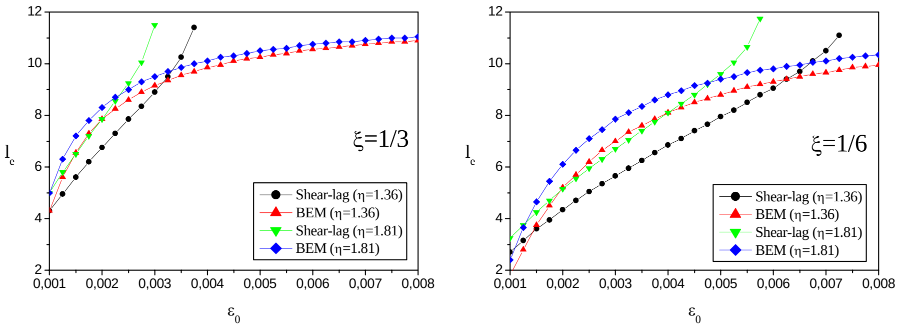

Fig.2.

The relative error of comparison between BEM and shear-lag debond length

$$r = \sqrt{(1/M)\sum_{i=1}^{M}\left(y_i^{theory}/y_i^{BEM} - 1\right)^2}.100\ (\%)\ \text{is:}$$

<u>C84/100Cr6</u>:	<u>Alumina/100Cr6</u>:
$\xi = 1/3$, $\eta = 1.36$, $r = 10.75\%$	$\xi = 1/3$, $\eta = 1.81$, $r = 9.22\%$
$\xi = 1/6$, $\eta = 1.36$, $r = 13.01\%$	$\xi = 1/6$, $\eta = 1.81$, $r = 11.64\%$

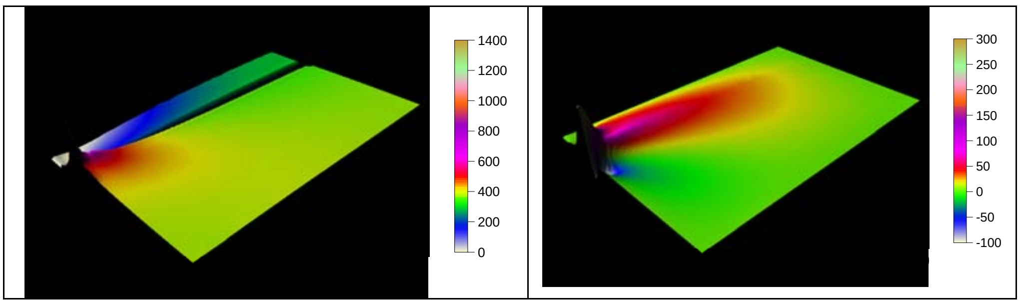

Fig. 3. Plots of the stresses $\sigma_{xx}(x,y)$, $\sigma_{xy}(x,y)$ [GPa] for pre-cracked bi-material structure

The investigations show that bigger the load, bigger the relative error. The decreasing of the thickness of the first plate also leads to increasing the relative error. It is a consequence of negligible thickness of the interface as well as that the approximate analytical shear lag model is 1D.

In Fig. 3 the numerical BEM results for stresses of bi-material structure for $\xi = 1/3$ and $\eta = 1.81$ are shown (as an example). The loading is $\varepsilon_0 = 0.0025$.

4. Discussion

In the paper the comparison between approximate shear-lag 1D method and 2D BEM for interface delamination of the bi-material structure under static load is done. The relative error between analytical and numerical results confirms the validity of the shear lag approach. The obtained predictions can be applied to a pre-cracked by an indentor bi-material structures under static tension for different mechanical behaviour and materials of plates.

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