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Wavelet approximations and statistical approach to random fluctuations of amplitude in backscattered ultrasonic signal

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Summary

The goal of this study was to find the macroscopic characteristics of the random nature of ultrasonic backscattering signals which would be sensitive to the temperature changes. The sample made of Polyvinyl Alcohol – Cryogel (PVA-C, the pre-freezing in one cycle aqueous solution of PVA) was heated in a water bath starting from the room temperature up to the temperature below the soft tissue ablation temperature. The RF signals were collected during the heating/cooling process and the signals envelopes had been calculated. The wavelet approximation of subsequent level worked as a low-pass filter what qualitatively improved the temperature estimating. The latter was realized by observing the variations of the shape parameter of K-distribution. The trend of the shape parameter variation with temperature was calculated including the wavelet decomposition and was compared with the real temperature changes measured by the thermometer. We have found that tracking changes in echoes envelope statistics allows to distinguish between heating and cooling process, and determine the time required to reach maximum temperature.

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1. Introduction

Therapeutic and surgical applications of low and/or high intensity focused ultrasound require monitoring of local temperature rises induced inside tissues, cf. [1, 2]. In this paper, one of the methods from the quantitative ultrasound research is used for the assessment of the soft tissue phantom microstructure changes caused by temperature increase/decrease. Namely, the dependencies between the variations in statistics of ultrasonic eches envelope and temperature variations are studied. The randomness of the RF signal envelope is described by the probability distribution function (PDF) of K-distribution. The PDF is characterized by two parameters, the scale and the shape parameter. In [3-5] the sensitivity of the shape parameter values to the number of scatterers within the resolution cell has been justified. Assuming existence of the direct relation between temperature level and scatterers statistics the values of the shape parameter of K-distribution probability function of the signals envelopes have been used to estimate the temperature changes, cf. [6]. In the paper the shape parameter values calculated for the different levels of wavelet approximations of different levels of the envelope signals are discussed from the point of view of their usefulness to improve the estimation of temperature. The wavelet analysis is also used to smooth the curve of the shape parameter temperature dependence, obtaining more accurate tracking of temperature process.

The backscattered FR signals had been registered during 1 hour of the temperature increase from T_0=20.6 ^{o}C to T_1=48.8 ^{o}C and subsequently temperature decrease from T_1=48.8 ^{o}C to T_3= 25.8 ^{o}C during next 2 hours. The description of the performed experiment and short introduction to the wavelet analysis are given in Section 2.

Then the role of the wavelet applications is presented in Section 3. There wavelet method is applied in two steps. First, the wavelet approximations of signals are the starting point for evaluation the shape parameter temperature dependence. It is demonstrated that the use of the wavelet approximations allows for better (less noisy) estimation of temperature dependence in comparison to the case when non-approximation signal is used. Secondly, the function describing the variation of the shape parameter with temperature is decomposed by the wavelet decomposition obtaining the approximations and the details. The comparison of the calculated temperature increase/decrease including the wavelet approximations, and the experimentally measured temperature changes is done in Section 4.

2. Elements of wavelets theory and experiment description

2.1. Wavelets

The main idea of this method is that any function $s(t)$ integrable with the second power may be represented in the form

$$s(t) = \sum_k g_{j_n,k}\varphi_{j_n,k} + \sum_{j \ge j_n} h_{j_n,k}\psi_{j_n,k}\ , \qquad (1)$$

where ψ is a chosen wavelet function, φ is the corresponding to ψ scaling function. The set of functions $\{\psi_{j,k}\}$ is constructed as

$$\psi_{j,k} = 2^{\frac{j}{2}}\psi\left(2^j t - k\right),$$

the set $\{\varphi_{j,k}\}$ may be obtained by using the additional relations for functions ψ and φ (see, e.g. [7]). The value of j_n denotes the level of decomposition whose scale coefficients g and detail coefficients h are calculated for each chosen wavelet family.

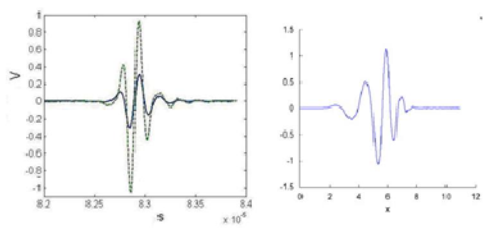

Fig. 1. Reflected ultrasonic impulse and Daubechies 6 wavelet.

The Daubechies 6 wavelets family [7] had been chosen as analyzing wavelet due to its pre-defined properties and the special form of the function which is similar to the shape of the transmitted ultrasonic signal (see Fig. 1). In the left image of the Fig. 1 the solid line represents the transmitted signal, the dash line represents the reflected impulse signal in water, in the right image there is Daubechies 6 wavelet function. For statistical analysis we use the signal wavelet approximation and reconstruction up to 5th level. The whole procedure is described in [6], and it was also used for the investigation the RF-signals in [8-9].

2.2. Experiment

The PVA-C sample was heated in a water bath during 1 hour and was cooled by 2 hours. The thermostat controlled the linear increase of temperature from 20.6^{o}C to 48.8^{o}C during one hour. Next the heating was switched off and after 2 hours temperature of water reached 25.8^{o}C. Measurements of RF signals had been carried out by means of ultrasound (ULTRASONIX SonixTOUCH, British Columbia, Canada) with the ultrasonic head (L14-5/38) was placed over the sample in the bath. Data were collected at frequency of 8 MHz every half minute (361 images), details are described in [10]. The transmitted pulse comprises 2 periods of the sine wave of 0.25 microseconds duration. Ultrasonic data, RF echoes have the form of the complex analytical signal, which module gives the envelope the RF signal that was stored as a $1001 \times 501 \times 361$ matrix. In calculation this data was limited to heated area only what reduced the matrix to the $601 \times 501 \times 361$ matrix.

2.3. Statistics

The K-distribution was chosen as the best estimation of the obtained data and according to the reults of the Kolmogorov-Smirnov two-way test at the significance level of 0.05.

The probability density function of K-distribution depending of two parameters has the form:

$$f(x) = \frac{2b}{\Gamma(\alpha)}\left(\frac{bx}{2}\right)^{\alpha} K_{\alpha-1}(bx), \qquad (2)$$

where x represents the amplitude of the signal, $a > 0$ and $b > 0$ are scale and shift distribution parameters respectively, Γ is the Euler gamma function, and $K_{\alpha-1}$ denotes the modified Bessel function of the second kind of order $\alpha - 1$. The follwing parameter was taken as a esimator of the number of scatterers

$$\alpha_0 = \frac{2}{r_4 - 2} = \frac{2}{\dfrac{E(x^4)}{\left(E(x^2)\right)^2} - 2}. \qquad (3)$$

Here the $E(x^2)$ and $E(x^4)$ denotes the second and the fourth moments of the signal's envelope amplitude respectively.

3. Wavelet applications

3.1. Wavelet approximations of signal

The procedure of pre-transformations described in [6] have been performed for the RF signal envelopes. Than there had been obtained the decomposition with Daubechies 6 wavelets up to the 5th level with respect to each image $t = 0, 1, \ldots 360$. There were constructed the functions $s_{app}(t)$ and, which denote the approximation and reconstruction, respectively. It is important to note that the reconstruction of the function $s_{rec}(t)$ may be considered as the function $s(t)$ because the estimated difference is of the order 10^{-7}, i.e.

$$|s(t) - s_{rec}(t)| < 10^{-7}.$$

Next the shape parameters of K-statistics $\alpha_{0,app}(t)$ and $\alpha_{0,rec}(t)$ were calculated with formula (3) for different levels of the wavelet approximation $s_{app}(t)$ and reconstruction $s_{rec}(t)$.

The dependence $\alpha_{0,app}(t,T)$ on the time and temperature simultaneously had been discussed in [6]. The values $\alpha_{0,app}(t)$ and $\alpha_{0,rec}(t)$ are examined in order to show that there is more precise to investigate the statistical papameters of wavelet approximation then the signal envelope itself as it was made in [10]. Hence we consider the functions $\alpha_{0,app}(t)$ and $\alpha_{0,rec}(t)$, $t = 0, 1, \ldots 360$ as the initial data. First the full decomposition with Daubechies 6 was made. According to the length of each dataset equal to 361, the full decomposition consisted of 8 levels as $256 = 2^8 < 361 < 512 = 2^9$. These decompositions are presented in the Figs. 2 - 3.

The notations the images are as follows: on the left *s* is the original, i.e. the $\alpha_0(t)$ dependence for each case, a_i are the i^{th} level approximation coefficients, $i = 1..5$. On the right: *cfs* represent the coefficients distribution at all 5 levels respectively, and d_i are the i^{th} level detail coefficients, $i = 1..5$. The approximation and detail coefficients form low-pass and high-pass filters correspondingly.

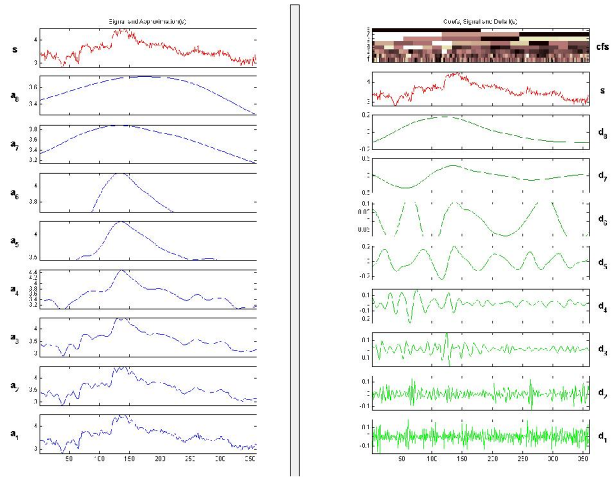

Fig. 2 The full decomposition of the parameter $\alpha_{0,app}(t)$.

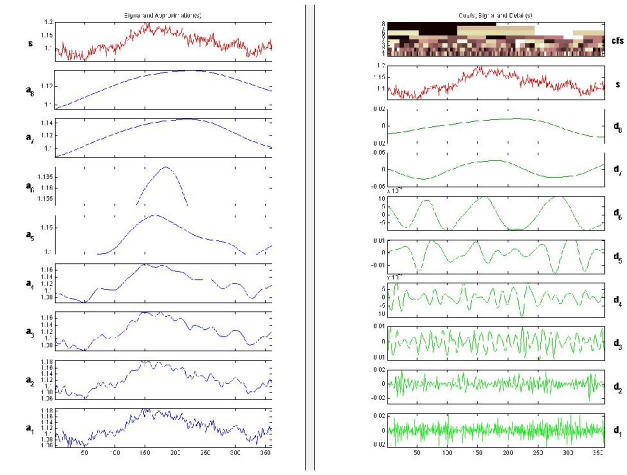

Fig. 3 The full decomposition of the parameter $\alpha_{0,rec}(t)$.

The initial function $s(t)$ may be composed from the approximation coefficient a_8 and detail coefficients d_j, $j = 1,\ldots 8$, by formula (1). Here $n = 8$, so

$$s_{rec} = a_8 + \sum_{j=1}^{8} d_j, \quad s_{app,8} = a_8,$$

where $a_8 = \sum_k g_{8,k}\varphi_{8,k}$ and $d_{j_n} = \sum_{j \ge j_n} h_{j_n,k}\psi_{j_n,k}$.

To approximate the parameters α_0 there was chosen the 5th level of decomposition (Figs. 4 - 5).

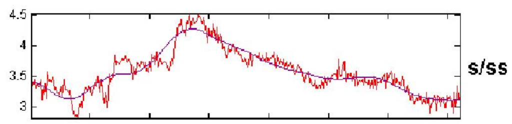

Fig. 4. The 5th level wavelet approximation for $\alpha_{0,app}(t)$.

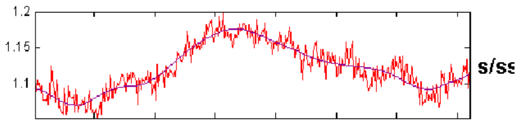

Fig. 5. The 5th level wavelet approximation for $\alpha_{0,rec}(t)$.

In Figs 4-5 s denotes the original functions $\alpha_{0,app}(t)$ and $\alpha_{0,rec}(t)$ and ss shows their 5th level approxiatiom which is called the synthesized signal.

The values of descriptive statistics $\alpha_{0,app}(t)$ and $\alpha_{0,rec}(t)$ itself, their 5th level approximation and the residuals as well as for their normalized values $\bar{\alpha}_{0,app}(t)=\frac{\alpha_{0,app}(t)}{\max\limits_{0\le t\le 360}\{\alpha_{0,app}(t)\}}$ and $\bar{\alpha}_{0,rec}(t)=\frac{\alpha_{0,rec}(t)}{\max\limits_{0\le t\le 360}\{\alpha_{0,rec}(t)\}}$ are given below in the tables. There were computed and analyzed the following characteristics: mean, minimum and maximum values, range, standard deviation (SD), mean absolute variation (MAV).

Table 1. Statistics on the $\alpha_0(t)$ and normalized $\alpha_0(t)$.

	Non-normalized		Normalized	
	$\alpha_{0,app}(t)$	$\alpha_{0,rec}(t)$	$\alpha_{0,app}(t)$	$\alpha_{0,rec}(t)$
Mean	0,357	1,12	0,7909	0,9345
Maximum	4,509	1,199	1	1
Minimum	2,835	1,054	0,6287	0,8789
Range	1,674	0,1452	0,3713	0,1211
SD	0,364	0,0337	0,0807	0,0281
MAV	0,289	0,0283	0,0641	0,0236

Table 2. 5th level approximation

	Non-normalized		Normalized	
	$\alpha_{0,app}(t)$	$\alpha_{0,rec}(t)$	$\alpha_{0,app}(t)$	$\alpha_{0,rec}(t)$
Mean	3,565	1,12	0,7907	0,7905
Maximum	4,277	1,176	0,9487	0,9544
Minimum	3,111	1,07	0,69	0,6799
Range	1,167	0,1062	0,2588	0,2746
SD	0,335	0,031	0,0742	0,0745
MAV	0,2695	0,263	0,0598	0,0622

Table 3. Residuals statistics.

	Non-normalized		Normalized	
	$\alpha_{0,app}(t)$	$\alpha_{0,rec}(t)$	$\alpha_{0,app}(t)$	$\alpha_{0,rec}(t)$
Mean	0,00084	0,00014	0,00019	0,000115
Maximum	0,3218	0,04194	0,07137	0,03498
Minimum	-0,4902	-0,0359	-0,1087	-0,02994
Range	0,812	0,07784	0,1801	0,06492
SD	0,1408	0,01362	0,03122	0,01136
MAV	0,11	0,01109	0,02441	0,009247

From the table 2 it follows that the parameters obtained for the wavelet approximations are better in the statistical sense, namely, it has less range, less variance and less mean absolute variation than the same obtained for the reconstructed signal.

Comparing the properties of the normalized signal it occurred that the wavelet approximation has less ratios of standard variation to range and mean absolute variation to range than the reconstructed signal (see table 4).

Table 4.

	$\alpha_{0,app}(t)$	$\alpha_{0,rec}(t)$
SD/Range	0,217	0,232
MAV/Range	0,173	0,195

Besides, the residuals also fulfill the same relations, i.e. residua of wavelet aproximation has less ratios of the stanard variation to the range and mean absolute variation to range than the residuals of reconstructed signal (see table 5).

Table 5.

Residuals	$\alpha_{0,app}(t)$	$\alpha_{0,rec}(t)$
SD/Range	0,173	0,175
MAV/Range	0,135	0,195

3.2 Wavelet approximation of $\alpha_0(t)$

Here the wavelet decomposition of $\alpha_{0,app}(t)$ and $\alpha_{0,rec}(t)$ at th 4th and 5th levels was performed and the comparison of the corresponding approximation and the temperature changes measured experimentally was made. The behavior of α_0 as a function of the time/temperature is depicted in Fig. 5.

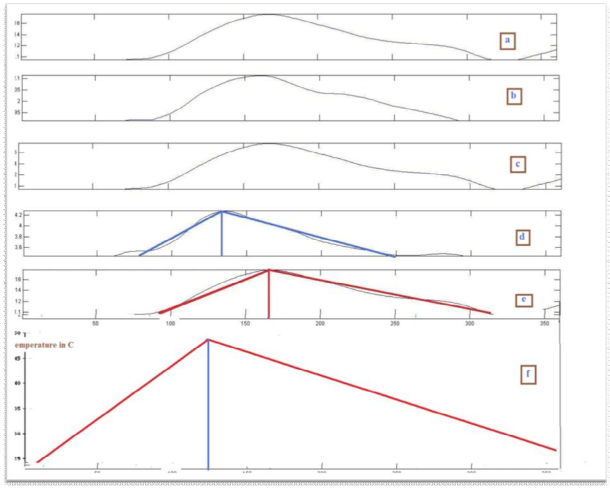

Fig. 5. The trend of the $\alpha_0(t)$

In Fig.5 there the trend of the $\alpha_0(t)$ versus variable time variable (minutes) are presented calculated for different levels of approximation:

a) $\alpha_{0,rec}(t)$ at the 4th approximation level;

b) $\alpha_{0,app}(t)$ at the 4th approximation level;

c) $\alpha_{0,rec}(t)$ at the 5th approximation level;

d) $\alpha_{0,app}(t)$ at the 5th approximation level

e) $\alpha_{0,rec}(t)$ at the 6th approximation level;

f) the experimentally measured temperature changes.

As it is seen from these figures, the $\alpha_{0,app}(t)$ at the 5th level (image d) represents the most correct temperature behaviour. This also explains the choice of the approximation level in the previous part.

4. Final remarks

In the first step of wavelet application we analyzed the approximations and reconstructions of the signal at different levels of wavelet decomposition. The successive approximations of signal envelopes at different levels are working as low-pass filters. That means they “throw away” information about the high frequency components in the backscattering field, called “details” in the wavelet decomposition. The details have the ranges of values one or more order of magnitude smaller than the range of approximations. In the full reconstruction they all are added and it is clear that neglecting the details in the signal approximation changes the shape of the histogram. The value of the shape parameter of K-distribution designated for this histogram has been increased. The approximation of the signal looks now like scattered signal from the ensemble of larger number of effective scatterers. The process of filtering is stopped on that level of approximation in which the approximation is still enough reach to design random scattering process. It was calculated by the Smirnov-Kolmogorov test that starting from the 6th level of approximation we lose the opportunity to prove the hypothesis that the approximation values are drawn from the K-distribution. It is a reason that an estimation of the whole amplitude the 5th level of wavelet approximation is considered. The comparative analysis between two different functions of the shape parameter temperature dependence have been performed. Namely, calculations performed on the whole data: $\alpha_{0,rec}(t)$, and on the basis of the 5th level of wavelet approximation $\alpha_{0,app}(t)$. The statistical analysis of shape parameter time dependence underlines the positive role of the wavelet filtering, the $\alpha_{0,app}(t)$ has less range, less variance and less mean absolute variation than the $\alpha_{0,rec}(t)$, so it is less noisy and will be better candidate for temperature marker.

In the second area of the wavelet application the full decomposition of the shape parameter of K-distribution α_0 as a function of time (temperature) has been calculated for two cases, the transformed signal approximation and the reconstructed signal, independently. The trend, understand here as 5th level of approximation, of the shape parameter time (temperature) dependence is more similar to temperature changes experimentally measured by the thermometer. Besides, the behavior of the detail coefficients in the cooling period is significantly different in the case α_0 being decomposition for signal 5th level approximation from the cooling period, there is no symmetry at all. In the second case (the signal 5th level reconstruction) there may be noted the symmetry of heating and cooling periods, what is in contradiction to the real temperature changes.

Additionally, let us notice, that the time point corresponding to the maximal temperature measured with thermometer is much more close to the maximal value of the $\alpha_{0,app}(t)$ at the 5th level than to the maximal values of the $\alpha_{0,rec}(t)$ at the same level. It is due to the next beneficial role of the wavelet application. The short wavelength contributions (in the wavelet decomposition this are details) in the backscattered amplitude provides to the fictive translation of the maximal temperature position. If they are thrown away, i.e. only approximations are used for calculations, the shape parameter much better fulfills the role of the “acoustical thermometer”.

Acknowledgement

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