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Sound absorption of porous layers of loosely-packed rigid spheres: multiscale modelling and experimental validation

Tomasz G. Zieliński
Institute of Fundamental Technological Research, Polish Academy of Sciences, ul. Pawinskiego 5B, 02-106 Warsaw, Poland.

Summary

Sound absorption in porous media with rigid structure and open porosity is most often modelled using the so-called fluid-equivalent approach, in which a porous medium is substituted by an effective dispersive fluid. There are many models of that kind. Perhaps the most frequently used and efficient one is the so-called Johnson-Champoux-Allard-Lafrage model, or its variations. This a rather advanced semi-phenomenological model with six to eight parameters; with enhancements by Pride and Lafarge, there are eight parameters, namely: the total open porosity, the tortuosity, the (viscous) permeability, the thermal permeability, the viscous and thermal characteristic lengths, and finally, the viscous and thermal tortuosities at low frequency limit (i.e., at 0 Hz). Although, most of these parameters can be measured, it is sometimes very problematic and requires various experimental facilities, which makes the idea of calculation of these parameters from the geometry of microstructure of porous medium very tempting – such multiscale modelling requires, however, some periodic yet sufficiently realistic representation of the actual porous geometry. In this paper such multiscale modelling is presented for the problem of sound absorption in layers composed of loosely-packed rigid spheres. Since the spheres are identical, the packing, although not dense, tends to be semi-regular. Therefore, some regular sphere packings are used to construct periodic Representative Volume Elements for such porous media – they are, however, modified a bit by shifting the spheres in order to fit exactly the actual measured porosity. Basing on such numerical representations of porous microstructure, all the necessary parameters are calculated from finite-element solutions of some relevant Boundary-Value Problems and the effective characteristics for equivalent fluid are determined. Then, the acoustic absorption coefficients are computed for a porous layer of specified thickness for some wide frequency range and the results are compared with the experimental curve obtained from the measurements of such layer carried out in the impedance tube using the so-called two-microphone transfer function method.

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1. Introduction

Sound propagation in air-saturated random packings of spherical beads were investigated experimentally by Charlaix et al. [1], and more thoroughly by Allard et al. [2] who also compared measurements with the prediction calculated from the Johnson-Champoux-Allard-Lafarge model [3]. This was a classic, purely macroscopic approach. A work where the sound absorption of a porous medium composed of rigid spheres is calculated from its microstructure was published in 2005 by Gasser et al. [4]; in this paper the very regular microgeometry of porous medium is represented by the face centred cubic (FCC) packing of spheres with porosity 26%. Similar microstructure-based approach has been applied by other authors to various porous and fibrous materials [5, 6, 7, 8, 9, 10]. Boutin and Geindreau [11, 12] used the homogenization of periodic media and the self-consistent method for porous media represented by sphere and polyhedron packings to study and estimate some characteristics relevant for sound absorption, namely: the dynamic and thermal permeability, diffusion and trapping constant. Lee et al. [13] used regular, close packings of spheres as microstructural geometry representations to compare between the so-called direct

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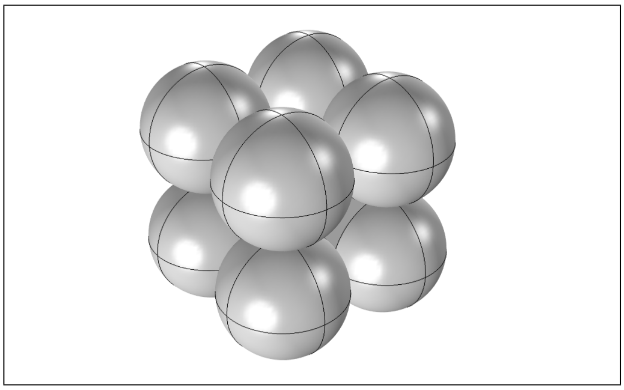

Figure 1. SC packing with slightly overlapping spheres.

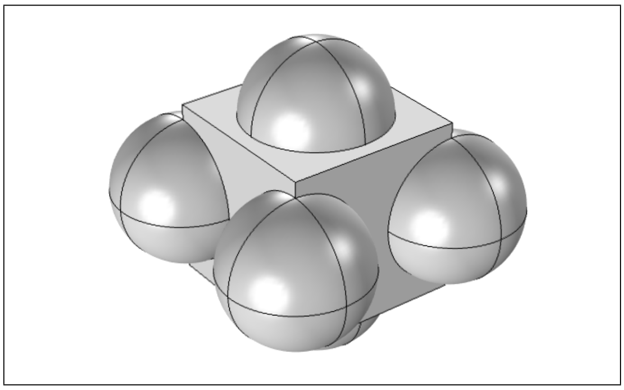

Figure 2. BCC packing with spheres slightly shifted apart.

and hybrid approach in microstructure-based modelling; in this purely numerical study the numerical calculations are also confronted with some results found in literature. The present work and its companion [14] apply the hybrid microstructure-based approach to model sound absorbing porous layers of loosely-packed rigid spheres and validate these approach with the results of original experimental tests.

2. Sphere packings and Representative Volume Elements

In order to apply the multiscale approach to modelling sound absorption of porous layers of loosely packed rigid spheres a typical geometry must be chosen and recognized as sufficiently representative for the microstructure of such layers. The considered porous layers are composed of identical spheres (with diameter of 5.9 mm) which suggests that regular sphere packings may be considered for the purpose of generation of the so-called Representative Volume Elements (RVEs), especially because the RVEs should be rather simple and small, that is, they should typically contain no more than a few spheres.

In the present paper two such packing are considered: the so-called Simple Cubic (SC) and Body-Centred Cubic (BCC). The porosity of SC packing is 47.6% and the porosity of BCC packing 32%. It was found that the first value is higher while the second one is lower than the actual porosity of 42% which had been experimentally found for the actual porous layers tested in laboratory. The actual value was found by estimating the number of spheres (based on their weight) contained in the cylindrical layer tested in the impedance tube and than subtracting their total volume from the volume of layer.

The porosity is an extremely important parameter. In order to match exactly the porosity of sphere packings with the actual value, in the SC case the spheres are slightly overlapping while in the BCC case they are slightly shifted apart. Such packings together with the corresponding cube of RVE are shown in Figures 1 and 2. The corresponding fluid domains of Representative Volume Elements and the finite-element meshes generated on them are presented in Figures 3 and 4. It should be noticed that the RVE with SC packing contains one whole solid sphere, whereas the one with BCC packning contains two whole spheres.

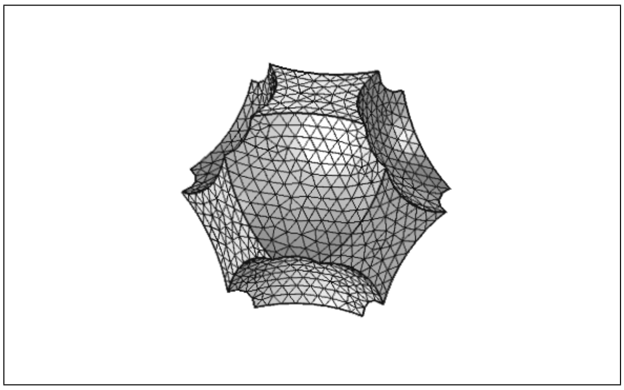

Figure 3. Finite element mesh of fluid domain in the RVE with the overlapping SC packing.

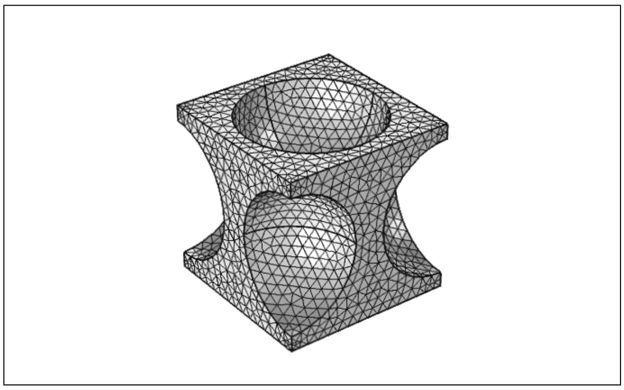

Figure 4. Finite element mesh of fluid domain in the RVE with the loosen BCC packing.

3. Multiscale modelling of sound absorbing porous media

The so-called hybrid approach for multiscale modelling of sound absorbing porous media have been recently applied to study various materials [7, 13, 8, 9]. It requires calculation from microstructure of several static parameters relevant for viscous or thermal effects in low- or high-frequency regimes. These parameters are computed from three steady, static (i.e., non-harmonic) analyses defined on the fluid domain of a Representative Volume Element, and eventually they are used to approximate some essential harmonic characteristics of a porous medium (the so-called dynamic tortuosities) in the whole considered frequency range.

The three static analyses defined on the fluid domain of a representative micro-structural geometry are:

- the Stokes flow (i.e., the steady viscous flow) caused by a unit pressure gradient constant throughout the fluid domain, with no-slip boundary conditions on the fluid-solid interface;
- the steady heat transfer caused by a unit heat source constant in the fluid domain, with isothermal boundary conditions on the fluid-solid interface;
- the Laplace problem (the potential flow) with a unit source constant in the fluid domain.

The microstructure-based parameters are determined from the rescaled solutions of those analyses, averaged over the fluid domain, and using some appropriate formulas. Thus, the viscous flow analysis allows to determine the parameter of static (viscous) permeability and the low-frequency (i.e., at 0 Hz) limit of viscous dynamic tortuosity. The heat transfer analysis permits to calculate the thermal analogue of static permeability and the low-frequency limit of the thermal dynamic tortuosity. Finally, from the Laplace analysis the classical parameter of tortuosity is calculated (which is in fact the high-frequency limit of the viscous dynamic tortuosity), as well as the viscous characteristic length. The thermal characteristic length is computed as the ratio of the doubled volume of fluid domain to the surface of fluid-solid interface.

The finite element meshes constructed on the fluid domains of two RVEs discussed in the previous Section were used to solve the analyses listed above using the Finite Element Method. Then, the geometric parameters were calculated. They are presented in Table I. These parameters, together with the parameter of total open porosity of 42%, served to compute the so-called viscous and thermal dynamic tortuosities of rigid porous media represented by the proposed RVEs. These frequency-dependent functions allowed to determine the effective density and bulk modulus of such media, and eventually, the effective frequency-dependent and complex-valued function of speed of sound. Such calculations were carried out for both considered representations of porous geometry.

Table I. Parameters computed from microstructure using two different RVEs with porosity 42%.

Parameter	**SC$_{42\%}$**	**BCC$_{42\%}$**
viscous permeability [m^2]	5.46×10^{-8}	4.52×10^{-8}
thermal permeability [m^2]	1.46×10^{-7}	8.03×10^{-8}
tortuosity (at ∞ Hz) [–]	1.5263	1.3245
viscous tort. at 0 Hz [–]	2.3052	1.9343
thermal tort. at 0 Hz [–]	1.4438	1.3141
viscous char. length [mm]	0.9900	1.1054
thermal char. length [mm]	1.5573	1.4268

4. Results of modelling and experimental testing

When the effective speed of sound was determined, the problem of plane harmonic acoustic wave propagation into a rigid porous layer set on rigid wall could be solved, and the surface acoustic impedance at normal incidence for porous layer with thickness of 106 mm was calculated using both RVEs, in the frequency range from 500 Hz to 6 kHz. Figures 5 and 6 show the real and imaginary parts of the ratio of surface acoustic impedance of porous layer to the characteristic impedance of air (i.e., the fluid in pores).

The surface acoustic impedance of porous layer and the characteristic impedance of air (which means now the fluid adjacent to the porous layer) were used to determine the so-called reflection coefficient, from which the acoustic absorption coefficient was calculated, see for example [15]. Figure 7 compares the results of those microstructure-based sound absorption calculations for SC$_{42\%}$ and BCC$_{42\%}$ RVEs with the acoustic absorption measured experimentally for the 106 mm-thick porous layer of rigid spheres set in the vertically positioned impedance tube, using the two-microphone transfer function method [16], in the frequency range from 500 Hz to 6 kHz. The agreement between the experimental and numerical results is good, although the latter ones are slightly underestimated with respect to the measured absorption. Moreover, it is easy to notice that with respect to the frequency the absorption peaks found experimentally are somehow between the corresponding peaks in the results obtained for the RVE with SC packing of overlapping spheres and the results computed from the RVE with BCC packing of spheres shifted apart; in general, the discrepancies increase with the frequency.

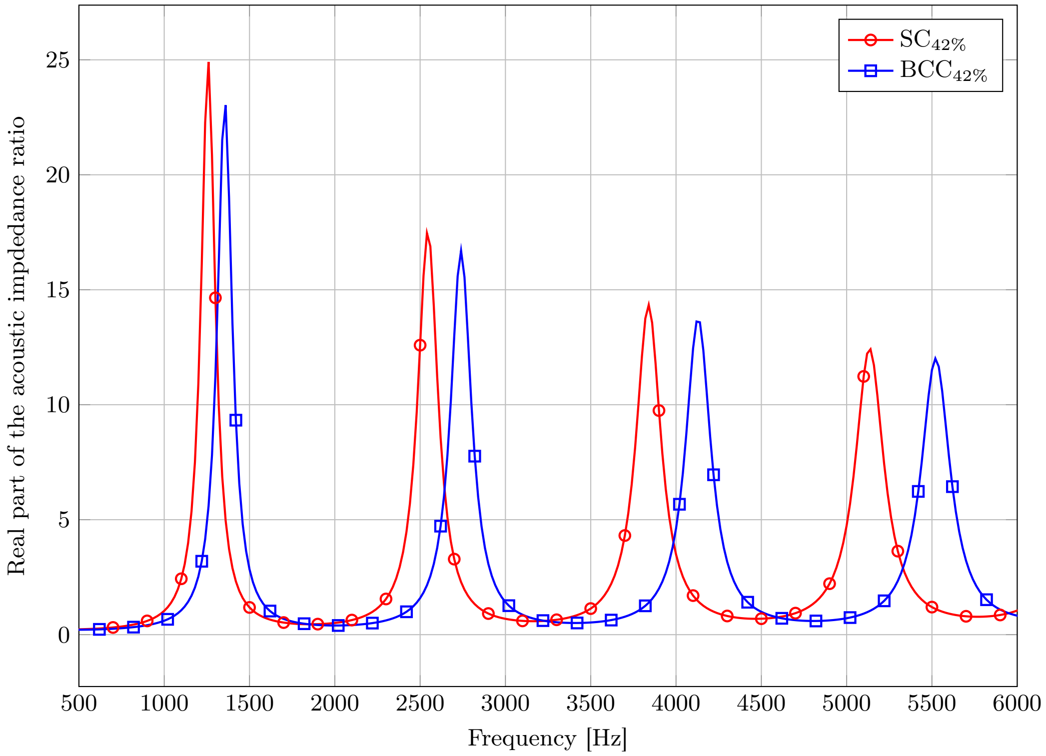

Figure 5. Real part of the ratio of surface acoustic impedance to the characteristic impedance of pore-fluid (air) for 106 mm-thick porous layer of identical spheres with diameter of 5.9 mm.

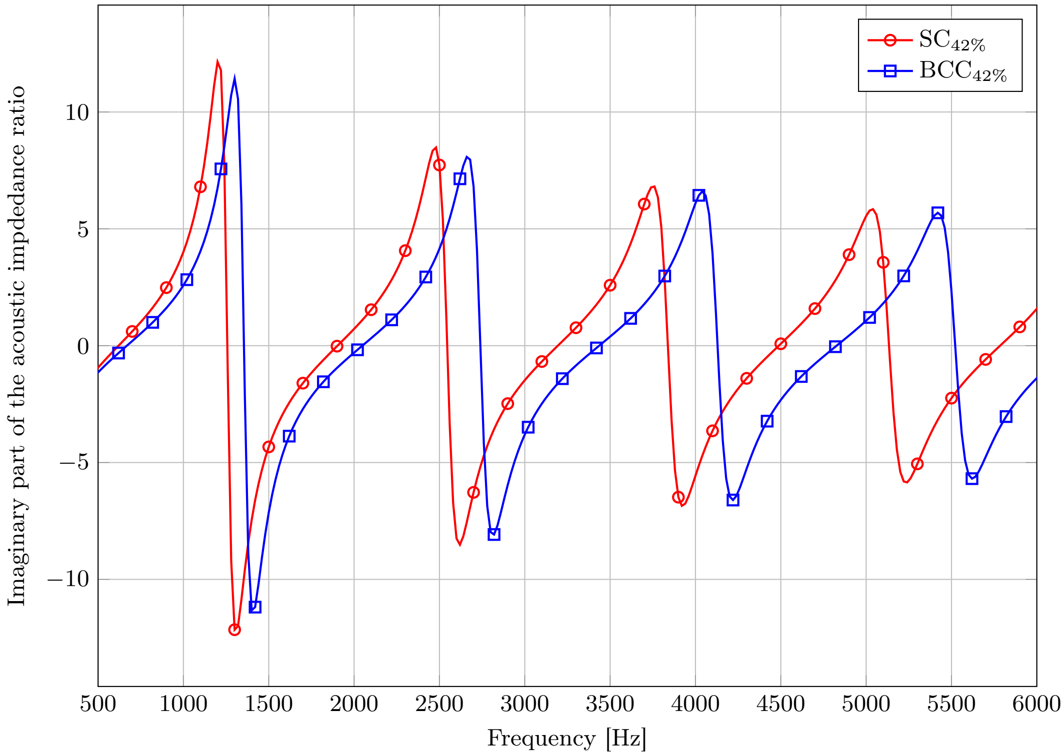

Figure 6. Real part of the ratio of surface acoustic impedance to the characteristic impedance of pore-fluid (air) for 106 mm-thick porous layer of identical spheres with diameter of 5.9 mm.

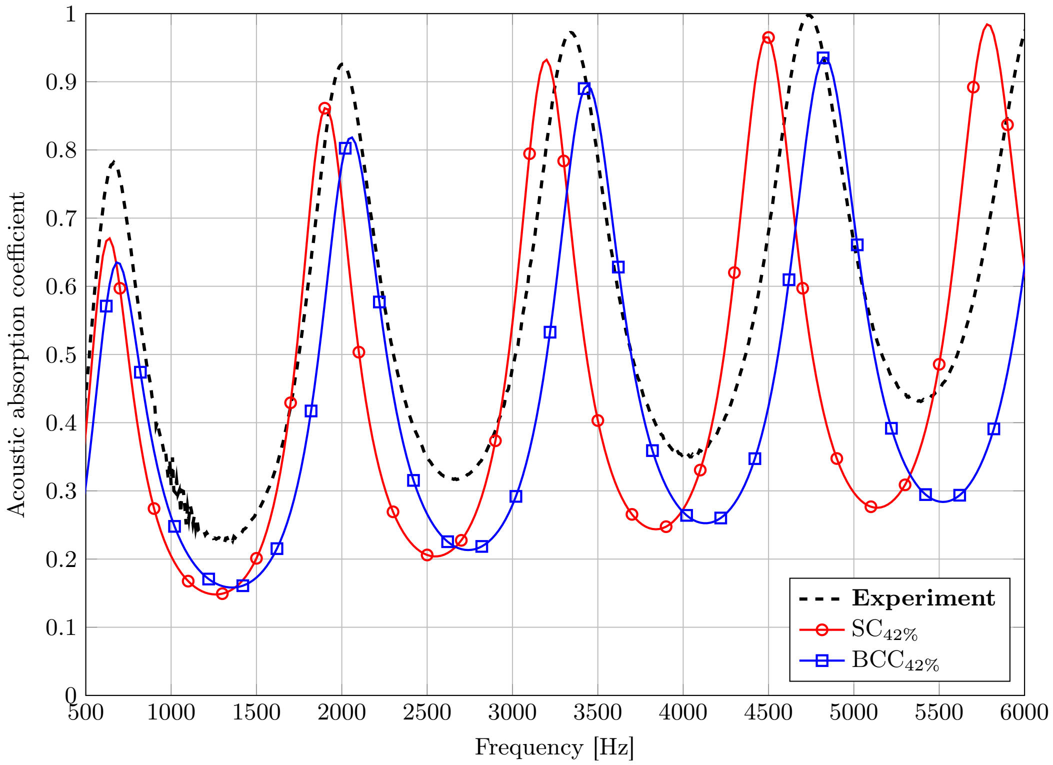

Figure 7. Acoustic absorption of 106 mm-thick porous layer of identical spheres with diameter of 5.9 mm.

5. Concluding remarks

- Two kinds of regular sphere packing – the SC type and BCC type – were used to represent the microstructural geometry of porous layers composed of loosely-packed rigid spheres of the same diameter of 5.9 mm.
- In order to fit exactly the actual porosity of layers of app. 42% (found from the estimated number of spheres in cylindrical layers of known size), the spheres in the SC geometry were allowed to slightly overlap, whereas in the BCC case the spheres were slightly shifted apart.
- For both types of packing, the Representative Volume Elements were constructed and used by finite element analyses to calculate seven microstructural parameters, which – together with the parameter of total porosity as well as some property parameters of air in pores – were then used to determined the effective density and bulk modulus, and eventually, the frequency-dependent complex effective speed of sound in the multiscale modelled porous media.
- The effective speed of sound allowed to calculate the surface acoustic impedances for porous layers at normal incidence, and then to determine the corresponding acoustic absorption coefficient. The numerical results obtained in that way for a layer with thickness 106 mm were compared with the measurements carried out for such layer in the impedance tube in the frequency range from 500 Hz to 6 kHz.
- It has been observed that the multiscale modelling provided quite correct though slightly underestimated results of acoustic absorption. The frequency related peaks in sound absorption performance are well represented by calculations based on both RVEs – they are rather close to each other for the two cases, especially at lower frequencies.
- Moreover, it is easy to notice that the experimental result is somehow in the middle between the one computed from the SC-based RVE with slightly overlapping spheres and the one obtained from the BCC-based RVE with spheres slightly shifted apart, however, the BCC case seems to be more accurate with respect to the experimental result.

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