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# MODELLING OF POROELASTIC LAYERS WITH MASS IMPLANTS IMPROVING ACOUSTIC ABSORPTION

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## ABSTRACT

The paper presents the modelling and frequency analysis of poroelastic layers with heavy solid implants where an improvement of acoustic absorption at lower frequencies is observed. To model the porous material the Biot's theory of poroelasticity is used while the solid implants are modelled in two ways: first, as small subdomains of elastic material (steel) situated inside the porous layer, and for the second time, in a more virtual manner (mathematically equivalent to the presence of masses in the given points), as some adequate inertial terms added directly to the weak (variational) formulation of the problem. Since the solid implants are very small the both ways give similar results. Obviously, the second approach is much more efficient to carry out numerical tests where the influence of the distribution of masses for the acoustic absorption of layers can be analysed. It seems that the improvement by distributed masses (implants) may be greater than the one due to the mass effect alone.

## 1 INTRODUCTION

Recent experimental investigations report a significant improvement of the insertion loss of standard acoustic blankets at lower frequencies by the addition of randomly placed masses to the poroelastic layers [1]. They show that the improvement by distributed masses (implants) tend to be greater than the one due to the mass effect alone. Therefore, there is a growing demand for an advanced modelling of porous media with distributed masses which should be at the same time sufficiently accurate and efficient to allow a reliable optimization of such poroelastic composites. Moreover, such modelling would be a first step for design of a new active composite where the vibrations of elastic porous skeleton with distributed masses are controlled by active implants to get a good acoustic performance at low frequencies. This paper presents the finite element modelling and frequency analysis of poroelastic layers with heavy solid implants where an improvement of acoustical absorption is observed. The Biot's theory of poroelasticity is used to model porous material and two techniques for considering mass implants are proposed and compared.

**Remarks on notation.** For the sake of brevity, symbols $\mathrm{d}\Omega$ and $\mathrm{d}\Gamma$ are skipped in all the integrals presented below since it is obvious that we integrate on the specified domain or boundary. The summation rule is used (for dummy indices $i, j, k$), and the (invariant) differentiation symbol which in the Cartesian coordinate system, simply reads: $(.)_{|i} = \frac{\partial (.)}{\partial x_i}$. The following notation rule applies for the symbol of variation (or test function): $\delta(v\,w) = v\,\delta w + w\,\delta v$, where $v$ and $w$ are two dependent variables (fields) and $\delta v$ and $\delta w$ their admissible variations. All problems considered in this paper are for the case of harmonic oscillations with the frequency $f$ and the angular frequency $\omega = 2\pi f$.

## 2 BIOT'S THEORY OF POROELASTICITY

### 2.1 Classical and mixed formulations of harmonic isotropic poroelasticity

The Biot's theory of poroelasticity [2, 3] provides a biphasic model of porous media: the so-called *solid phase* is used to describe the behavior of ("smeared") elastic skeleton whereas the *fluid phase* pertains to the fluid in the pores. The both phases are coupled homogeneous continua. The most frequently used version of poroelasticity assumes besides that the both phases are isotropic. Moreover, the fluid is modelled as perfect (i.e., inviscid), though viscous forces, are taken into account but only when modeling interaction between the fluid and the solid frame.

**The classical displacement formulation.** In the classical formulation [2, 3] a state of poroelastic medium is described by the displacements of solid, $\boldsymbol{u} = \{u_i\}$, and fluid phase, $\boldsymbol{U} = \{U_i\}$. The Biot's equations for a local dynamic equilibrium of poroelastic material link partial stress tensors associated with the skeleton particle ($\sigma^{\mathsf{s}}_{ij}$) and the macroscopic fluid particle ($\sigma^{\mathsf{f}}_{ij}$) with the solid and fluid macroscopic displacements. In the case of harmonic oscillations (with angular frequency $\omega$) these equations read

$$\sigma^{\mathsf{s}}_{ij|j} + \omega^2 \tilde{\varrho}_{\mathsf{ss}}\, u_i + \omega^2 \tilde{\varrho}_{\mathsf{sf}}\, U_i = 0\,, \qquad \sigma^{\mathsf{f}}_{ij|j} + \omega^2 \tilde{\varrho}_{\mathsf{ff}}\, U_i + \omega^2 \tilde{\varrho}_{\mathsf{sf}}\, u_i = 0\,, \tag{1}$$

where the *frequency-dependent effective densities*, $\tilde{\varrho}_{\mathsf{ss}}$, $\tilde{\varrho}_{\mathsf{sf}}$, and $\tilde{\varrho}_{\mathsf{ff}}$, are introduced. These densities are responsible not only for the inertia of solid or fluid phase particles but also for the combined inertial and viscous coupling (interaction) of both phases. They depend on the *viscous drag coefficient*, $\tilde{b}$, and the normal *effective densities*, $\varrho_{\mathsf{ss}}$, $\varrho_{\mathsf{ff}}$, $\varrho_{\mathsf{sf}}$. The latter quantities in turn depend on the porosity, $\phi$, the tortuosity of pores, $\alpha_\infty$, the density of the material of skeleton, $\varrho_{\mathsf{s}}$, and the density of saturating fluid, $\varrho_{\mathsf{f}}$. The adequate formulas may be found in [3].

The partial solid and fluid stress tensors are linearly related to the partial strain tensors prevailing in the skeleton and the interstitial fluid. This is given by the following linear and isotropic *constitutive equations* of the Biot's theory of poroelasticity:

$$\sigma^{\mathsf{s}}_{ij} = \mu_{\mathsf{s}}\,(u_{i|j} + u_{j|i}) + \Big(\tilde{\lambda}_{\mathsf{s}}\, u_{k|k} + \tilde{\lambda}_{\mathsf{sf}}\, U_{k|k}\Big)\delta_{ij}\,, \quad \sigma^{\mathsf{f}}_{ij} = \Big(\tilde{\lambda}_{\mathsf{f}}\, U_{k|k} + \tilde{\lambda}_{\mathsf{sf}}\, u_{k|k}\Big)\delta_{ij}\,. \tag{2}$$

Four material constants are involved here, namely $\mu_{\mathsf{s}}$, $\tilde{\lambda}_{\mathsf{s}}$, $\tilde{\lambda}_{\mathsf{f}}$, and $\tilde{\lambda}_{\mathsf{sf}}$. The first two of them resemble the two Lamé coefficients of isotropic elasticity. Moreover, $\mu_{\mathsf{s}}$ is the shear modulus of the poroelastic material and consequently the shear modulus of the frame since the fluid does not contribute to the shear restoring force. The three dilatational constants, $\tilde{\lambda}_{\mathsf{s}}$, $\tilde{\lambda}_{\mathsf{f}}$ and $\tilde{\lambda}_{\mathsf{sf}}$ are frequency-dependent and are functions of $K_{\mathsf{b}}$, $K_{\mathsf{s}}$, and $\tilde{K}_{\mathsf{f}}$ ($\tilde{\lambda}_{\mathsf{s}}$ depends also on $\mu_{\mathsf{s}}$), where: $K_{\mathsf{b}}$ is the bulk modulus of the frame at constant pressure in the fluid, $K_{\mathsf{s}}$ is the bulk modulus of the elastic solid from which the frame is made, and $\tilde{K}_{\mathsf{f}}$ is the bulk modulus of the fluid. The adequate formulas to compute the poroelastic material constants can be found in [3]. Finally, the *total stress tensor* of poroelastic medium is defined as a simple sum of the partial, i.e. phasic, stress tensors, whereas the *total displacement vector* sums up porosity-dependent contributions of the displacements of both phases:

$$\sigma^{\mathsf{t}}_{ij} = \sigma^{\mathsf{s}}_{ij} + \sigma^{\mathsf{f}}_{ij}\,, \qquad u^{\mathsf{t}}_i = (1-\phi)\,u_i + \phi\,U_i\,. \tag{3}$$

The equations of equilibrium (1) together with the constitutive relations (2) form the *displacement formulation* of linear, isotropic poroelasticity for harmonic oscillations. Notice that the first equations from the both pairs refer to the solid phase whereas the second ones to the fluid phase. Nevertheless, the both phases are strongly coupled by the *viscous-inertial coupling coefficient*, $\tilde{\varrho}_{\mathsf{sf}}$, and the *constitutive coupling constant*, $\tilde{\lambda}_{\mathsf{sf}}$. In this classical formulation the unknown fields are the solid and fluid phase displacements, that is, 6 degrees of freedom in a node of 3-dimensional model.

**The mixed displacement–pressure formulation.** The fluid phase stress tensor can be expressed as

$$\sigma^{\mathsf{f}}_{ij} = -\phi\, p\, \delta_{ij} \tag{4}$$

where $p$ is the pressure of fluid in the pores (it should not be mistaken for the pressure of fluid phase which equals $\phi p$). Using this relation for the harmonic Biot's poroelasticity the fluid phase displacements can be expressed as functions of the pressure in the pores, and so eliminated from the equations (replaced by $p$). This results in the mixed displacement–pressure formulation [4, 5] where the dependent variables are the three solid phase displacements and the pore-fluid pressure. Therefore, 3-dimensional models have now only 4 degrees of freedom in a node.

### 2.2 Weak integral form of the mixed formulation of poroelasticity

Let $\Omega_{\mathsf{p}}$ be a domain of poroelastic material and $\Gamma_{\mathsf{p}}$ its boundary with $n_i$ being the components of the unit vector normal to the boundary and pointing outside the domain. The harmonic poroelasticity problem can be described in this domain by the mixed formulation which uses as dependent variables the solid phase displacements, $u_i$, and pore-fluid pressure, $p$. The corresponding weak form [6] reads (for every admissible $\delta u_i$ and $\delta p$)

$$
\begin{aligned}
\mathcal{WI}_{\mathsf{p}} = & -\int\limits_{\Omega_{\mathsf{p}}} \sigma^{\mathsf{ss}}_{ij}\,\delta u_{i|j} + \int\limits_{\Omega_{\mathsf{p}}} \omega^2 \tilde{\varrho}\,u_i\,\delta u_i - \int\limits_{\Omega_{\mathsf{p}}} \frac{\phi^2}{\omega^2\tilde{\varrho}_{\mathsf{ff}}}\,p_{|i}\,\delta p_{|i} + \int\limits_{\Omega_{\mathsf{p}}} \frac{\phi^2}{\tilde{\lambda}_{\mathsf{f}}}\,p\,\delta p \\
& + \int\limits_{\Omega_{\mathsf{p}}} \phi\left(1+\frac{\tilde{\varrho}_{\mathsf{sf}}}{\tilde{\varrho}_{\mathsf{ff}}}\right)\delta(p_{|i}\,u_i) + \int\limits_{\Omega_{\mathsf{p}}} \phi\left(1+\frac{\tilde{\lambda}_{\mathsf{sf}}}{\tilde{\lambda}_{\mathsf{f}}}\right)\delta(p\,u_{i|i}) + \mathcal{BI}_{\mathsf{p}} = 0\,,
\end{aligned}
\tag{5}
$$

where

$$
\sigma^{\mathsf{ss}}_{ij} = \mu_{\mathsf{s}}\,(u_{i|j}+u_{j|i}) + \left(\tilde{\lambda}_{\mathsf{s}} - \frac{\tilde{\lambda}^2_{\mathsf{sf}}}{\tilde{\lambda}_{\mathsf{f}}}\right)u_{k|k}\,\delta_{ij} \quad \text{and} \quad \tilde{\varrho} = \tilde{\varrho}_{\mathsf{ss}} - \frac{\tilde{\varrho}^2_{\mathsf{sf}}}{\tilde{\varrho}_{\mathsf{ff}}}\,, \tag{6}
$$

$\mathcal{BI}_{\mathsf{p}}$ is the boundary integral

$$
\mathcal{BI}_{\mathsf{p}} = \int\limits_{\Gamma_{\mathsf{p}}} \sigma^{\mathsf{t}}_{ij}\,n_j\,\delta u_i + \int\limits_{\Gamma_{\mathsf{p}}} \phi\,(U_i-u_i)\,n_i\,\delta p\,, \tag{7}
$$

whereas $\delta u_i$ and $\delta p$ are test (or weighting) functions, that is, arbitrary yet admissible virtual displacements and pressure. Below, the two most relevant boundary conditions of poroelastic medium are discussed [5, 6].

### 2.3 Relevant boundary conditions for poroelastic medium

**Imposed displacement field.** A displacement field, $\hat{u}_i$, applied on a boundary of poroelastic medium describes, for example, a case of a piston in motion acting on the surface of the medium. Here, we assume that the solid skeleton is fixed to the surface of piston while the fluid obviously cannot penetrate into the piston. Therefore, on $\Gamma^u_{\mathsf{p}}$:

$$
u_i = \hat{u}_i\,, \qquad (U_i - u_i)\,n_i = 0\,. \tag{8}
$$

The first condition expresses the continuity between the imposed displacement vector and the solid phase displacement vector. The second equation expresses the continuity of the normal displacements between the solid phase and the fluid phase. Using these conditions and the fact that the variations of the known solid displacements are zero ($\delta u_i = 0$) the boundary integral reduces to zero (on the relevant part of the boundary, $\Gamma^u_{\mathsf{p}}$):

$$
\mathcal{BI}_{\mathsf{p}} = 0. \tag{9}
$$

**Imposed pressure field.** A harmonic pressure field of amplitude $\hat{p}$ is imposed on the boundary of poroelastic domain which means that it affects at the same time the fluid in the pores and the solid skeleton. Therefore, the following boundary conditions must be met on $\Gamma^p_{\mathsf{p}}$:

$$
p = \hat{p}\,, \qquad \sigma^{\mathsf{t}}_{ij}\,n_j = -\hat{p}\,n_i\,. \tag{10}
$$

The first condition is of Dirichlet type and must be applied explicitly. It describes the continuity of pressure in the fluid. It means also that the pressure variation is zero ($\delta p = 0$) at the boundary. The second condition expresses the continuity of the total normal stress. All this, when used for Equation (7), leads to the following boundary integral

$$
\mathcal{BI}_{\mathsf{p}} = \int\limits_{\Gamma_{\mathsf{p}}} \sigma^{\mathsf{t}}_{ij}\,n_j\,\delta u_i = -\int\limits_{\Gamma^p_{\mathsf{p}}} \hat{p}\,n_i\,\delta u_i\,. \tag{11}
$$

## 3 WEAK FORM OF ELASTICITY AND COUPLING TO POROELASTIC MEDIA

### 3.1 Weak form for an elastic solid

Let $\Omega_{\mathsf{e}}$ be an elastic solid domain with mass density $\varrho_{\mathsf{e}}$ and boundary $\Gamma_{\mathsf{e}}$, and $n^{\mathsf{e}}_i$ the components of unit vector normal to the boundary and pointing outside the domain. Assuming zero body forces and the case of harmonic oscillations the weak variational form of the problem of elasticity expressing the principle of virtual work reads (for every admissible $\delta u_i^{\mathrm{e}}$)

$$\mathcal{WI}_{\mathrm{e}} = -\int_{\Omega_{\mathrm{e}}} \sigma_{ij}^{\mathrm{e}}\, \delta u_{i|j}^{\mathrm{e}} + \int_{\Omega_{\mathrm{e}}} \omega^2 \varrho_{\mathrm{e}}\, u_i^{\mathrm{e}}\, \delta u_i^{\mathrm{e}} + \int_{\Gamma_{\mathrm{e}}} \sigma_{ij}^{\mathrm{e}}\, n_j^{\mathrm{e}}\, \delta u_i^{\mathrm{e}} = 0 \tag{12}$$

where $\delta u_i^{\mathrm{e}}$ is the arbitral yet admissible variation of displacements; the elastic stress tensor $\sigma_{ij}^{\mathrm{e}} = \sigma_{ij}^{\mathrm{e}}(\boldsymbol{u}^{\mathrm{e}})$ substitutes here a linear function of elastic displacements $\boldsymbol{u}^{\mathrm{e}} = \{u_i^{\mathrm{e}}\}$. In the case of the linear isotropic elasticity it can be expressed as follows

$$\sigma_{ij}^{\mathrm{e}} = \mu_{\mathrm{e}}\,(u_{i|j}^{\mathrm{e}} + u_{j|i}^{\mathrm{e}}) + \lambda_{\mathrm{e}}\, u_{k|k}^{\mathrm{e}}\, \delta_{ij} \tag{13}$$

where the well-known Lamé coefficients: the shear modulus, $\mu_{\mathrm{e}}$, and the dilatational constant, $\lambda_{\mathrm{e}}$, appear.

### 3.2 Elastic solid boundary conditions

For the sake of brevity, only von Neumann and Dirichlet boundary conditions will be discussed here (the Robin type involves the technique of Lagrange multipliers and will be skipped). The Neumann (or natural) boundary conditions describe the case when forces $\hat{t}_i^{\mathrm{e}}$ are applied on a boundary, that is,

$$\sigma_{ij}^{\mathrm{e}}\, n_j^{\mathrm{e}} = \hat{t}_i^{\mathrm{e}} \text{ on } \Gamma_{\mathrm{e}}^t\,, \tag{14}$$

whereas the displacements, $\hat{u}_i^{\mathrm{e}}$, are prescribed by the Dirichlet (or essential) boundary conditions

$$u_i^{\mathrm{e}} = \hat{u}_i^{\mathrm{e}} \text{ on } \Gamma_{\mathrm{e}}^u\,. \tag{15}$$

According to these conditions the boundary is divided into two (directionally disjoint) parts, i.e., $\Gamma_{\mathrm{e}} = \Gamma_{\mathrm{e}}^t \cup \Gamma_{\mathrm{e}}^u$. There is an essential difference between the two kinds of conditions. The displacement constraints form the kinematic requirements for the trial functions, $u_i^{\mathrm{e}}$, while the imposed forces appears in the weak form; thus, the boundary integral, that is, the last of the integrals of Equation (12), equals

$$\mathcal{BI}_{\mathrm{e}} = \int_{\Gamma_{\mathrm{e}}} \sigma_{ij}^{\mathrm{e}}\, n_j^{\mathrm{e}}\, \delta u_i^{\mathrm{e}} = \int_{\Gamma_{\mathrm{e}}^t} \hat{t}_i^{\mathrm{e}}\, \delta u_i^{\mathrm{e}}\,. \tag{16}$$

Here, the property $\delta u_i^{\mathrm{e}} = 0$ on $\Gamma_{\mathrm{e}}^u$ has been used.

### 3.3 Poroelastic–elastic coupling

Let $\Gamma_{\text{p-e}}$ be the interface between poroelastic and elastic media. Let $n_i$ be the components of the unit vector normal to the interface and pointing outside the poroelastic domain into the elastic one. The coupling integral combines boundary integral terms resulting from both, poroelastic and elastic, weak forms (Equations (5)-(7) and (12)-(13), respectively):

$$\mathcal{CI}_{\text{p-e}} = \int_{\Gamma_{\text{p-e}}} \sigma_{ij}^{\mathrm{t}}\, n_j\, \delta u_i + \int_{\Gamma_{\text{p-e}}} \phi\,(U_i - u_i)\, n_i\, \delta p + \int_{\Gamma_{\text{p-e}}} \sigma_{ij}^{\mathrm{e}}\, n_j^{\mathrm{e}}\, \delta u_i^{\mathrm{e}} \tag{17}$$

where $n_i^{\mathrm{e}} = -n_i$ are the components of the unit normal vector pointing outside the elastic domain (and into the poroelastic medium). Now, the following coupling conditions must be met at the interface $\Gamma_{\text{p-e}}$:

$$\sigma_{ij}^{\mathrm{t}}\, n_j = \sigma_{ij}^{\mathrm{e}}\, n_j\,, \qquad (U_i - u_i)\, n_i = 0\,, \qquad u_i = u_i^{\mathrm{e}}\,, \tag{18}$$

The first condition states the continuity of total stress tensor, the second expresses that there is no mass flux across the interface, and the last one assumes the continuity of the solid displacements [6]. This last condition means also the equality of the variations of displacements, $\delta u_i = \delta u_i^{\mathrm{e}}$. Now, applying the coupling conditions for the coupling integral (17) results in

$$\mathcal{CI}_{\text{p-e}} = 0\,. \tag{19}$$

## 4 FINITE ELEMENT MODELLING AND RESULTS OF ANALYSES

### 4.1 Two approaches in modelling of small solid implants

To model a domain of porous layer the mixed formulation of poroelasticity is used while the solid implants are modelled in two ways. Firstly, they can be very accurately modelled as small elastic subdomains in the poroelastic domain. Then, the Galerkin finite element model uses the following weak integral:

$$\mathcal{WI}_{\mathsf{p}} + \mathcal{CI}_{\mathsf{p\text{-}e}} + \mathcal{WI}_{\mathsf{e}} = 0\,. \tag{20}$$

Let us remind that the weak form $\mathcal{WI}_{\mathsf{p}}$ of the mixed formulation of poroelasticity (5) ensures that the coupling of the two media is naturally handled [6] (i.e., $\mathcal{CI}_{\mathsf{p\text{-}e}} = 0$): and so only the continuity between the solid phase displacements and the elastic subdomain(s) displacements must be ensured, that is, $u_i = u_i^{\mathsf{e}}$ on $\Gamma_{\mathsf{p\text{-}e}}$. However, since the implants are small the finite element mesh around them becomes dense and the FE model is significantly enlarged (the poroelastic domain has 4 nodal DoF in 3D, or 3 nodal DoF in 2D). But the predominant effect of the solid implants (attached to the elastic skeleton of porous medium) is caused by their mass since they are very small though heavy (and practically rigid) comparing to the poroelastic medium. Therefore, another approach may be proposed: it consists in adding some adequate inertial terms directly to the weak (variational) formulation of the poroelastic problem, that is,

$$\mathcal{WI}_{\mathsf{p}} + \mathcal{MI} = 0 \qquad \text{where} \qquad \mathcal{MI} = \int\limits_{\Omega_{\mathsf{p}}} \omega^2\, m\, u_i\, \delta u_i \tag{21}$$

is the added (concentrated) mass term. Here, $m = m(\boldsymbol{x})$ is a (local) distribution of additional mass added to the solid phase. In general, this approach (mathematically equivalent to the presence of concentrated masses in the given localizations) is effective if the mass is concentrated in points, and particularly in the nodes of FE mesh so to preserve it simple. Thus, for a concentrated point-mass $M_0$ (added in the point $\boldsymbol{x}_0$) one may formally write: $m(\boldsymbol{x}) = M_0\,\delta(\boldsymbol{x} - \boldsymbol{x}_0)$, where $\delta(.)$ is the Dirac delta function.

### 4.2 Acoustic absorption of poroelastic layer

The main purpose of the present analysis of poroelastic layers with solid implants is to asses how the heavy implants influence the acoustic absorption of layers. The acoustic absorption of a poroelastic layer fixed to a rigid wall and subject to a plane acoustic wave propagating in the air onto the layer surface at normal incidence will be computed as follows [3]. First, the acoustic impedance at normal incidence is determined at the interface between the poroelastic layer and the air:

$$Z = \frac{p_0}{v}\,, \quad \text{where} \quad v = \mathrm{j}\,\omega\, u_1^{\mathsf{t}} = \mathrm{j}\,\omega\left[(1-\phi)\,u_1 + \phi\, U_1\right]. \tag{22}$$

Here, $v$ is the velocity of propagating wave at the layer–air interface (continuous across this boundary) whereas $p_0$ is the wave pressure. Now, the reflection coefficient in this point is computed:

$$R = \frac{Z - Z_0}{Z + Z_0}\,, \tag{23}$$

where $Z_0 = \varrho_0\, c_0$ is the characteristic impedance of the air ($\varrho_0$ is the air density and $c_0$ the speed of sound). Finally, knowing the reflection coefficient the acoustic absorption coefficient can be determined:

$$A = 1 - |R|^2\,. \tag{24}$$

This final property is real-valued (unlike the reflection coefficient $R$, and the impedance $Z$, which are complex).

### 4.3 Results of analyses

Several finite element analyses of poroelastic layers with solid implants were carried out for the configuration presented in Figure 1 [left]. Poroelastic material data for two different high-porosity polyurethane foams (termed A and B) was used for this configuration. The thickness of layer is 24 mm. At $x_1 = 0$ mm the layer is fixed to a rigid wall whereas at $x_1 = 24$ mm the plane harmonic acoustic wave propagates onto the interface between the poroelastic layer and the air. At the

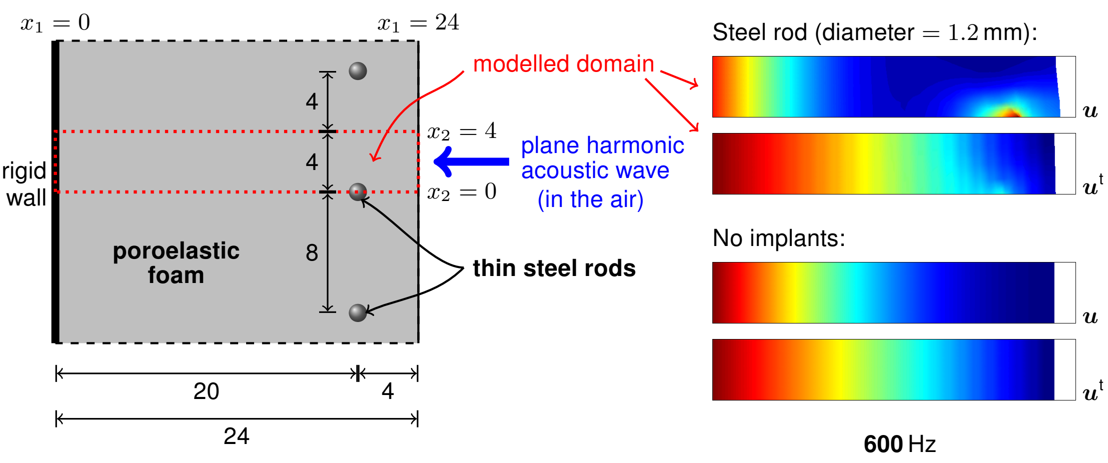


Figure 1: [left] A 24 mm-thick layer of poroelastic foam (fixed to a rigid wall) with small steel implants (regularly spaced steel balls or rods), with the modelled subdomain shown. [right] Solid phase ($\boldsymbol{u}$) and total displacement ($\boldsymbol{u}^{\mathrm{t}}$) in the modelled subdomain of poroelastic layer, with and without mass implant, at $f = 600$ Hz.

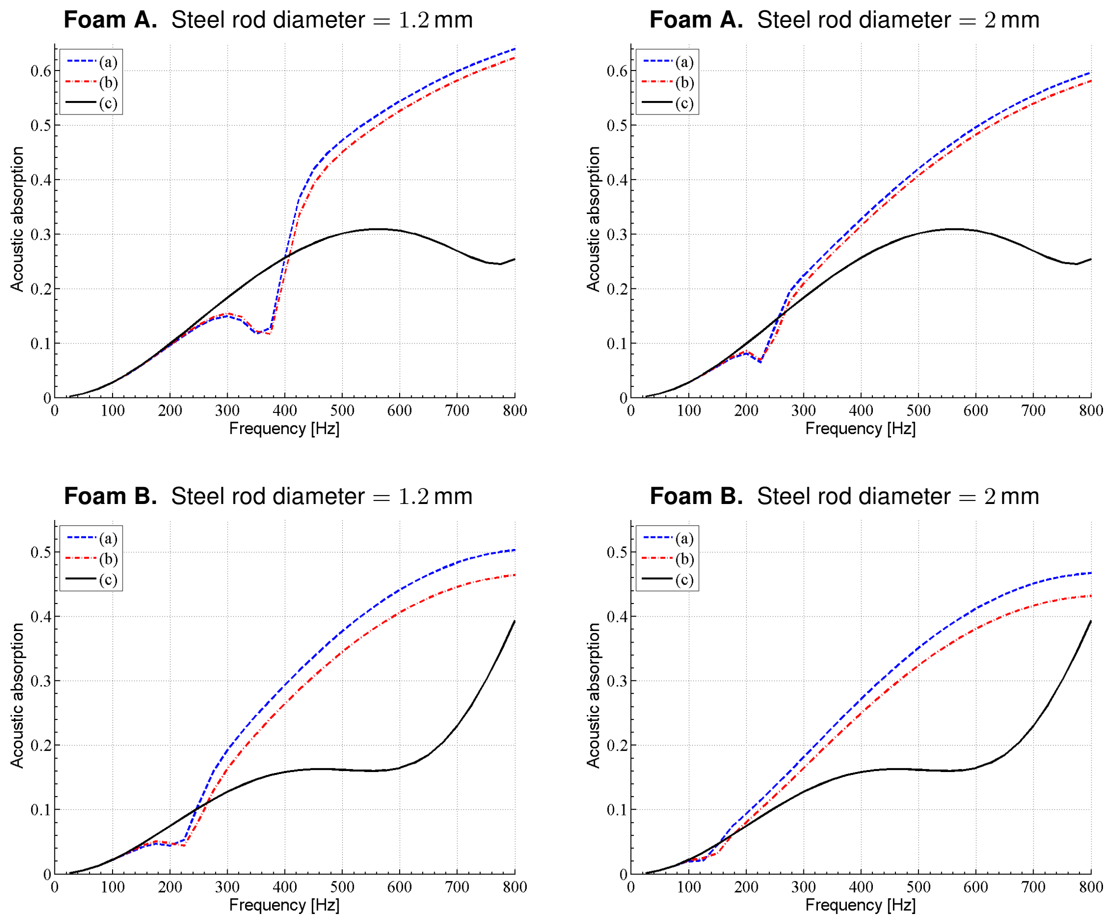


Figure 2: The acoustic absorption of the composite poroelastic layers at $x_1 = 24$ mm and: (a) $x_2 = 0$ mm (facing a rod), or (b) $x_2 = 4$ mm (between two rods). Also the absorption of the (homogenous) layers with no implants (c). The frequency range: $f = 25$-$800$ Hz.

depth of $4$ mm from the incident surface, and spaced by $\Delta x_2 = 8$ mm, thin steel rods are planted along the $x_3$-axis. Two cases of rod diameters were considered: $1.2$ mm and $2$ mm. The problem is modelled as two-dimensional: in the $x_1 x_2$-plane. Moreover, the symmetry (regularity along the $x_2$-axis) makes it possible to model only a rectangular slice (of width $\Delta x_2 = 4$ mm) of the layer comprising only a half of one implant (see Figure 1).

There were two purposes of the analysis. First, the conformity between accurately modelled solid implants (which involves a locally denser FE mesh) and the implants considered as additional weak mass terms was investigated. It was found that for the present configuration the discrepancy between the both models is small enough to allow the usage of the second, simpler model. An obvious conclusion is that the smaller are the implants the better is the conformity. Figure 1 [right] presents the solid phase and total displacements of the modelled slice (for two cases: the layer with the implants of diameter $1.2$ mm, and the layer without implants). The plots are independently scaled and so they are rather only qualitative: the imposed boundary conditions, the presence (or absence) of mass implants are visualized.

The second analysis consisted in determining the acoustic absorption of the poroelastic composite. To this end, the results of FE analysis ($u^{\mathrm{t}}$ at the layer's surface) were used by the analytical formulas for the impedance, the reflection and absorption coefficients (see Section 4.2). These formulas result from a one-dimensional analysis of the plane wave propagation which is slightly violated if the solid implants are present. Therefore, the absorption coefficient was computed in two points of the layer surface: at $x_2 = 0$ mm and $x_2 = 4$ mm (see Figure 1 [left]), providing two limit-values. These values are plotted (for the range of frequency $25$-$800$ Hz) in Figure 2 as curves (a) and (b), respectively. Moreover, the curves (c) show the acoustic absorption for the homogeneous layer (i.e., no implants). The results were obtained for the both versions of poroelastic data: foam A (upper graphs) and foam B (lower graphs), and for the both versions of the steel rod diameter: $1.2$ mm (left graphs) and $2$ mm (right graphs). Notice that in this latter case the implant mass is $2.78$ times bigger than in the case of thinner rods.

Although, the choice of configuration and materials was quite arbitral the following observations should prove to be general:

- the presence of mass implants significantly increases the acoustic absorption of porous layers (especially, in medium frequency),
- there is a lower frequency range, however, where the presence of implants deteriorate the absorption,
- this effect is significantly reduced if the implant mass is bigger; moreover, the range is then narrowed and shifted to even lower frequencies,
- below this range (i.e., for very low frequencies) the mass implants have no noticeable effect (there is the same very poor performance of acoustic absorption).

## 5 CONCLUSIONS

Layers of porous material with heavy solid implants may be modelled as poroelastic media with adequate point-masses if the implants are very small (and sufficiently heavy). Such approach should be very efficient for the optimization of composite configuration where the influence of the distribution of masses for the acoustic absorption of layers is analysed.

The presence of mass implants may significantly increase the acoustic absorption of porous layers, especially, in medium frequency. It seems that the improvement by distributed masses (implants) may be greater than the one due to the mass effect alone (that is, by a thin, heavy layer). Therefore, more numerical tests where this influence is analysed should be carried out.

In lower frequency range the passive vibroacoustic attenuation by mass implants ceases to work and an active approach to the problem proves to be necessary. However, it seems that the most promising concept should combine active implants, distributed masses and possibly other solid implants, that is, to create an active poroelastic composite able to significantly dissipate the energy of acoustic waves also in low frequencies (where it should rely on an active control, whereas in the high and medium frequency range an excellent passive acoustic absorption would be guaranteed thanks to the designed absorbing properties of poroelastic composite).

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