FINITE STRAIN BEHAVIOUR OF A POLYCRYSTALLINE CERAMIC COMPOSITE WITH INTERGRANULAR LAYERS

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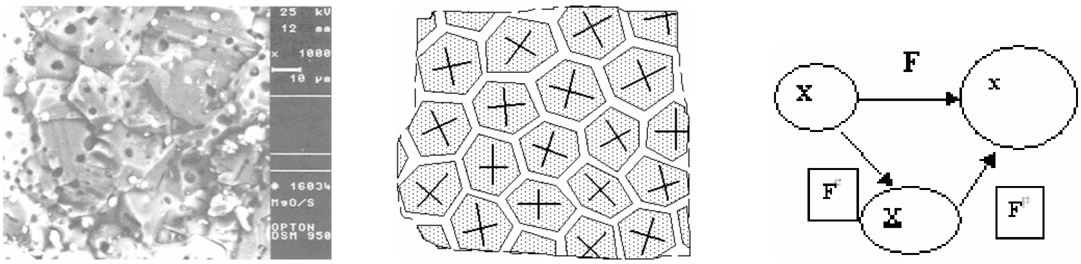
ABSTRACT- This paper presents results from a study of the mechanical behaviour of a two-phase ceramic polycrystalline material subjected to in-plane static and dynamic loading with the assumptions of the finite strains. The material is idealized based on observations from SEM images.

INTRODUCTION: An application of polycrystalline materials is the fabrication of cutting tools. These tools are working in conditions of dynamic and temperature loading. The two-phase material under consideration consists of elastic grains and ductile interfaces. Interest is focused on the failure loadings in the cases of static and dynamic loading. The small strains behaviour of the composite with initial voids and pores has previously been presented by Sadowski *et al* [2005].



(a) (b) (c)

Fig 1. (a) SEM image, (b) idealization, (c) gradient decomposition

FORMULATION: The static problem is presented in the form of a FE discretized non-linear incremental equation of equilibrium fulfilling the boundary and initial conditions valid for elasto-plasticity with non-linear geometry, Zienkiewicz and Taylor [2000], Kleiber [1989].

$$\left(\int_{\Omega^t} \mathbf{B}_L^{'T}\, {}_t^t\bar{\boldsymbol{\tau}}\, \mathbf{B}_L^{'}\, d\Omega^t\right)\Delta\mathbf{q} + \int_{\Omega^t} \mathbf{B}_L^{T}\Delta\mathbf{S}\, d\Omega^t = \int_{\Omega^t} \mathbf{N}^{T}\Delta\mathbf{f}\; d\Omega^t + \int_{\partial\Omega^t} \mathbf{N}^{T}\Delta\mathbf{t}\, d\left(\partial\Omega^t\right) \qquad (1)$$

where $\mathbf{B}_L^{'T}$ is the large displacements operator, $\mathbf{B}_L^{T}$ is the linear operator, ${}_t^t\bar{\boldsymbol{\tau}}$ is the Cauchy stress matrix, $\Delta\mathbf{S}$ is the stress increment, $\mathbf{N}$ is the shape functions matrix, $\Delta\mathbf{q}$ is the displacements increment vector, $\Delta\mathbf{f}$ is the body forces increment vector and $\Delta\mathbf{t}$ is the tractions external load increment vector. This equation is integrated implicitly. The nonlinear dynamic problem takes the form:

$$\mathbf{Mq} + \mathbf{Cq} = \mathbf{F} - \mathbf{R} \tag{2}$$

where $\mathbf{M}$ is the diagonal mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{q}$ is the nodal displacement vector and $\mathbf{F}$ and $\mathbf{R}$ are the external and internal nodal force vectors respectively. An explicit integration procedure is used because the loading is of short duration, Owen and Hinton [1980], Bathe [1996].

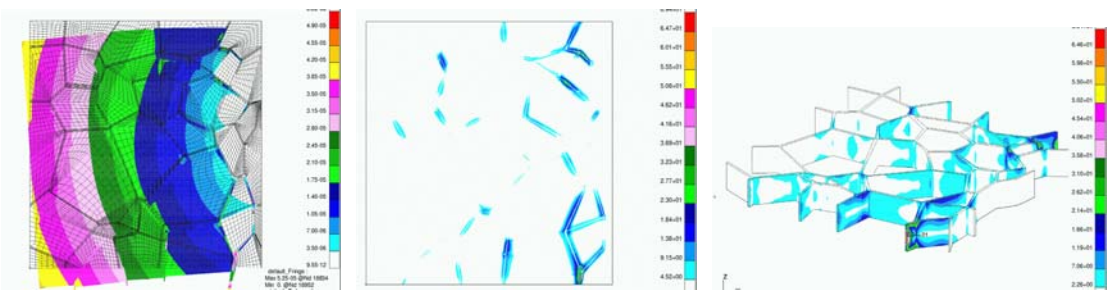
CONSTITUTIVE MODELS: When considering the finite strains effect, the gradient $\mathbf{F} = \partial(\mathbf{X} + \mathbf{u}) / \partial\mathbf{X}$ is decomposed into its elastic and plastic parts, $\mathbf{F} = \mathbf{F}^e\mathbf{F}^p$, see Fig 1(a). To integrate the constitutive relations, the deformation increment $\Delta\mathbf{D}$ is rotated to the un-rotated configuration by means of a rotation matrix obtained from the polar decomposition $\mathbf{F} = \mathbf{VR} = \mathbf{RU}$, $\Delta\mathbf{d} = \mathbf{R}^T_{n+1}\Delta\mathbf{D}\mathbf{R}_{n+1}$, then the radial return is performed and stresses are transformed to the Cauchy stresses at $n+1$, $\boldsymbol{\sigma}_{n+1} = \mathbf{R}_{n+1}\boldsymbol{\sigma}^u_{n+1}\mathbf{R}^T_{n+1}$.

The constitutive models assume elasto-plastic behaviour with hardening and the Gurson [1977]- Tvergaard [1990] model with the yield condition as follows

$$\sigma_y = \left(\frac{\sigma^M}{\overline{\sigma}}\right)^2 + 2q_1 f \cosh\left(\frac{3q_2\sigma_m}{2\overline{\sigma}}\right) - \left(1 + q_3 f^2\right) = 0 \tag{3}$$

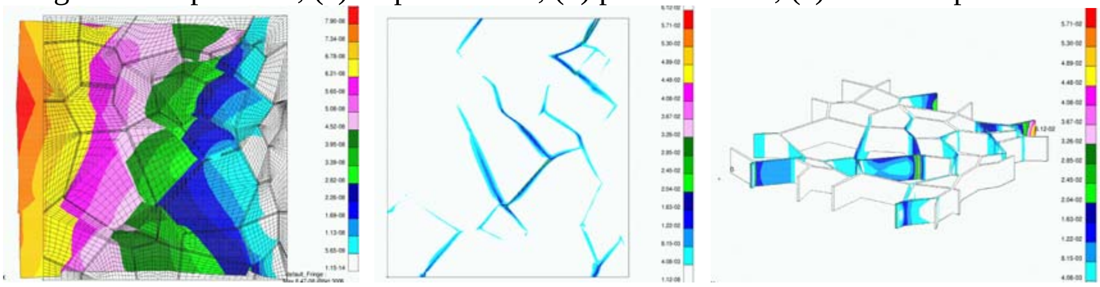
where σ^M is the von Mises stress, σ_m is the mean stress, $\overline{\sigma}$ is the von Mises stress in the matrix, *f* is the void ratio and q_1, q_2, q_3 are the Tvergaard coefficients.

NUMERICAL EXAMPLE: A SEM image of the material sample is given in Fig. 1(a) and its idealization is shown in Fig 1(b). The elastic grains are made of Al_2O_3 (Young modulus 410000 MPa) and the interfaces are made of *Co* with Young modulus 210000 MPa, yield stress 297 MPa and q-coefficients 1.25, 1.0 and 1.56. Two characteristic loading cases are chosen, namely, shear loading along the free edge and uniaxial pressure applied dynamically. The reference pressure σ_o is 400 MPa. The dimensions of the sample are 100x100x10 µm. In the case of shear loading, the sample is loaded up to $0.09\sigma_o$. The shape of the sample during failure is shown in Fig. 2(a). The plastic strains are localized at the interfaces close to the fixed edge, see Fig 2 (b) and Fig 2(c).



(a) (b) (c)

Fig 2. Shear pressure, (a) displacements, (b) plastic strains, (c) interface plastic strains



(a) (b) (c)

Fig 3. Dynamic pressure, (a) displacements, (b) plastic strains, (c) interface plastic strains

The dynamic load is applied instantenously and kept constant at the reference level throughout the process. The shape of the sample in Fig. 3(a) tends to exhibit the necking phenomenon. The grains start to slide along the interfaces. This is particularly visible close to the free edges of the sample. The interfaces which yield are mostly concentrated along a 45^o lines, see Fig. 3(b) and Fig 3(c). Considering both cases, i.e. Fig 2(c) and 3(c), the plastic strains are the highest where individual interfaces meet. The plastic strains start to develop here and then propagate along the interfaces between the grains. The plastic strains in the interfaces between the grains first appear at the boundary between the grains and the interfaces and then spread into the interface material itself.

FINAL REMARKS: The plastic strains are localized in the thin interfaces and they are sensitive to imperfections. However, when observing the global behaviour of the sample, it is possible to notice phenomena such as necking and the formation of plastic hinges.

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