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# LARGE STRAIN THERMO-ELASTO-PLASTICITY: SIMULATION OF SHEAR BANDING FOR DIFFERENT STRESS STATES

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## 1. Introduction

The aim of the paper is to simulate numerically the shear band formation in a plate in tension, induced by geometrical and thermal softening included in the employed large strain model of thermoplasticity. In the adopted model the following assumptions hold: statics, hyperelasticity, rate-independent plasticity with associated flow rule and strain hardening with one internal variable, non-stationary heat flow with Fourier-type heat conduction. Three-dimensional formulation is employed. In the paper two stress states are considered: plane strain and plane stress. The influence of heat conduction on the results is examined.

## 2. Finite thermoplasticity model

Fully coupled thermoplasticity models have been considered among others in the papers of Simo and Miehe, see [1]. The model applied in the presented research is described in [2] and its basis is summarized below.

The multiplicative decomposition of the deformation gradient into its mechanical (elastic and plastic) and thermal parts is considered in the form  $\mathbf{F} = \mathbf{F}^\theta \mathbf{F}^e \mathbf{F}^p$ , where  $\mathbf{F}^\theta = (J^\theta)^{1/3} \mathbf{I}$ . The thermal part is purely volumetric with unit tensor  $\mathbf{I}$  and  $J^\theta = \exp[3\alpha_T(T - T_0)]$ , where  $T$  is the absolute temperature,  $T_0$  is the reference temperature and  $\alpha_T$  is the coefficient of thermal expansion.

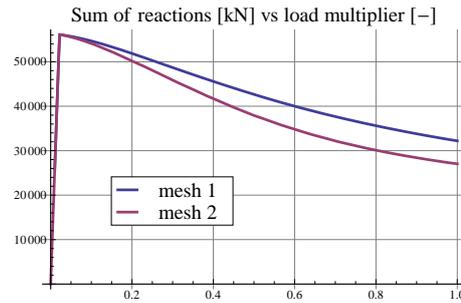
The free energy potential is assumed in the additive form  $\psi(\mathbf{b}^e, T, \gamma) = \psi^e(\mathbf{b}^e) + \psi^\theta(T) + \psi^p(\gamma)$ , where the respective parts represent elasticity with thermal expansion, thermal process and plastic hardening,  $\mathbf{b}^e = \mathbf{F}^e(\mathbf{F}^e)^T$  is the elastic left Cauchy-Green tensor and  $\gamma$  is a plastic strain measure.

The yield condition is formulated in a classical way  $F_p(\boldsymbol{\tau}, \gamma, T) = f(\boldsymbol{\tau}) - \sqrt{2/3}\sigma_y(\gamma, T) \leq 0$ , where  $f(\boldsymbol{\tau})$  is an equivalent Kirchhoff stress function and  $\sigma_y(\gamma, T)$  denotes the yield strength including isotropic strain hardening and thermal softening. The model includes self-heating due to plastic dissipation. The two governing equations (balance of linear momentum and balance of energy), written in Euler description, can be found in [2].

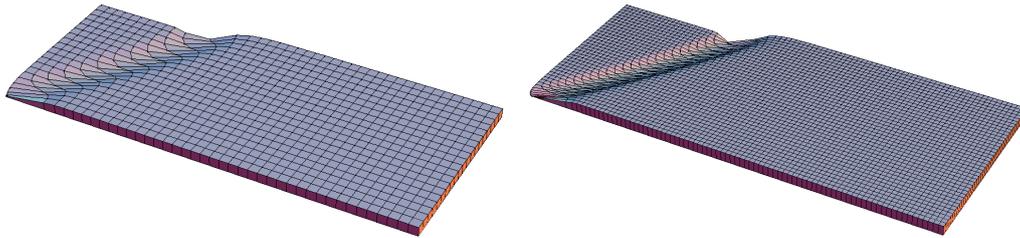
## 3. Analysis method and scope

The finite element implementation is based on appropriate potentials. The numerical simulations are carried out using the symbolic-numerical packages *AceGen* and *AceFEM* [3] for *Wolfram Mathematica*. The important advantage of the package is automatic differentiation which enables an easy derivation of the tangent operator.

In the paper shear banding in a plate in tension is examined. Symmetry of the plate is exploited. The deformation in the length direction is imposed and in the width direction is free. For plane strain the displacements in the direction normal to the plate are constrained, for plane stress they are free. For plane strain and isothermal case (thus when only geometrical softening is involved), a diffuse localization mode (necking) is observed. For thermal softening strain localization is observed in the form of a shear band. When heat transfer is taken into account (non-adiabatic case), the value of the heat conduction coefficient  $k$  influences the width of the shear band, see [4, 2].



**Figure 1.** Sum of reactions vs displacement factor for isothermal plane stress (3D) case and two FE meshes



**Figure 2.** Deformed meshes at the end of elongation process

In Figures 1-2 pilot results are presented for the plane stress case, ideal plasticity and isothermal conditions. Further research is necessary to find whether the mesh sensitivity of the results is pathological due to softening or just reflects the standard discretization error due the high strain gradients. Moreover, the effect of the definition of the equivalent Kirchhoff stress function  $f$  on the direction of the shear band and the influence of the stress state on the instability mode are assessed in the paper.

#### 4. Conclusions

The large strain fully coupled thermoplastic model is used in the paper to analyze shear banding in a plate under tension, either constrained by the plane strain assumption or free to deform (plane stress). When instabilities result in localized deformation modes an important difficulty occurs in mathematical modelling and numerical modelling: the considered boundary value problem can lose well-posedness and simulation results are sensitive to discretization density in a spurious manner. A localization limiter is then necessary. In the isothermal and adiabatic case the limiter should be provided by a non-local enhancement of the constitutive description, while in the thermomechanical model with significant heat conduction localization due to thermal softening is regularized by the coupling.

#### 5. References

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