Cardigan Bay

Middle Wales 

Middle of Cardigan bay coast

Middle of Nowhere ☺


© 2006 Janet Baxter






PD's
DINERO

How Hydraulic Fracture is paying back to Fracture Mechanics

Gennady Mishuris

Aberystwyth University (AU)

in collaboration with

G. Da Fies, D. Peck (AU; UK), M. Dutko (RockField, AU; UK)

A. Piccolroaz (UNITN; IT), M. Wrobel (Uni Cyprus; Cyprus)

Current Research Interests

- *Mathematical and numerical modelling of Hydraulic Fracture*
- "Transformable waves" in discrete structures
- W-H Factorisation of Matrix-Functions
- Biomechanics
- Multiphysics phenomena in thin layers
- Plasticity and Viscoplasticity
- Various Industrial applications

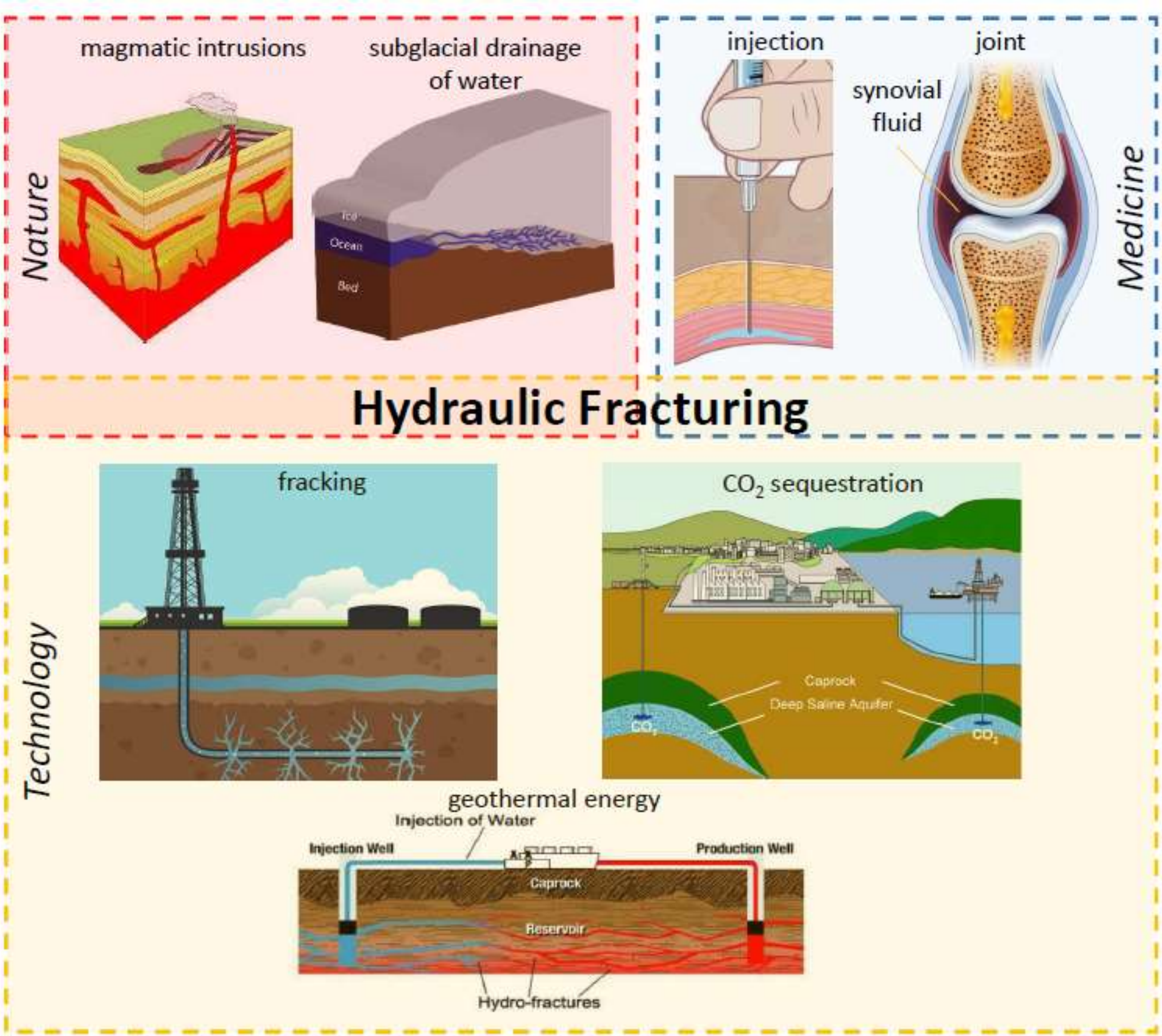
EffectFact

INTERCRACKS, OA-AM, FAANON

Plan for the talk

- ❑ **Introduction**
- ❑ **Development of AU-UA solvers** [1D (in space & time) utilising an *explicit* fracture tracing algorithm and appropriate asymptotic network]
- ❑ **Accounting for the shear traction induced by the fluid** on the crack surfaces
- ❑ **Generalisation of ERR** (appropriate for all LEFM problems with line defects)
- ❑ **Variable toughness** (how to address the issue?)
  - ✓ Preliminary results (simulations)
  - ✓ Suggested averaging strategies (not homogenisation!) & its verification
  - ✓ Conclusions & Discussions

HF is much more than “Fracking”



Challenges in addressing HF Problem

- **Complex geometry (3D)**
- **Coupled fields**
- **Nonlinearity**
- **Moving boundary**
- **Nonlocal effects**
- **Various propagation regimes**
- **High computational stiffness,**
- **Multiscaling,**
- **Degeneration at the boundary**
- **Multifracturing/shadowing**
- **Heterogeneity**
- **Lag, regimes (lam/turb)…….**

➢ Nature
- subglacial drainage of water
- magma driven dykes

➢ Technology
- fracking
- geothermal reservoirs exploitation
- coal mine degassing
- CO_2 sequestration
- geological insulation of radioactive waste and chemical contaminants

➢ Medicine - Biomechanics
- Injections
- Fluid- tissues interactions (cartilage…)
- Negative pressure healing therapy

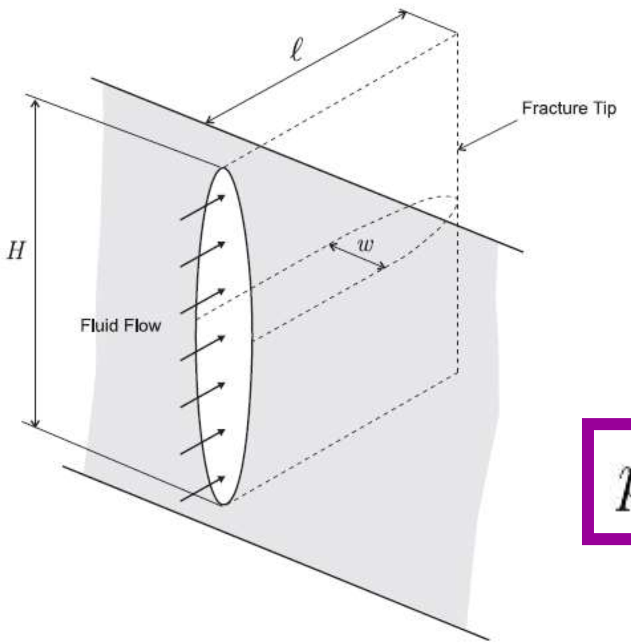
⇒ **Reasonable and motivated simplifications**

⇒ **Effective numerical simulators**

Classic 1-D HF models

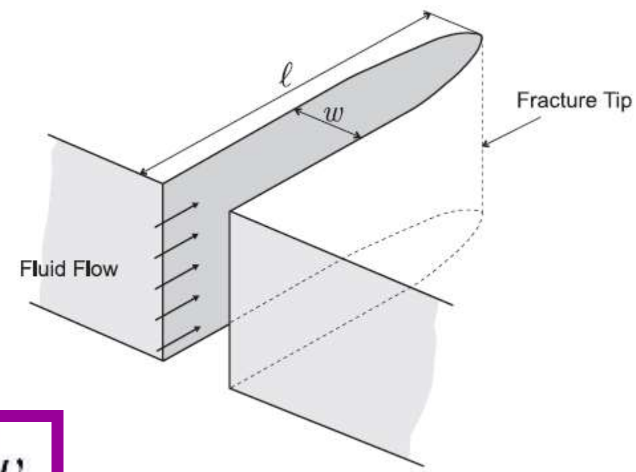
Local models

PKN model



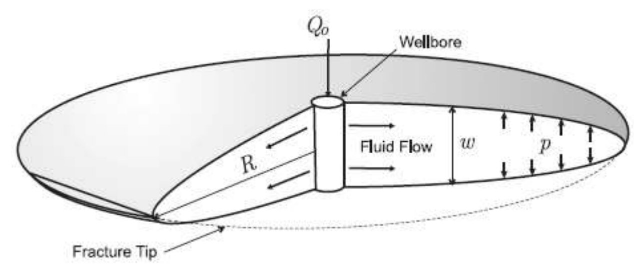
Non-local models

KGD fracture geometry



$$\boxed{p = \mathcal{A}w}$$

Radial fracture geometry



P3D, PE models

Pseudo PKN model

Explicit AU-Universal Algorithm

- **AU-UA** universal 1D (in space) - PKN, KGD, Radial,…
- All propagation regimes (viscosity / toughness / leak-off dominated);
- (***w***, ***v***) - crack opening, fluid velocity [*pressure – postprocessing*];
- an *explicit* fracture tracing algorithm (utilising velocity at the crack tip);
- Time-space adaptive algorithm;
- nonlinear non-singular elasticity operator acting on the velocity (not pressure);
- the asymptotic network (thank you … Minnesota Mafia … ☺);
- Predefined accuracy of the computations.

AU-UA solver has allowed us

- to verify most of semi-analytical solutions and to propose new ones.
- To analyse some phenomena that have not been previously addressed.

AU-UA solver development

[1] Mishuris, G., Wróbel, M., Linkov, A. (2012). On modeling hydraulic fracture in proper variables: Stiffness, accuracy, sensitivity. *IJES,* 61, 10-23.

[2] Linkov, A., Mishuris, G. (2013). Modified Formulation, ε-Regularization and the Efficient Solution of Hydraulic Fracture Problems. In "Effective and Sustainable Hydraulic Fracturing", book edited by A. Bunger, J. McLennan, R. Jeffrey, ISBN 978-953-51-1137-5.70.

[3] Wróbel, M., Mishuris, G. (2013). Efficient pseudo-spectral solvers for the PKN model of hydrofracturing. *IJF*, 184 (1-2), 151-170.

[4] Kusmierczyk, P., Mishuris, G., Wróbel, M. (2013). Remarks on application of different variables for the PKN model of hydrofracturing: various fluid-flow regimes. *IJF*, 184(1), 185-213.

[5] Wrobel, M. Mishuris, G. (2015) Hydraulic fracture revisited: Particle velocity-based simulation. *IJES,* 94, 23-58.

[6] Perkowska, M., Wrobel, M., Mishuris, G. (2016). Universal hydrofracturing algorithm for shear-thinning fluids: Particle velocity based simulation. *Comp. & Geotech*. 71, 310-337.

[7-8] Peck, D., Wrobel, M., Perkowska, M., Mishuris, G. (2018) Fluid velocity based simulation of hydraulic fracture: a penny shaped model. Part I: the numerical algorithm. *Meccanica,* 53 (15), 3615-3635. Part II: new, accurate semi-analytical benchmarks for an impermeable solid. *Meccanica*, 53(15), 3637-3650.

[9] Da Fies, G. (2020) Effective Time-Space Adaptive Algorithm for Hydraulic Fracturing, PhD thesis, Aberystwyth.

Plan for the talk

- ❑ **Introduction**
- ❑ **Development of AU-UA solvers** [1D (in space & time) utilising an *explicit* fracture tracing algorithm and appropriate asymptotic network]
- ❑ **Accounting for the shear traction induced by the fluid** on the crack surfaces
- ❑ **Generalisation of ERR** (appropriate for all LEFM problems with line defects)
- ❑ **Variable toughness** (how to address the issue?)
  - ✓ Preliminary results (simulations)
  - ✓ Suggested averaging strategies (not homogenisation!) & its verification
  - ✓ Conclusions & Discussions

Impact of the fluid induced shear stress

$\tau \sim p_0 w_0 r^{-2/3}$

What were the reasons to neglect the shear stress?

(A) $p \gg \tau \equiv |\boldsymbol{\tau}| = \frac{1}{2} w |\nabla p|$ **?**

(B) symmetrical shear stress (not important in the classic LEFM)

Toughness regime ($K_{IC} > 0$)

$$\boxed{\tau \sim p_0 \gamma K_I r^{-1/2}}$$

$p \sim p_0 \log r$, $\quad w \sim \gamma K_I \sqrt{r}$, $\quad V_* < \infty$

Viscosity regime ($K_{IC} = 0$)

$$\boxed{\tau \sim p_0 w_0 r^{-2/3}}$$

$p \sim -p_0\, r^{-1/3}$, $\quad w \sim w_0 r^{2/3}$, $\quad V_* < \infty$

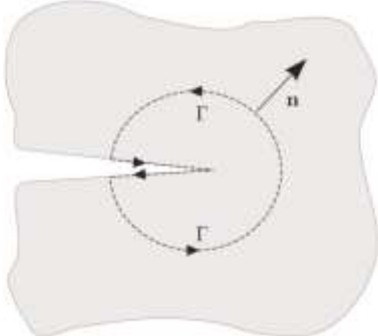
Answers:

Wrobel, M., Mishuris, G., Piccolroaz, A. (2017) Energy release rate in hydraulic fracture: Can we neglect an impact of the hydraulically induced shear stress? *Int. J. Engng Sci.*, 111, 28-51. **=>** ***Only toughness singularity "survives"***

Impact of the fluid induced shear stress

ERR Fracture Criterion for HF:

$$J = \int_\Gamma \left\{ \frac{1}{2}(\boldsymbol{\sigma} \cdot \boldsymbol{\varepsilon}) n_x - \boldsymbol{t}_n \cdot \frac{\partial \boldsymbol{u}}{\partial x} \right\} ds,$$



$$J = \frac{1}{E^*}\left(K_I^2 + aK_I K_f\right) = \frac{1}{E^*} K_{Ic}^2 \qquad a = \frac{4}{1+\nu^*} = 4(1-\nu).$$

$$\boxed{K_f = \varpi K_I,} \quad \varpi = \frac{p_0}{G - p_0} > 0, \qquad \begin{array}{l} \varpi \to 0, \quad K_{IC} \gg 1, \\ \varpi \to \infty, \quad K_{IC} \ll 1. \end{array}$$

$$K_I = \boxed{\frac{K_{IC}}{\sqrt{1 + a\varpi}}},$$

Apparent toughness

Large Toughness:

$K_I \to K_{IC},\ J \to J_C,\ (K_f \to 0),\ K_{IC} \gg 1,$

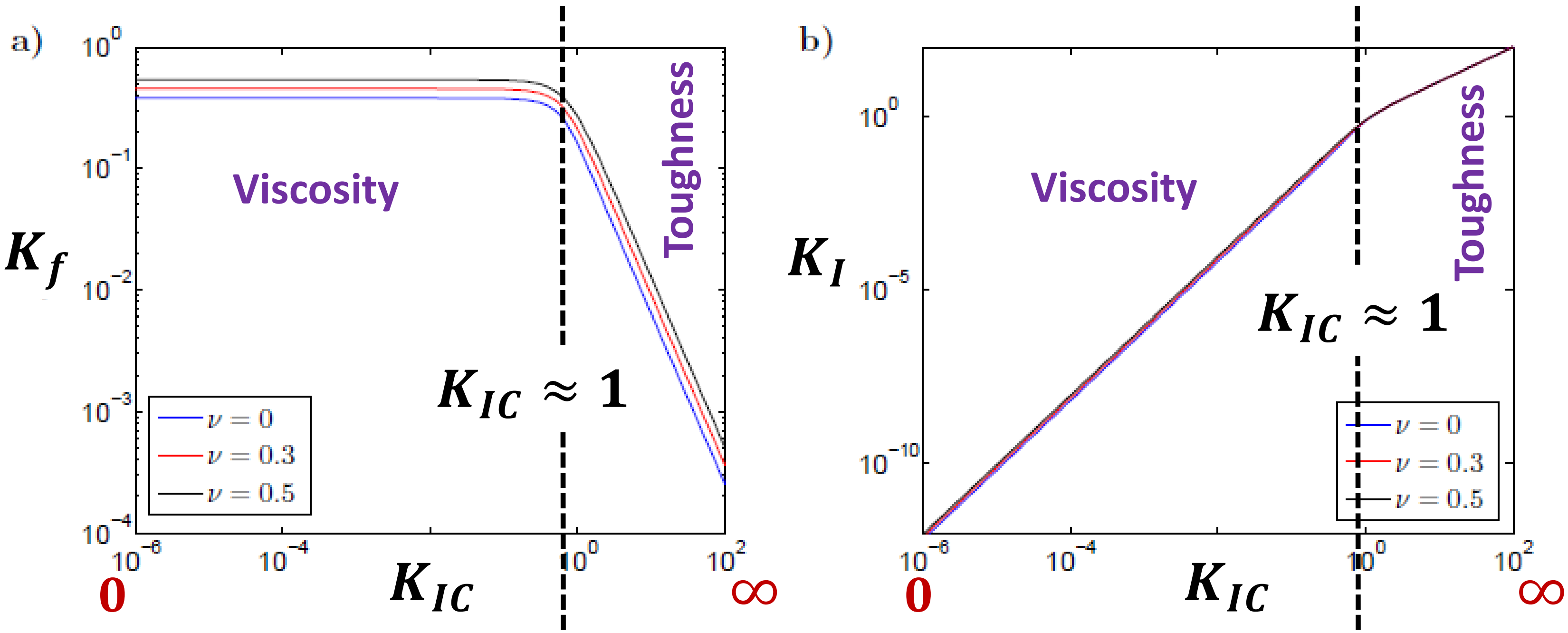
Small Toughness:

$K_I \to 0,\ J \to 0,\ (K_f \to K_* < \infty),\ K_{IC} \ll 1.$

Impact of the fluid induced shear stress

Normalised dimensionless SIFs K_f, K_I *versus the normalised fracture toughness* K_{IC} *for various values of the Poisson ratio*

KGD



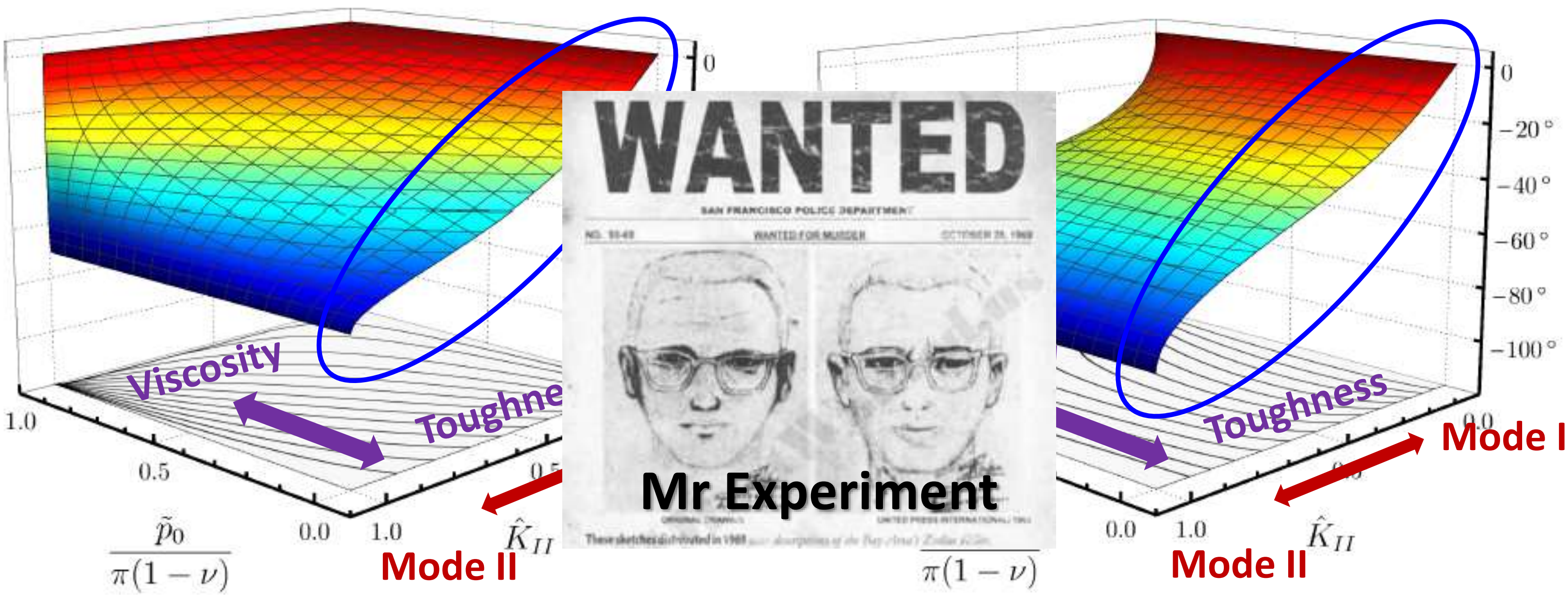
Combination of the accurate ERR criterion and the universal $1/\sqrt{r}$ - stress singularity is in a sense a natural HF regulariser

Impact on HF redirection (mixed mode I-II)

$$\sigma_{\theta\theta}(r,\theta) = \frac{K_{IC}}{\sqrt{2\pi r}}\left[\hat{K}_I \Psi_{11}^{\theta\theta}(\theta) + \hat{K}_{II}\Psi_{21}^{\theta\theta}(\theta) + 2(1-\nu)\hat{K}_f \Psi_{\tau_0}^{\theta\theta}(\theta)\right]$$

ERR: $K_I^2 + K_{II}^2 + 4(1-\nu)K_I K_f = K_{IC}^2.$ $\hat{K}_f = \tilde{\varpi}\hat{K}_I,$

Maximum Circumferential Stress **Minimum Strain Energy Density**



✓ ***Formulation accounting for the shear stress induced by fluid essentially influences the fracture propagation direction !!!***

Impact on HF redirection (mixed mode I-II) taking into account also local plastic zone

Only elastic effects:

- **Maximum Circumferential Stress**
- **Minimum Strain Energy Density**

Accounting for local plastic effects (via leading asymptotic terms) (not solving full elasto-plastic problem!):

- **Maximum Dilatational Strain Energy Density (MDSED)**
- **Modified Maximum Circumferential Stress (MMCS)**
  - **a) von Mises yield criterion**
  - **b) Drucker-Prager yield criterion**
  - **c) Tresca yield criterion**
  - **d) Mohr-Coulomb yield criterion**

✓ ***Formulation accounting for the shear stress induced by fluid significantly influences the fracture propagation direction !!!***

Impact on the HF Solver/s

- ✓ ***AU-UA solver built for the formulation accounting for the shear stress is completely universal (no "regime change" needed);***
- ✓ ***Computational performance appears the same regardless of the HF propagation regime;***
- ✓ ***Hybrid model (adjusted ERR and "old" elasticity) has all advantages of the full revised model;***
- ✓ ***Thus… it easy to implement into any existing "toughness" solver;***
- ✓ ***It brings new light on the direction of the fracture propagation;***
- ✓ ***It may allow for efficient and accurate 2D-3D front tracking!***

Shear traction induced by the fluid on the crack surfaces plays a role of a natural HF regulariser

Related references

[1] Peck, D, Da Fies, G. (2022) Shear traction induced by the fluid in hydraulic fracture (Penny-Shaped crack). (TBS)

[2] Wrobel, M., Mishuris, G, Papanastasiou, P. (2021) On the influence of fluid rheology on hydraulic fracture. IJES, 158, 103426.

[3] Wrobel, M., Piccolroaz, A., Papanastasiou, P., Mishuris. G. (2021) Redirection of a crack driven by viscous fluid taking into account plastic effects in the process zone. *Geomechanics for Energy and the Environment*. 26, 100147.

[4] Wrobel, M., Mishuris, G., Piccolroaz, A. (2018) On the impact of tangential traction on the crack surfaces induced by fluid in hydraulic fracture: Response to the letter of A.M. Linkov. *IJES*, 127, 220–224

[5] Perkowska, M., Piccolroaz, A., Wrobel, M. Mishuris, G. (2017) Redirection of a crack driven by viscous fluid, *IJES*, 121, 182-193

[6] Wrobel, M., Mishuris, G., Piccolroaz, A. (2017) Energy release rate in hydraulic fracture: Can we neglect an impact of the hydraulically induced shear stress? *IJES*, 111, 28-51.

Plan for the talk

- **Introduction**
- **Development of AU-UA solvers** [1D (in space & time) utilising an *explicit* fracture tracing algorithm and appropriate asymptotic network]
- **Accounting for the shear traction induced by the fluid** on the crack surfaces
- **Generalisation of ERR** (appropriate for all LEFM problems with line defects)
- **Variable toughness** (how to address the issue?)
  - ✓ Preliminary results (simulations)
  - ✓ Suggested averaging strategies (not homogenisation!) & its verification
  - ✓ Conclusions & Discussions

Discussions of the shear stress impact

- Wrobel, M., Mishuris, G., Piccolroaz, A. 2018 On the impact of tangential traction on the crack surfaces induced by fluid in hydraulic fracture: Response to the letter of AM Linkov. IJES. (2018) 127, 217-219, IJES. 127, 220-222.
- Garagash, D. 2018 Private correspondence: **what about Irwin's crack closure integral ?**

$$\mathcal{G} = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} \int_0^{\Delta a} \boxed{\langle \sigma_{2i}(a) \rangle (r) [\![u_i(a)]\!] (\Delta a - r)\, dr.}$$

$$\mathcal{G} = \frac{1-\nu^2}{E} \left(K_I^2 + \cancel{(3-2\nu)} K_I K_f + 2(1-\nu) K_f^2 \right).$$

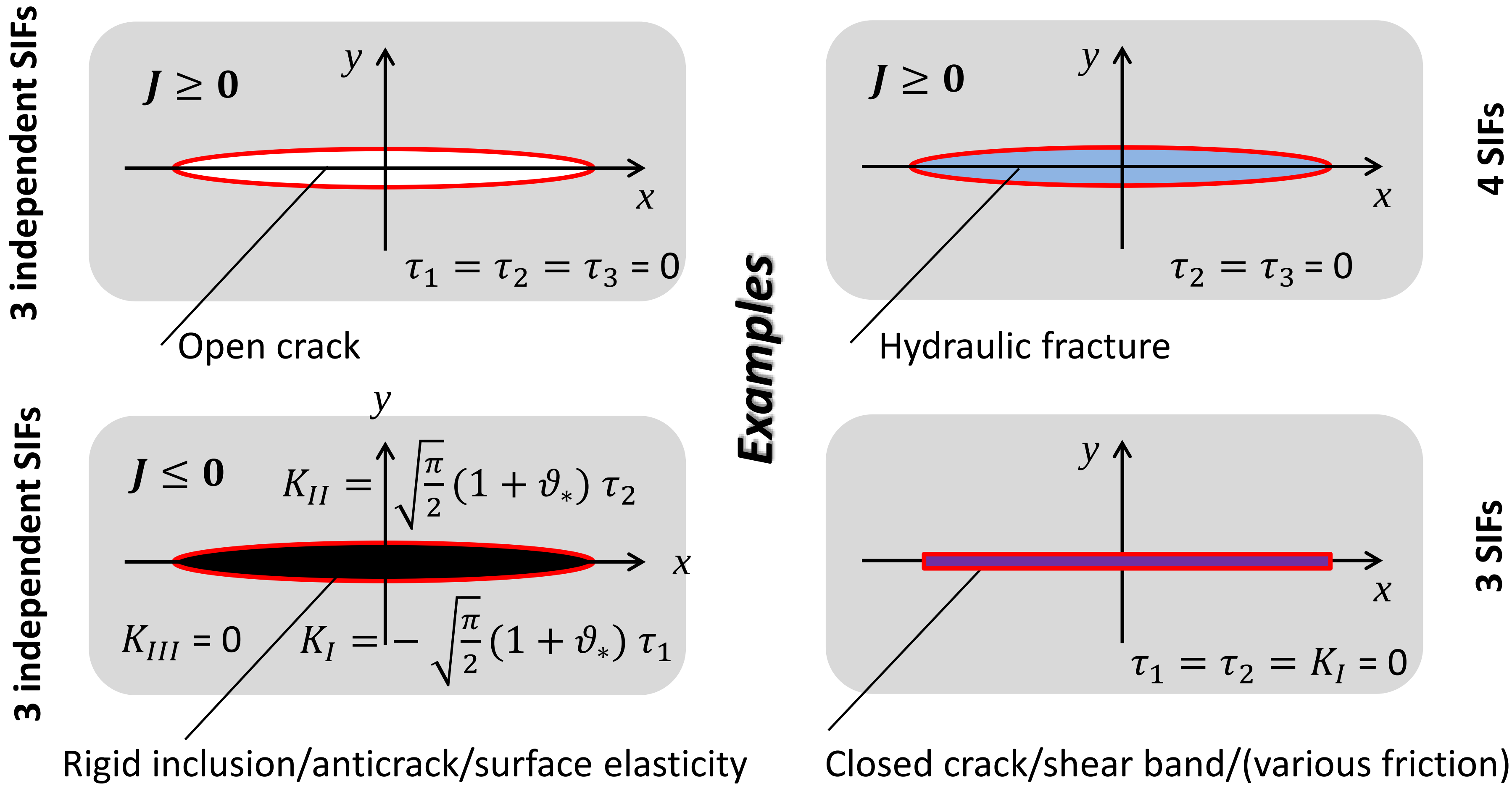
$$J = \frac{1}{E^*} \left(K_I^2 + a K_I K_f \right) \qquad a = \frac{4}{1+\nu^*} = 4(1-\nu).$$

$$\boxed{J = \mathcal{G}} \quad !$$

$$\mathcal{G} = \lim_{\Delta a \to 0} \frac{1}{2\Delta a} \int_0^{\Delta a} \left\{ \begin{aligned} &\langle \sigma_{\theta r}(a)(r) \rangle [\![-u_r(a)(\Delta a - r)]\!] + \langle u_r(a)(r) \rangle [\![\sigma_{\theta r}(a)(\Delta a - r)]\!] + \\ &\langle \sigma_{\theta \theta}(a)(r) \rangle [\![-u_\theta(a)(\Delta a - r)]\!] + \langle u_\theta(a)(r) \rangle [\![\sigma_{\theta \theta}(a)(\Delta a - r)]\!] + \\ &\langle \sigma_{\theta z}(a)(r) \rangle [\![u_z(a)(\Delta a - r)]\!] - \langle u_z(a)(r) \rangle [\![-\sigma_{\theta z}(a)(\Delta a - r)]\!] \end{aligned} \right\} dr.$$

Impact on the Fracture Mechanics

Where can one use this generalised ERR formula?



$$J = \frac{1}{E_*}\left(K_I^2 + K_{II}^2 + 2\sqrt{2\pi}K_I\tau_1 - 2\sqrt{2\pi}K_{II}\tau_2\right) + \frac{1+\nu_*}{E_*}\left(K_{III}^2 - 2\pi\tau_3^2\right).$$

Conclusion

Finite ERR for “arbitrary” Boundary Conditions

in a neighbourhood of line/planar defect tip/front

can be computed with use of

six SIFs: $K_I, K_{II}, K_{III}, \tau_1, \tau_2, \tau_3$

$$\boxed{J = \frac{1}{E_*}\left(K_I^2 + K_{II}^2 + 2\sqrt{2\pi}K_I\tau_1 - 2\sqrt{2\pi}K_{II}\tau_2\right) + \frac{1+\nu_*}{E_*}\left(K_{III}^2 - 2\pi\tau_3^2\right).}$$

[1] Piccolroaz, A., Peck, D., Wrobel, M., Mishuris, G. Energy Release Rate, the crack closure integral and admissible singular fields in Fracture Mechanics.(2021), IJES. 164, 103487

Plan for the talk

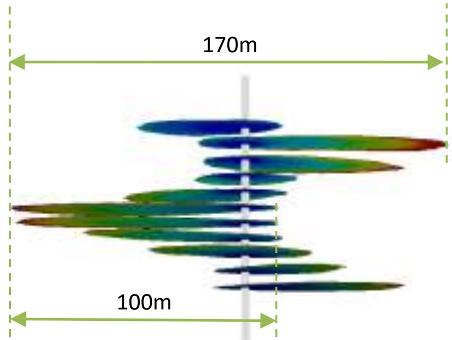
- **Introduction**
- **Development of AU-UA solvers** [1D (in space & time) utilising an *explicit* fracture tracing algorithm and appropriate asymptotic network]
- **Accounting for the shear traction induced by the fluid** on the crack surfaces
- **Generalisation of ERR** (appropriate for all LEFM problems with line defects)
- **Variable toughness** (how to address the issue?)
  - ✓ Preliminary results (simulations)
  - ✓ Suggested averaging strategies (not homogenisation!) & its verification
  - ✓ Conclusions & Discussions

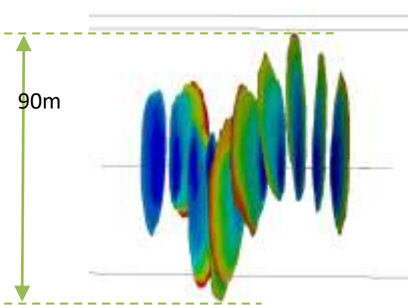
Sêr Cymru Industrial Fellowship

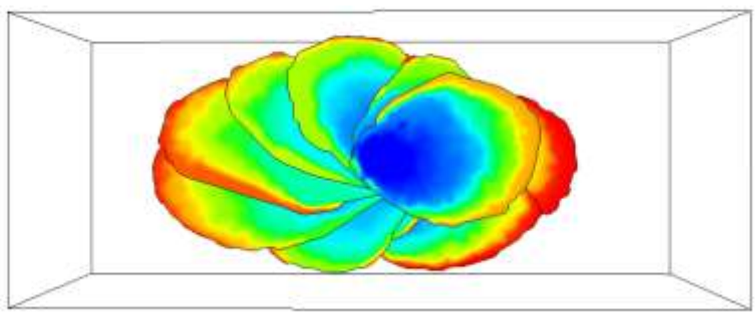
Aberystwyth University – RockField (Technology background)

Rockfield's software is based on **full physics, multi-field** using combined Finite / Discrete Element Method (FE/DEM)

Example:
Enhancement of SRV (Stimulated Reservoir Volume)







Evolution of complex 3D fractures.

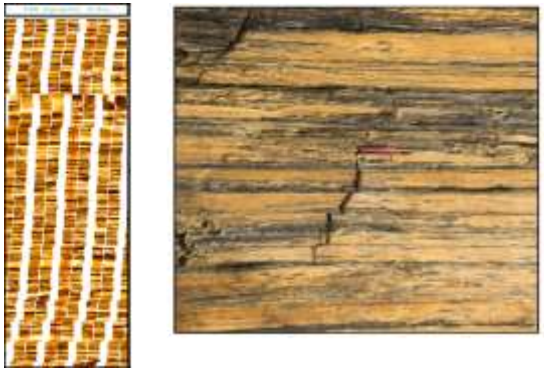
- ❑ Influence of the stress shadowing on the fracture propagation direction & fracture shape
- ❑ Significant asymmetry of fractures in all directions
- ❑ Fractures are not planar
- ❑ A helical pattern of fractures emerges

Heterogeneity: Observations & Motivation

- **Observations**
  - Unconventional reservoirs are **highly heterogeneous** – both in terms of in-situ horizontal stress and layer & interlayer material properties
  - The heterogeneity is one of the key challenges for **HF numerical simulations**
  - New technology measurements provide the scale 10 cm or even 1 cm!
  - Usual commercial software can compute in a reasonable time at least 1 m size (out of 1 km)

- **Motivations**
  - Advanced HF simulators (commercial & academic, can capture complex physical phenomena, but need data upscaling to perform the computations.
  - The validity of simulations may be questionable if inputs do not honour **reasonable ???** scale heterogeneity in critical formation properties

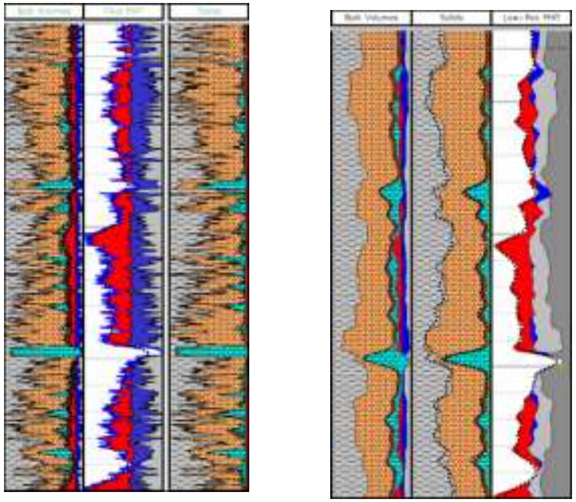
Presented by Adam Bere (Rockfield), ARMA Robe Talks, May 2020, https://www.youtube.com/watch?v=QgK49fgrqJM



Real life (From Galliot 2020, Richards, 2020)

High Resolution

Low Resolution



Measured field material data (Crawford, 2020)

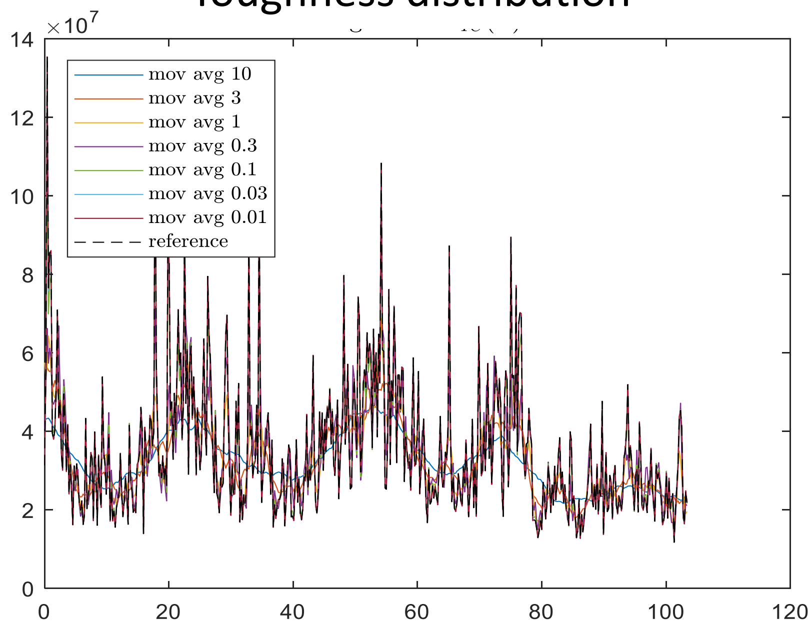
Sensitivity analysis (moving average)

How to implement the available data to be on a safe side in predictions?

It is also known that toughness homogenesation is available in LEFM…

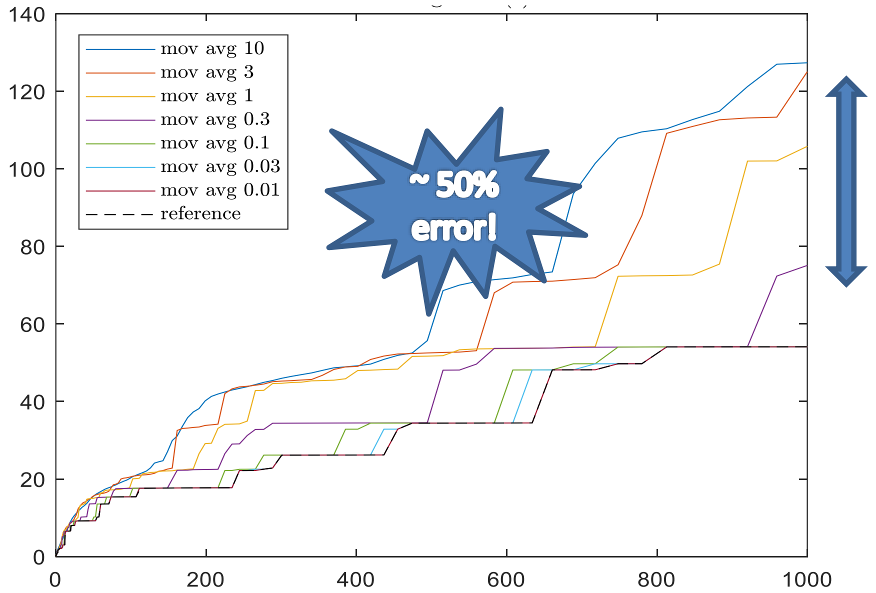
AU-UA can do any steps for 1D cases (<u>only toughness changes</u>)! KGD model

Toughness distribution



Random toughness

Crack length for different averaging

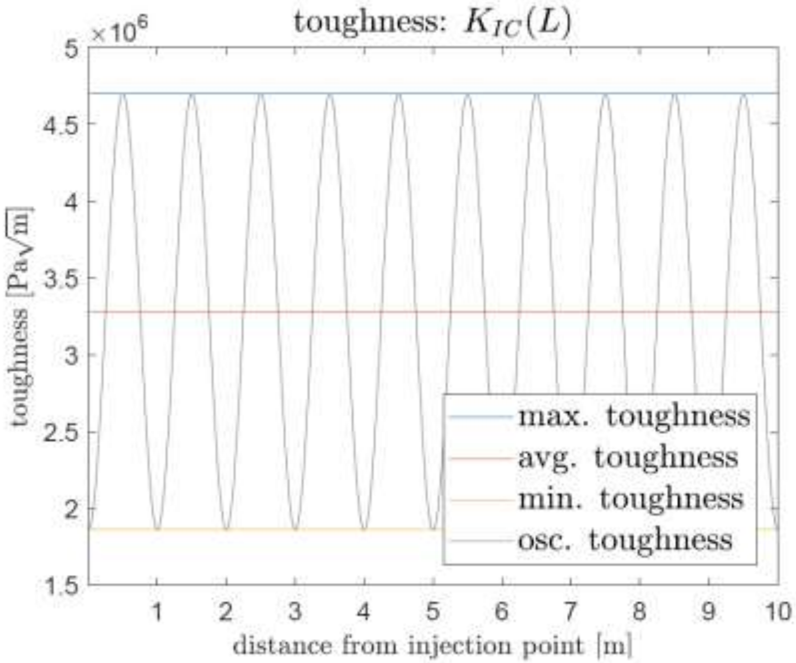


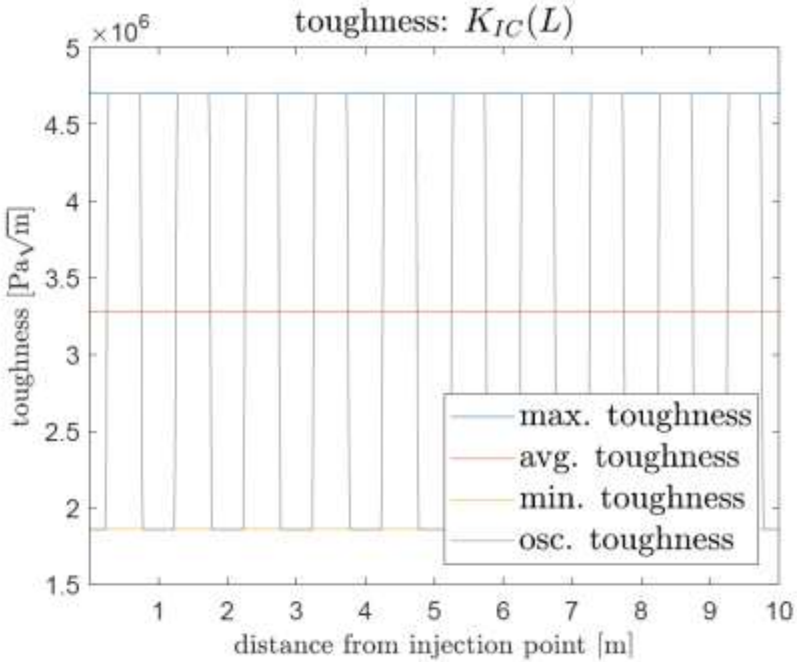
Treatment time

Preliminary tests (periodic toughness distribution)

Simulations with AU-UA: symmetrical KGD, Period = 1 m. KGD, realistic HF parameters

Sinusoidal **Toughness distribution** Step-wise





Defining Maximum / Minimum toughness "produces" combinations of the regimes:

a) toughness-toughness; **b)** toughness-viscosity; **c)** viscosity- viscosity.

- **Maximum Toughness Criterion** (**MTC**) proposed by Dontsov et.al. (2021)

Preliminary tests (periodic toughness distribution)

- Notations $w(t,x) = w_{vis}(t,x) + w_{tou}(t,x);$
  - Inverted elasticity equation (Wrobel, Mishuris 2015)

$$w_{vis} = \frac{1}{E'}\frac{4}{\pi}\int_0^{l(t)} \frac{\partial p}{\partial s}(t,s)\, l(t) Ker(x,s,t) ds\,; \qquad w_{tou} = \frac{K_{IC}(x)}{E'}\frac{4}{\sqrt{\pi l(t)}}\sqrt{l(t)^2 - x^2}$$

- Local propagation regime:

$$\delta(t) = \frac{V_{tou}(t)}{V_{vis}(t)}$$

Viscosity dominated: $\delta(t) \ll 1,\quad \delta_{\mathcal{K}} \sim c_K \mathcal{K},\quad \mathcal{K} \ll 1,$

Toughness dominated: $\delta(t) \gg 1,\quad \delta_{\mathcal{M}} \sim c_M/\mathcal{M},\quad \mathcal{M} \ll 1$

- Three simulation configurations

Data-Set	K_{IC}^{max} [Pa.m$^{1/2}$]	K_{IC}^{min} [Pa.m$^{1/2}$]	$\delta_{Max}/\delta_{Min}$	**Regime**
DS 1	8.42 e+06	4.50 e+06	100 / 10	Both K_{IC}^{max} & K_{IC}^{min} inside toughness dominated regime
DS 2	4.50 e+06	1.77 e+06	10 / 1	K_{IC}^{max} toughness dominated, K_{IC}^{min} intermediate regime
DS 3	1.77 e+06	3.11 e+05	1 / 0.1	K_{IC}^{max} intermediate, K_{IC}^{min} viscosity dominated regime

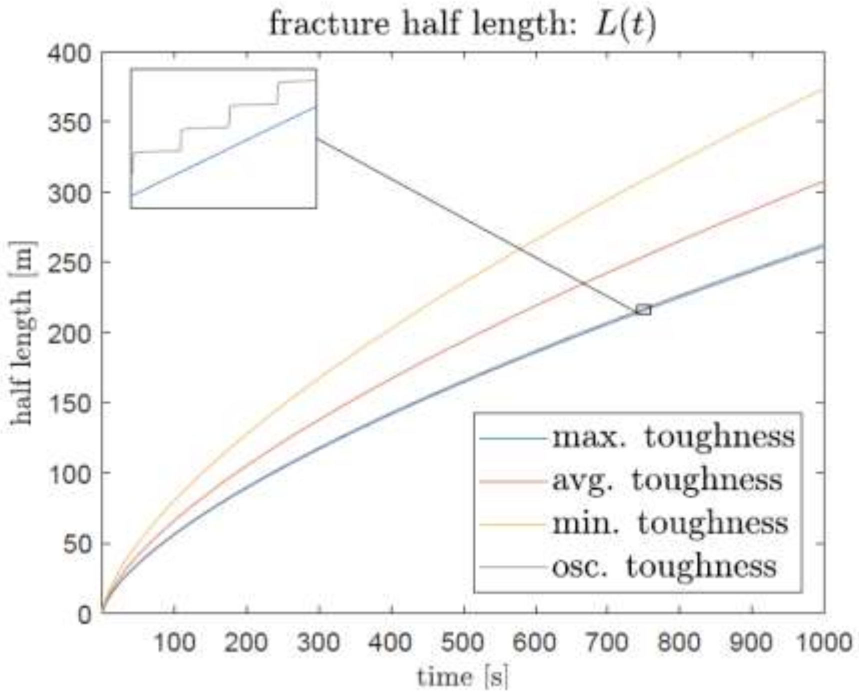
- Further data
  - Injection rate - $Q_0 = 6.62 \cdot 10^{-2}\ \mathrm{m}^3/s$; Young's Modulus - $E = 2.81 \cdot 10^{10}$ Pa ; Poisson's radio - $\nu = 0.25$; fluid viscosity - $\mu = 10^{-3}$ Pa.s, crack height - $H = 15$ m

Preliminary tests (Toughness-toughness)

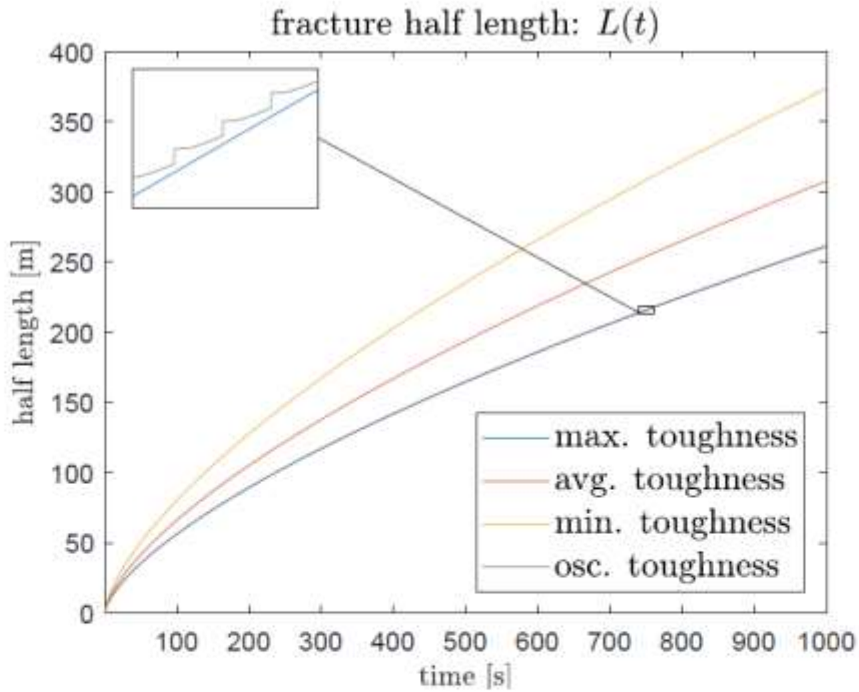
Regime indicator (max and min): $\delta_{max} = 100 \text{ and } \delta_{min} = 10$

Crack Length

Sinusoidal



Step-wise



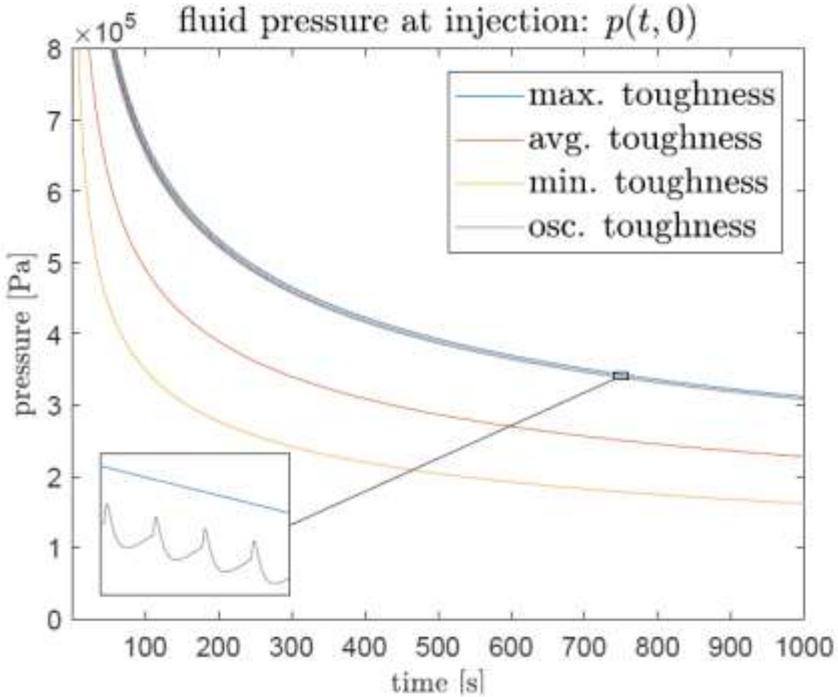
MTC works nicely!

Preliminary tests (Toughness-toughness)

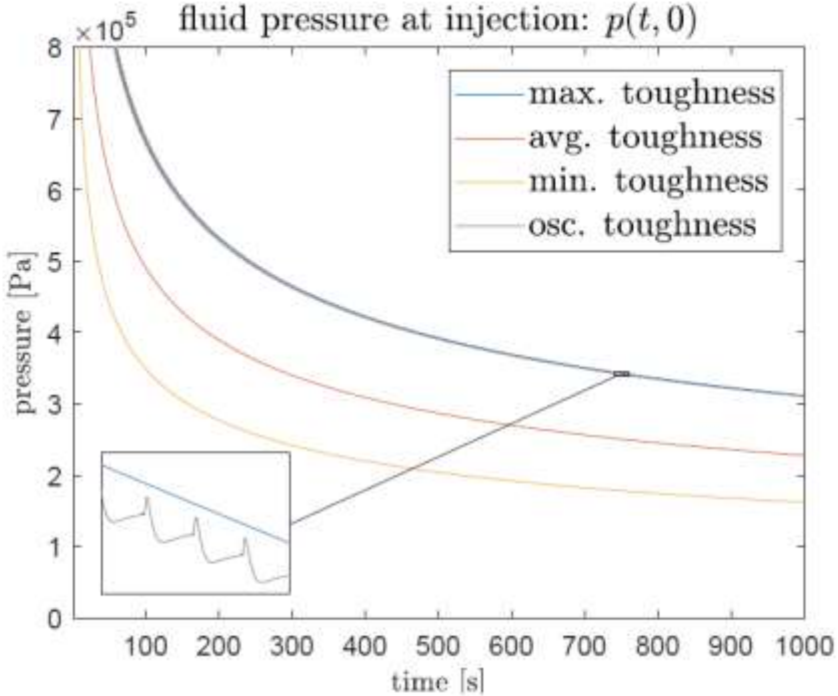
Regime indicator (max and min): $\delta_{max} = 100 \text{ and } \delta_{min} = 10$

Pressure at injection point

Sinusoidal



Step-wise



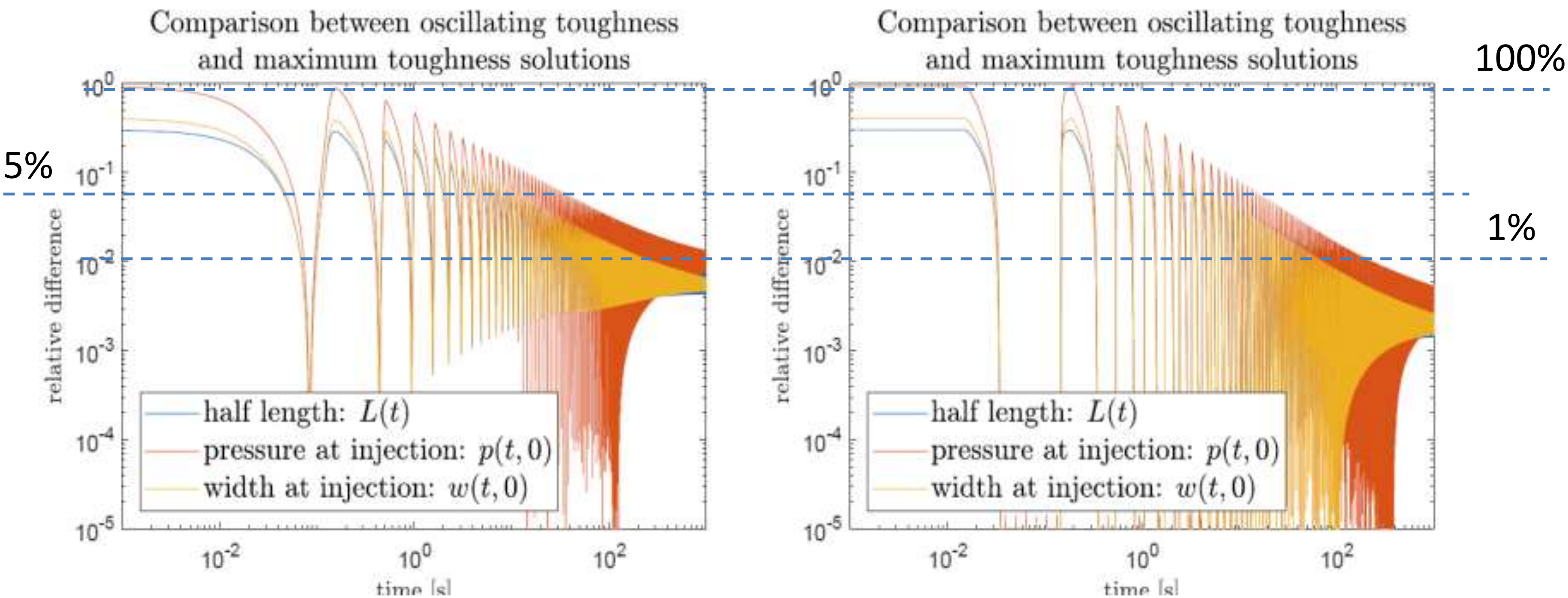
MTC looks like the perfect replacement strategy!

Preliminary tests (Toughness-toughness)

Regime indicator (max and min): $\delta_{max} = 100 \text{ and } \delta_{min} = 10$

MTC relative error in comparison with real toughness distribution

Sinusoidal | Step-wise



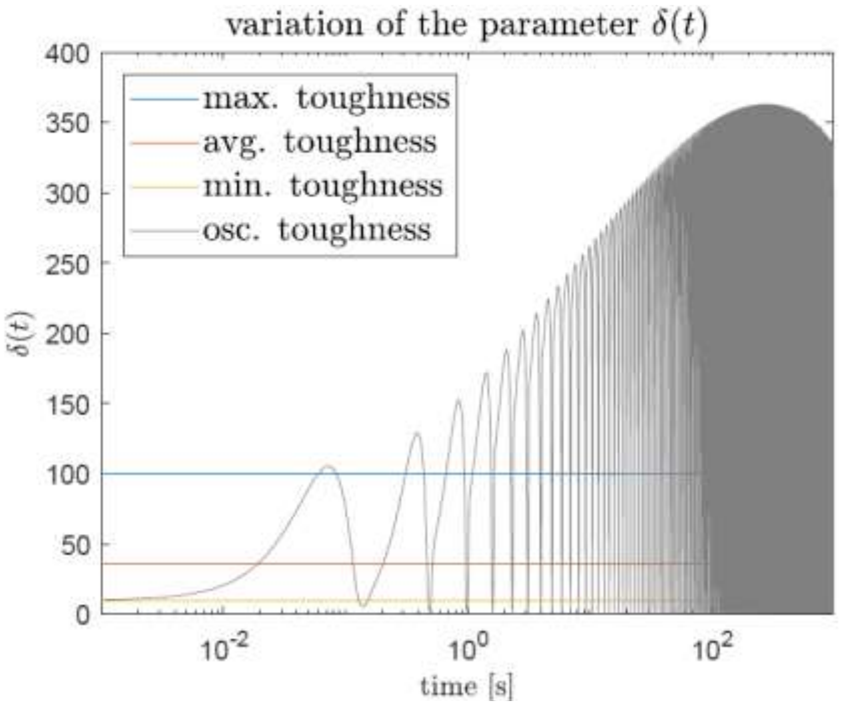
- *Well… For large time (length) yes, but for small… Reason? Cure ???*
- *Interesting fact: step-wise distribution tends faster to MTC limit !*

Preliminary tests (Toughness-toughness)

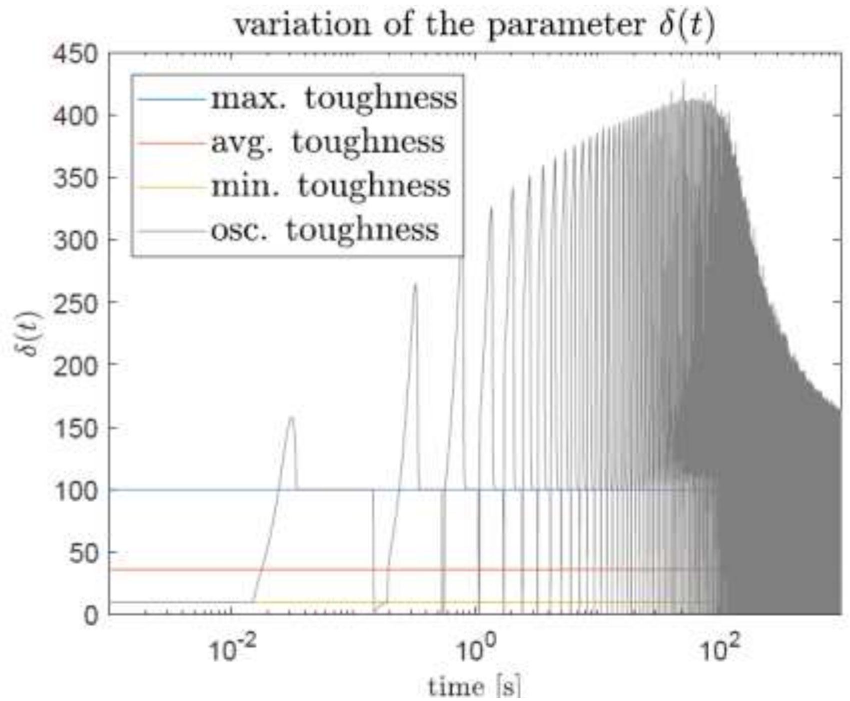
Regime indicator (max and min): $\delta_{max} = 100 \text{ and } \delta_{min} = 10$

Local (in time) regime indication $\delta(t)$

Sinusoidal



Step-wise



$\delta \ll 1$ – local viscosity dominated regime $\delta \gg 1$ – high (local) toughness

$\cancel{\delta_{min} < \delta(t) < \delta_{max}}$?? ***NO*** ☹

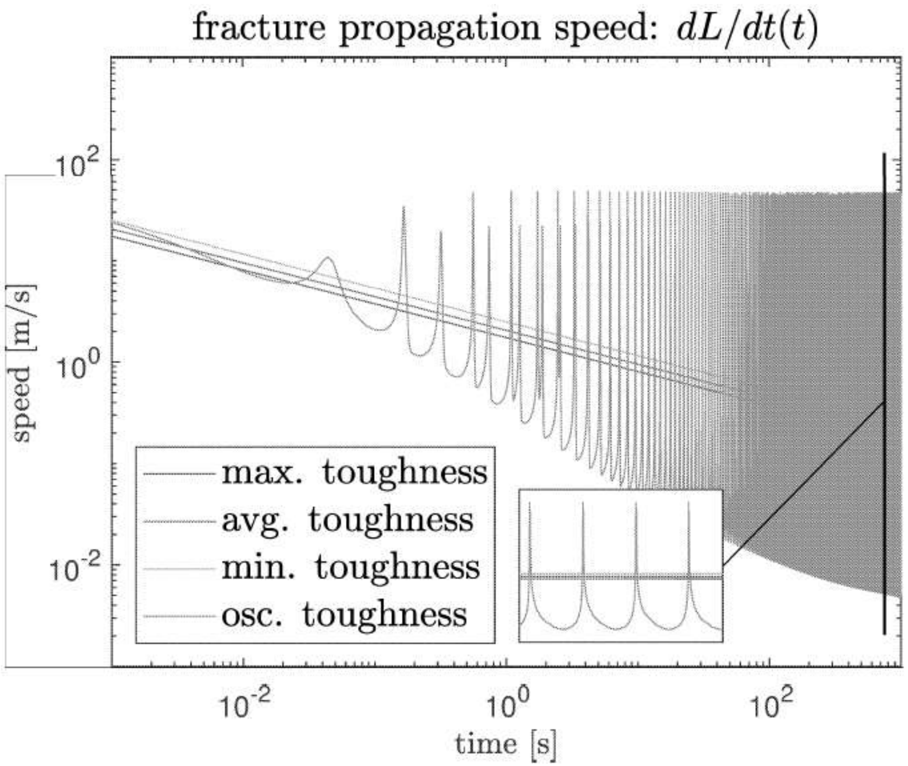
DS 1

Preliminary tests (Toughness-toughness)

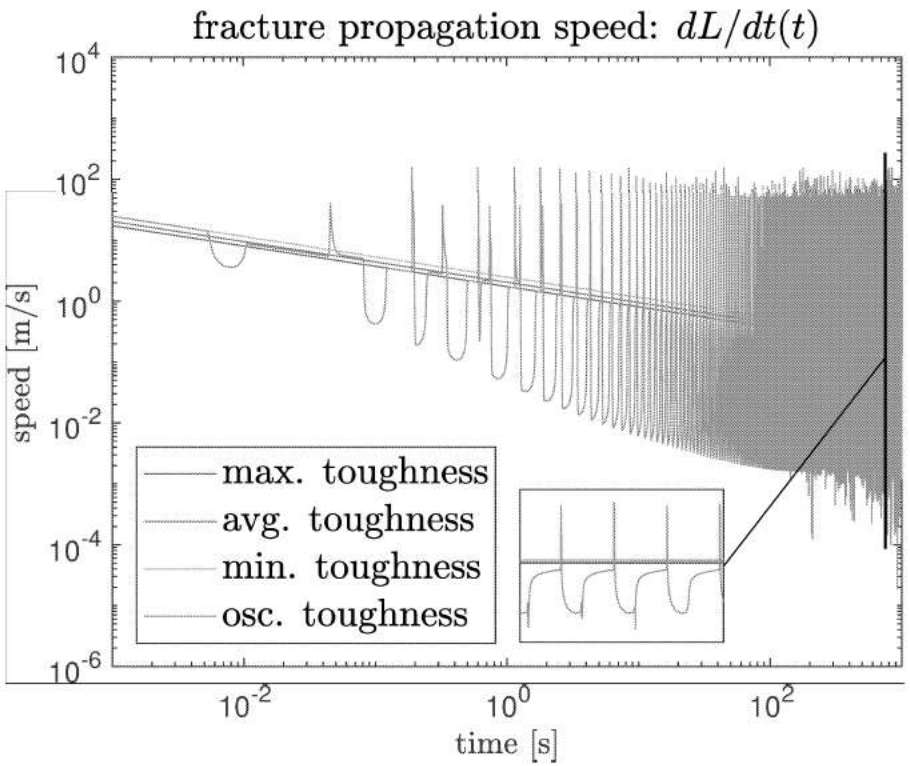
Regime indicator (max and min): $\delta_{max} = 100 \text{ and } \delta_{min} = 10$

Crack speed distribution (in time)

Sinusoidal



Step-wise



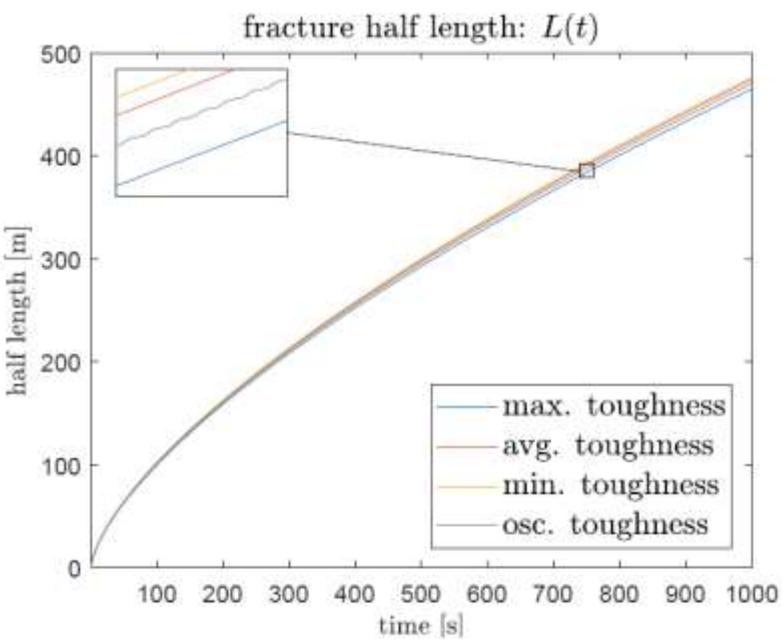
Dramatic crack speed gradient! (acceleration / deceleration)

Preliminary tests (Viscosity-viscosity)

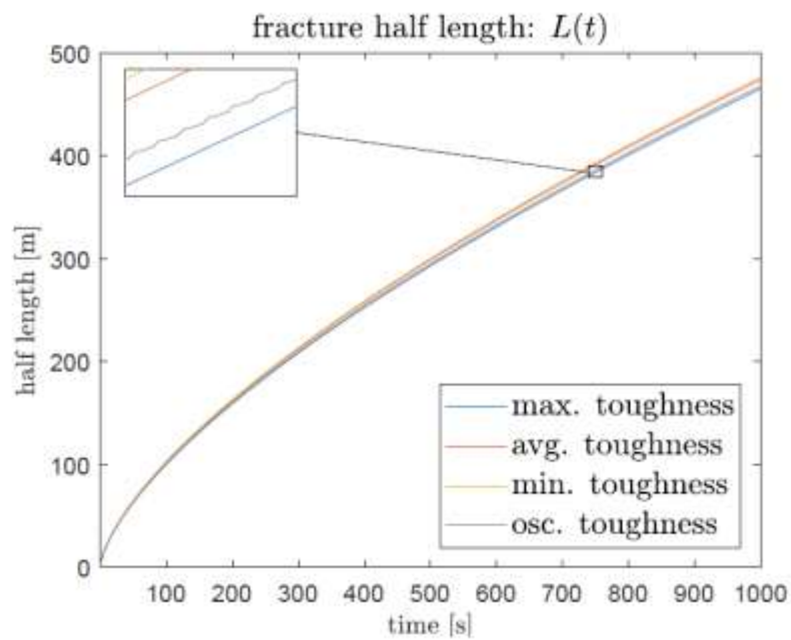
Regime indicator (max and min): $\delta_{max} = 1$ and $\delta_{min} = 0.1$

Crack Length

Sinusoidal



Step-wise

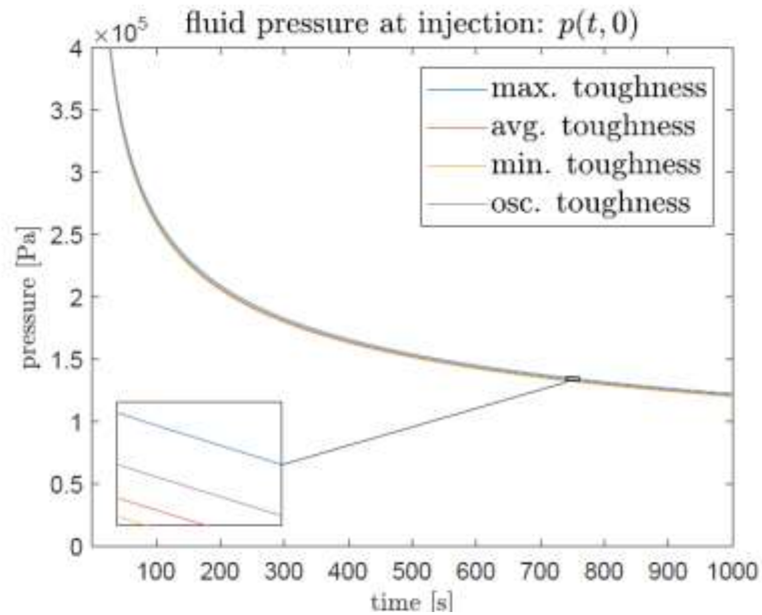


Preliminary tests (Viscosity-viscosity)

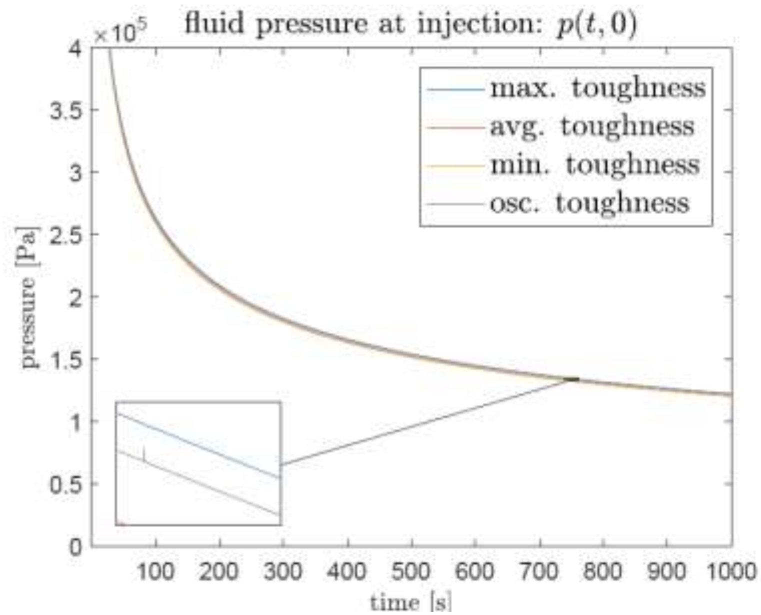
Regime indicator (max and min): $\delta_{max} = 1 \text{ and } \delta_{min} = 0.1$

Pressure at injection point

Sinusoidal



Step-wise



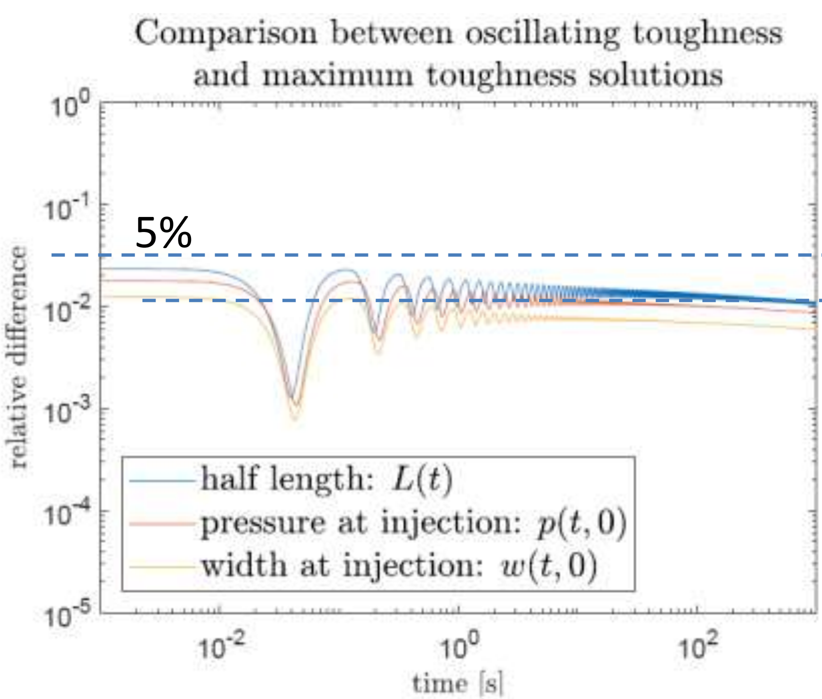
Toughness not so important – one can choose any strategy!

Preliminary tests (Viscosity-viscosity)

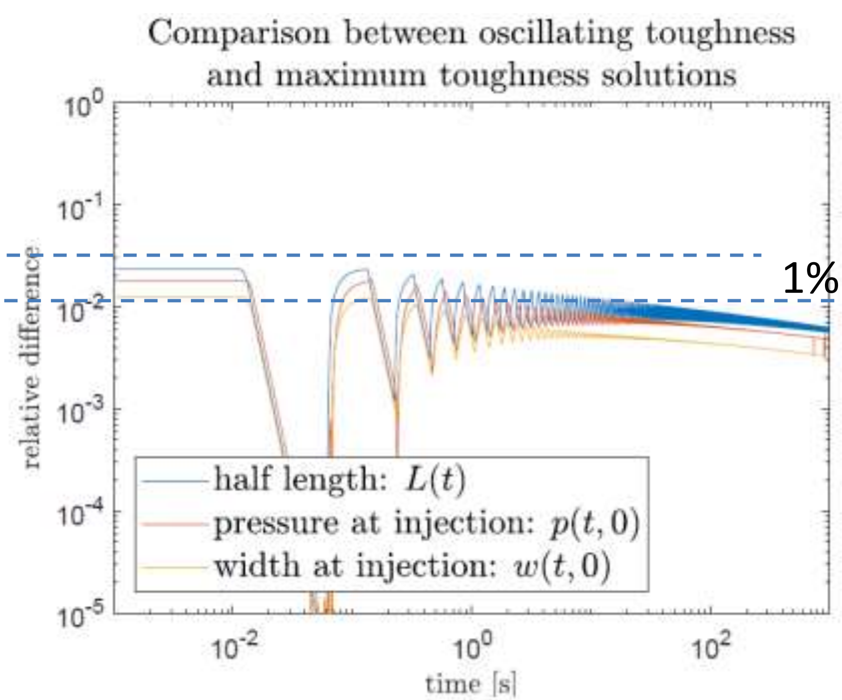
Regime indicator (max and min): $\delta_{max} = 1 \text{ and } \delta_{min} = 0.1$

MTC relative error in comparison with real toughness distribution

Sinusoidal



Step-wise



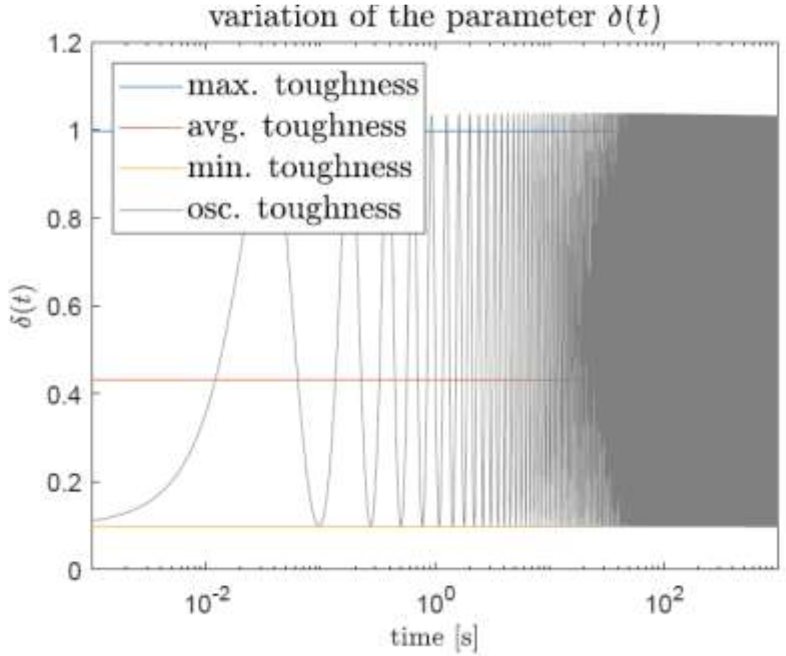
Less than 5% for entire process, any criterion enough for applications!

Preliminary tests (Viscosity-viscosity)

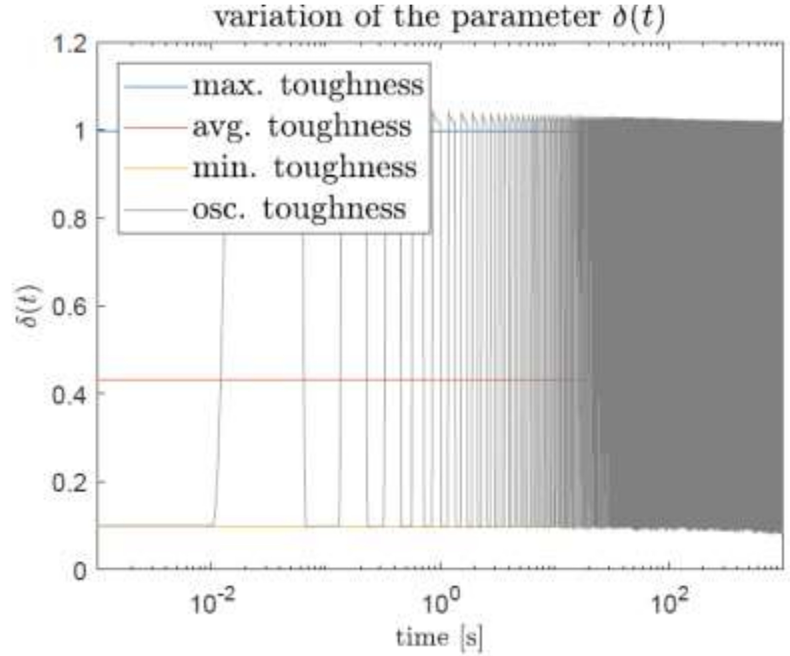
Regime indicator (max and min): $\delta_{max} = 1 \text{ and } \delta_{min} = 0.1$

Local (in time) regime indication

Sinusoidal



Step-wise



$\delta \ll 1$ – local viscosity dominated regime $\delta \gg 1$ – high (local) toughness

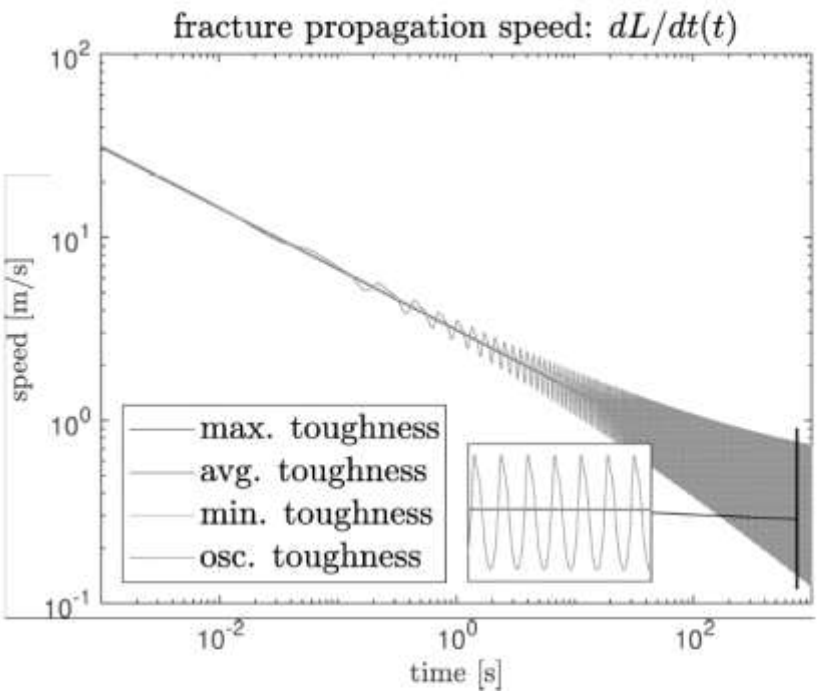
$\delta_{min} < \delta(t) < \delta_{max}$?? ***Practically YES !***

Preliminary tests (Viscosity-viscosity)

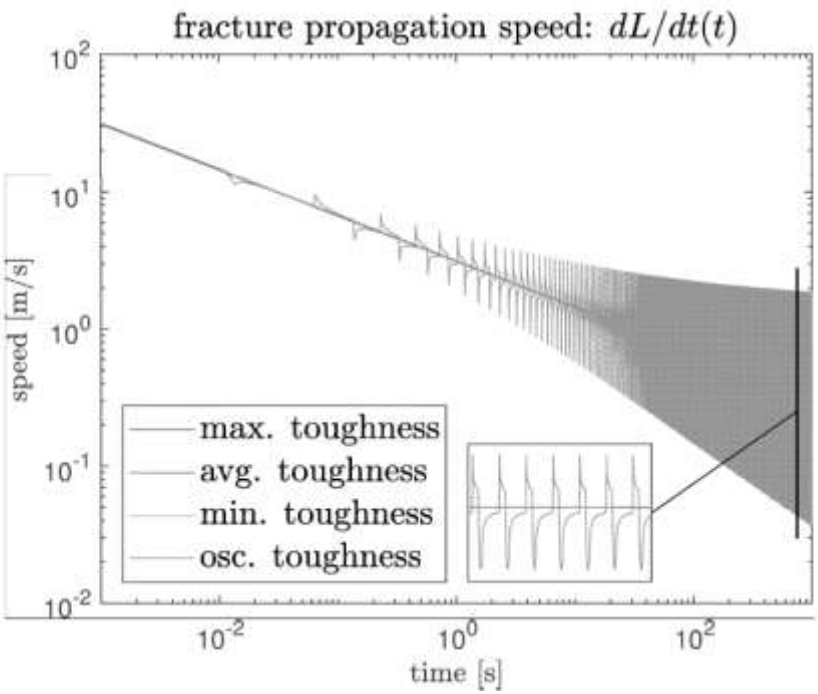
Regime indicator (max and min): $\delta_{max} = 1 \text{ and } \delta_{min} = 0.1$

Crack speed distribution (in time)

Sinusoidal



Step-wise

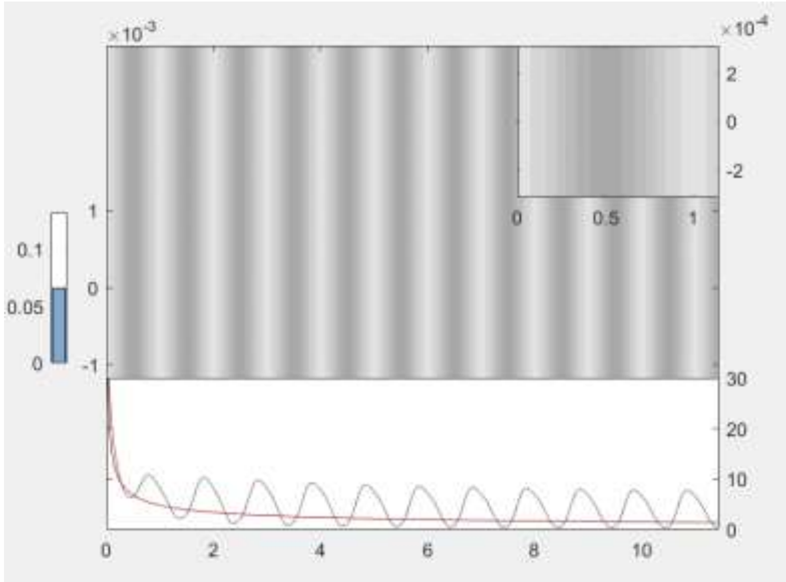


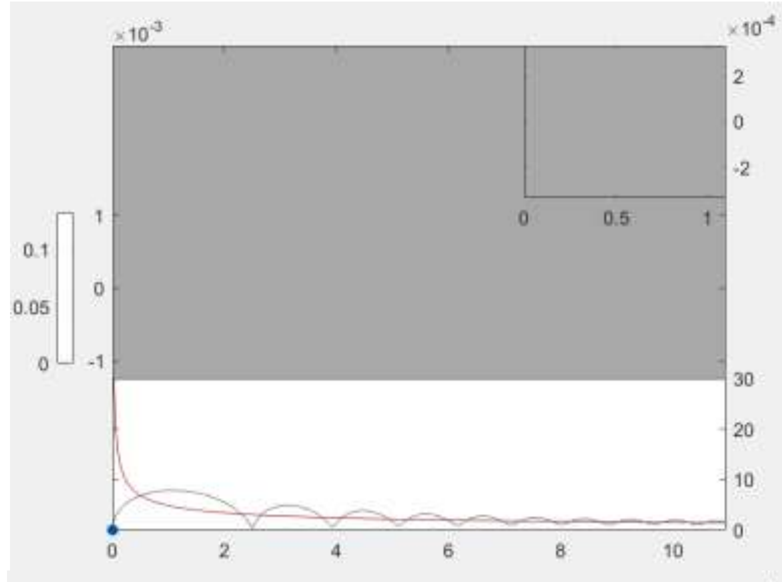
Crack speed gradient (acceleration / deceleration) still significant

DS 2 (moderate toughness – moderate viscosity)

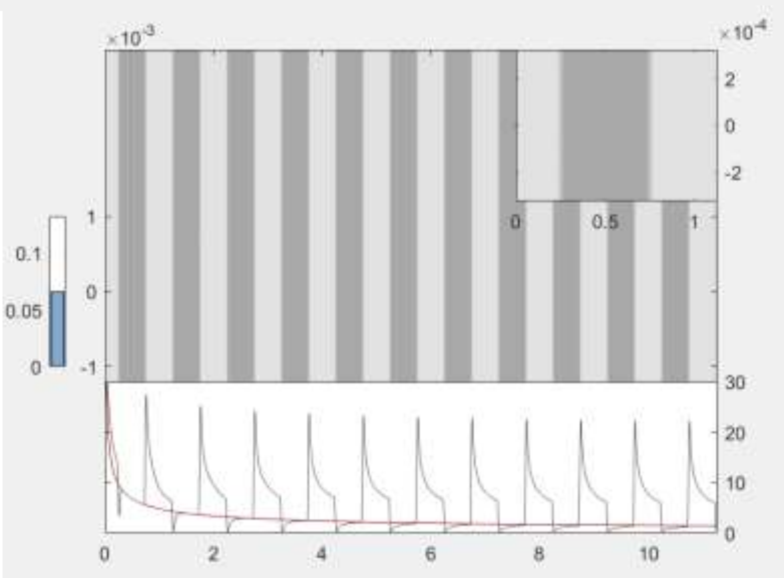
Variable toughness vs Variable injection rate

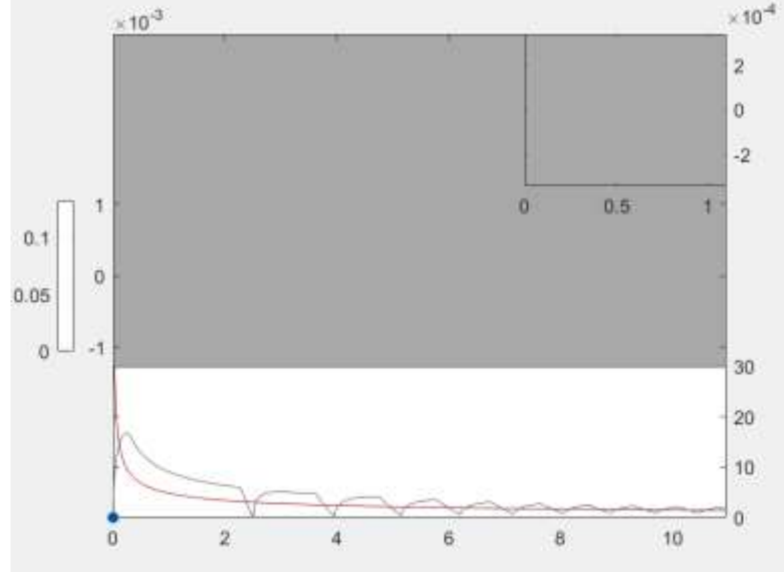
Sinusoidal





Step – wise

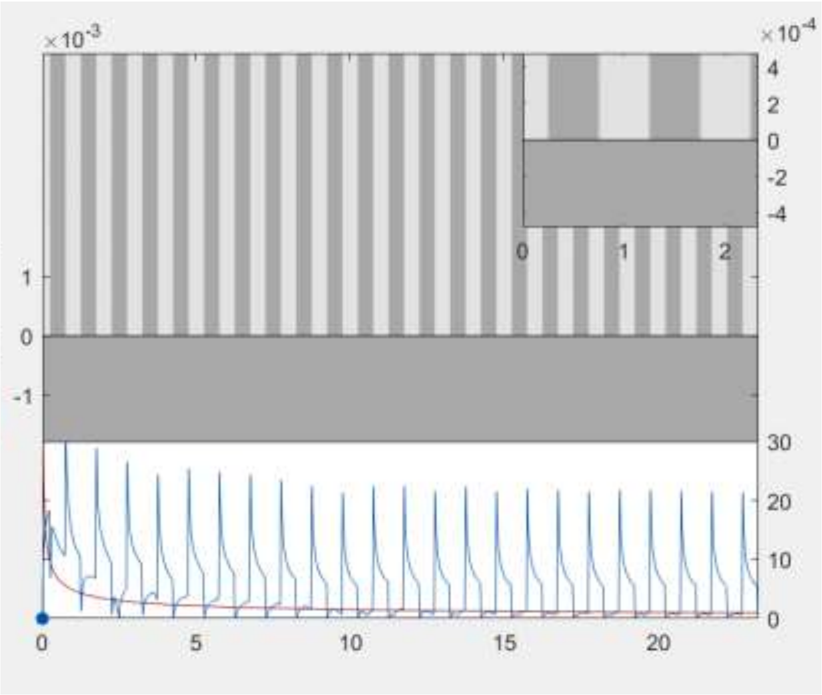




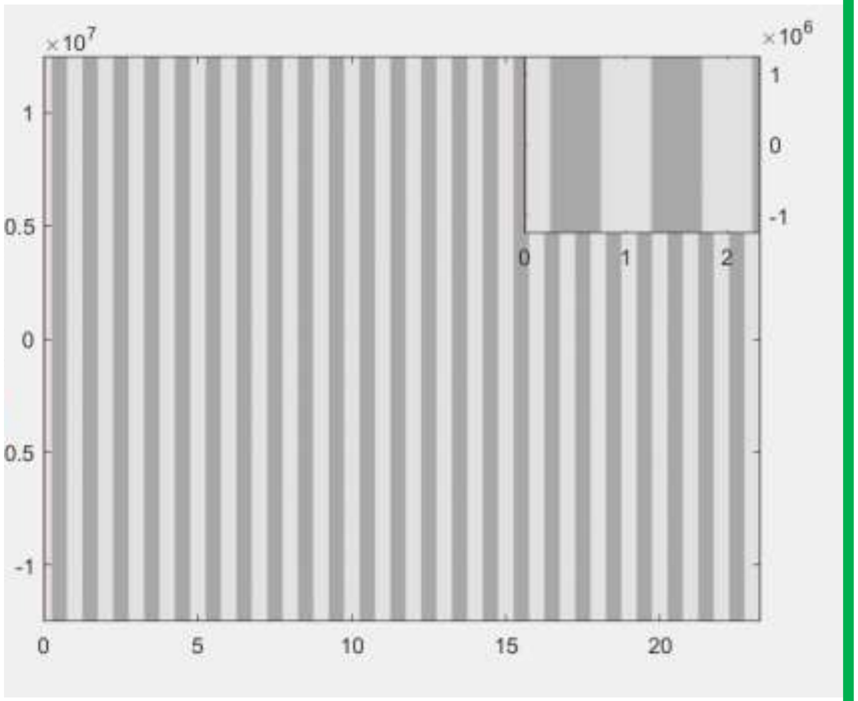
DS 2 (moderate toughness – moderate viscosity)

Variable step-wise toughness vs **MTC (maximal toughness)**

Fracture profile



Net pressure

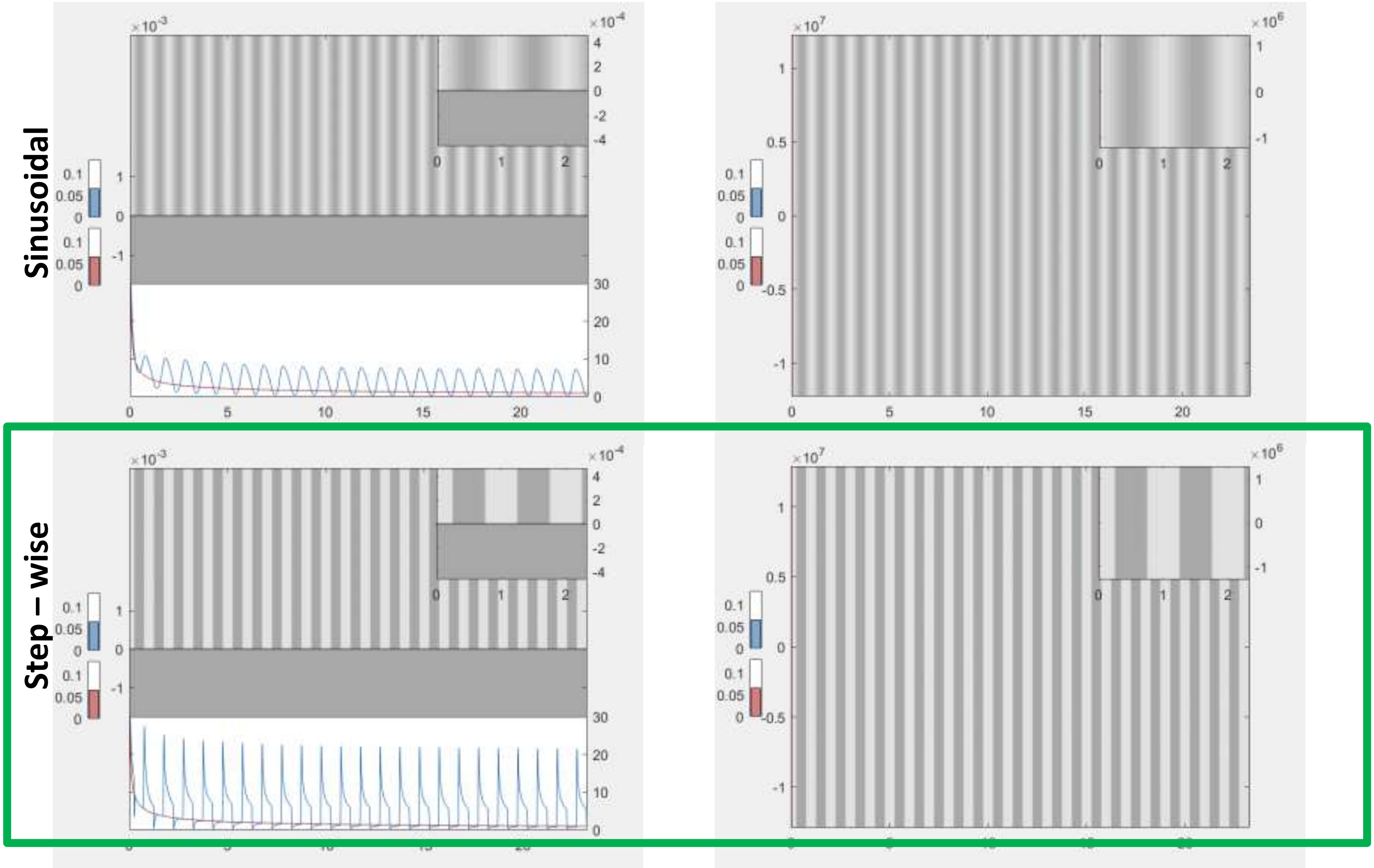


Equivalent (constant) injection rates in both simulations

DS 2 (moderate toughness – moderate viscosity)

Crack profile

Net pressures profile

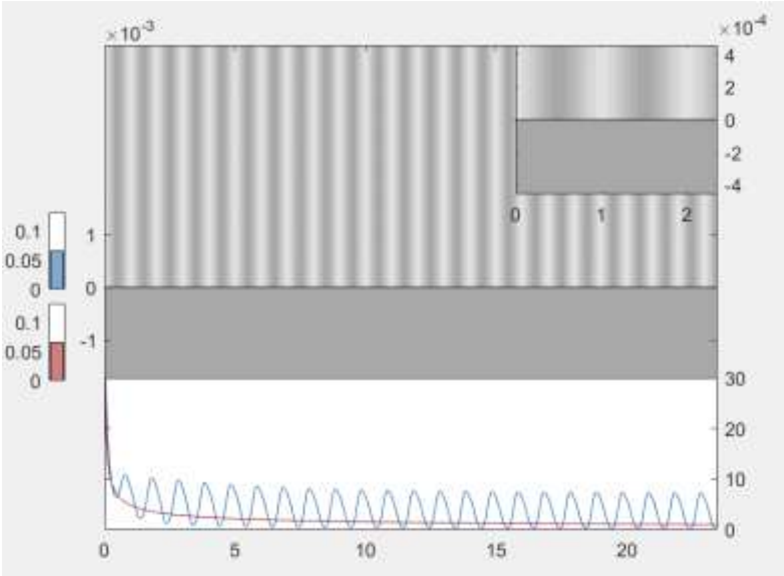


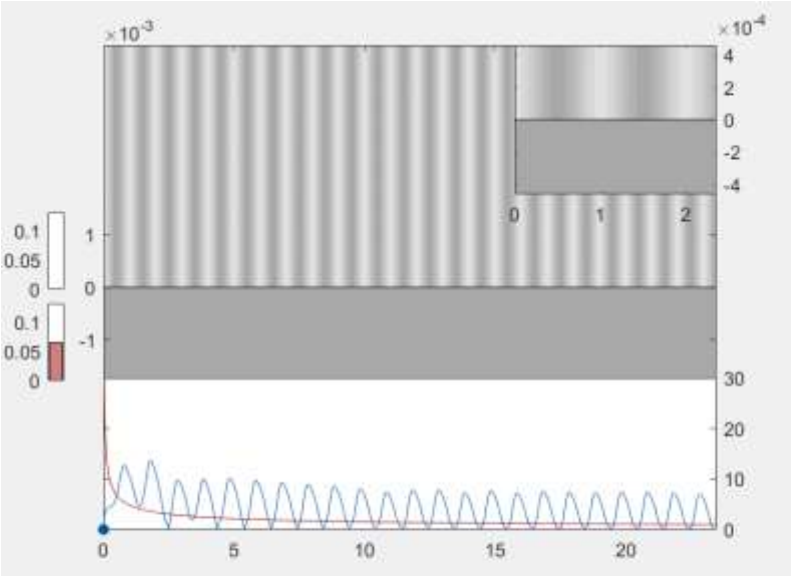
DS 2 (moderate toughness – moderate viscosity)

Crack profile

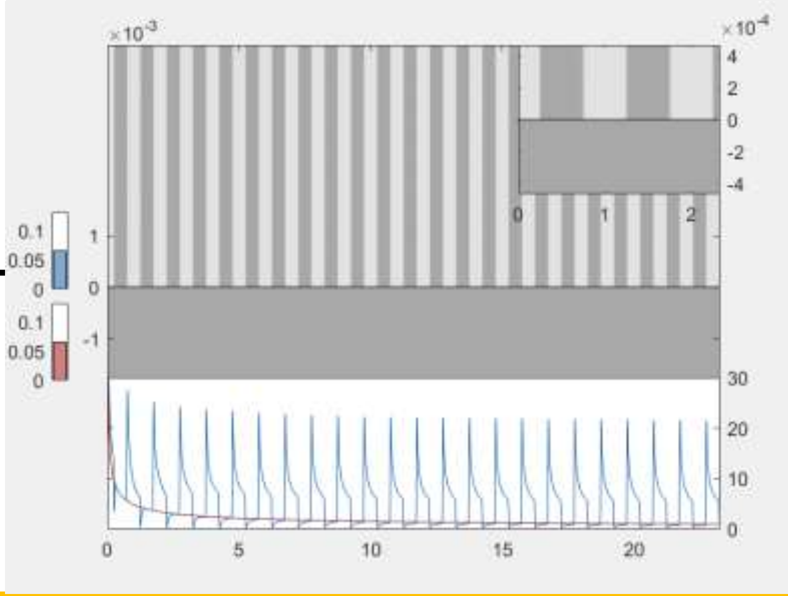
Oscillating injection rate

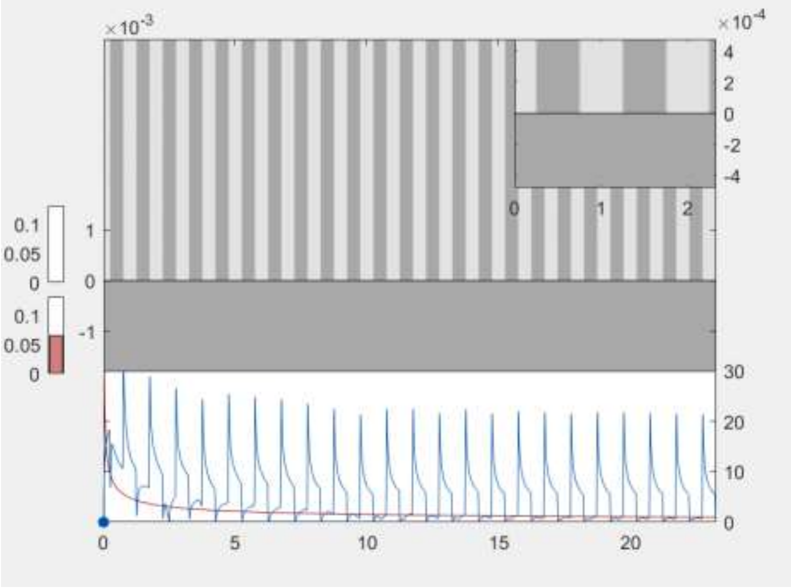
Sinusoidal





Step – wise





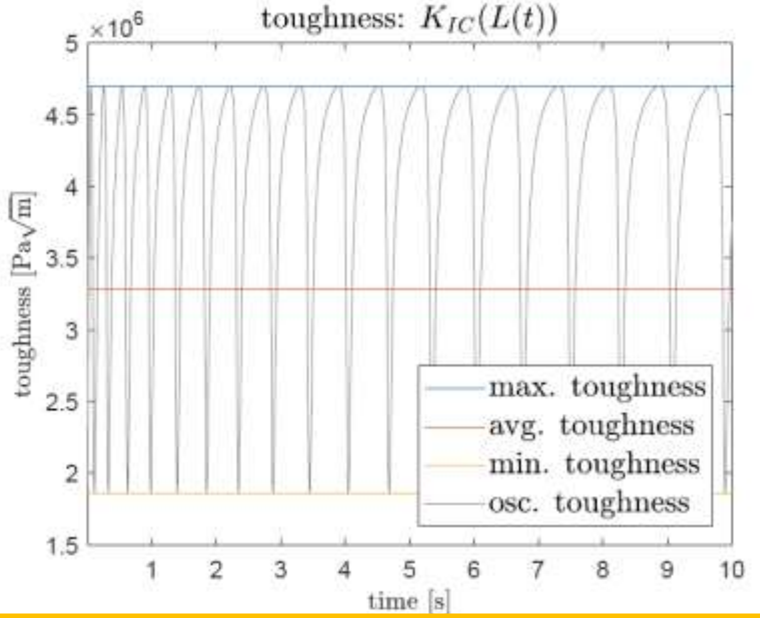
Questions raised

- ***Even though homogenesation is not applicable notion within LEFM, MTC (Maximum Toughness Criterion, Dontsov et.al. 2021) is a good approximation for the process modelling for large time (regardless of the regime(s) for different reasons though)***
- ***What is the reason behind that MTC miracle?***
- ***Can one improve MTC strategy for small and moderate time?***

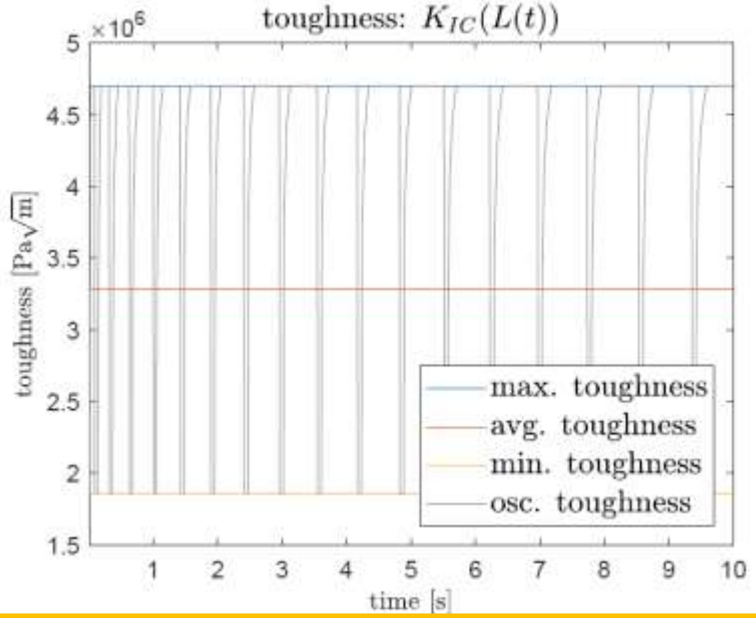
Helpful tip? : How the crack tip "feels" the toughness?

Let's "sit" at the crack tip and observe things around us [IN TIME]

Sinusoidal



Step-wise



Conjecture:

Averaging or approximation (but not a homogenisation!) <u>should be performed in time</u> NOT in space?

$$\langle K_{IC}\rangle_1(t) = \frac{1}{t}\int_0^t K_{IC}(L(\xi))d\xi.$$

$$\langle K_{IC}\rangle_2(t) = \frac{1}{\Delta t}\int_t^{t+\Delta t} K_{IC}(L(\xi))d\xi,$$

They are both process dependent parameters and NOT a material property only!!!

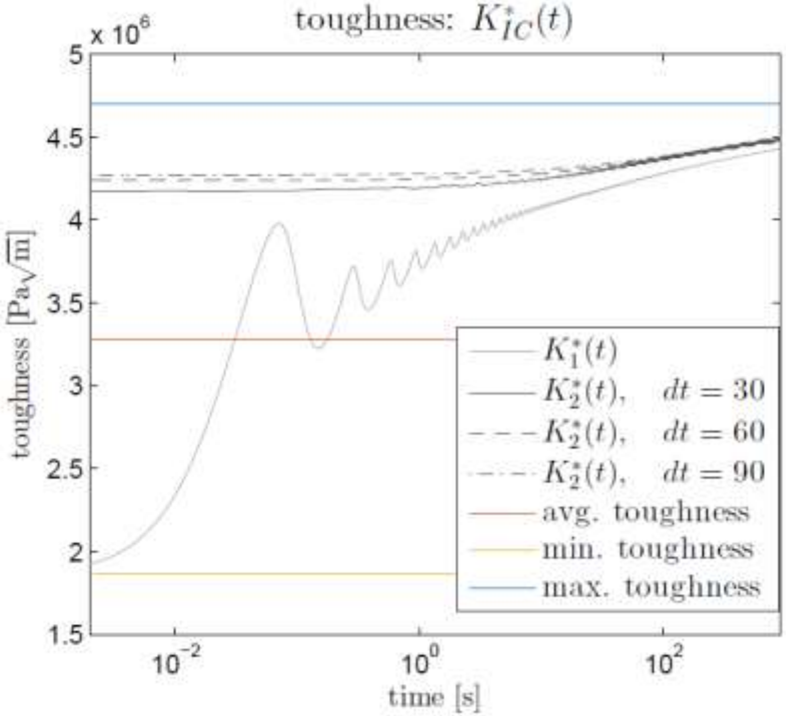
Let's check this conjecture?

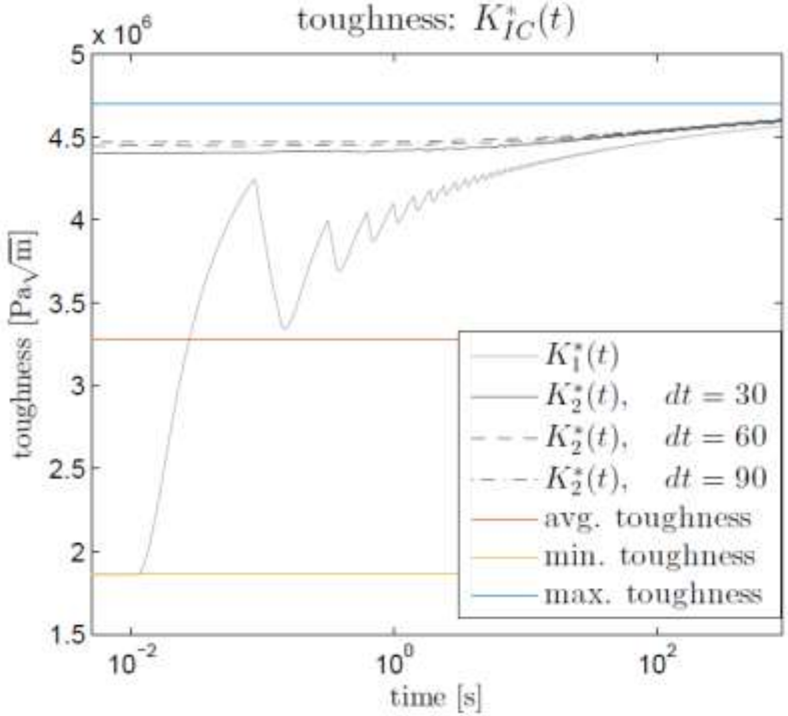
We have results of those computations…

Proposed averaging strategies (in time)

Sinusoidal | Step-wise

Regime indicator (max and min): $\delta_{max} = 10$ and $\delta_{min} = 1$





Both averaging are promising (tend to the maximum toughness) BUT…

- ***Moving average does not have a clear time frame (what to do with it?)***
- ***Which of those averages is better for predictions?***

Equivalent Conjecture:

Averaging (or approximation) <u>can be performed in space</u>

<u>but should be weighed by the reciprocal crack speed:</u>

$$K_1^*(L) = \left(\int_0^L \frac{dx}{v(x)} \right)^{-1} \int_0^L K_{IC}(x) \, \frac{dx}{v(x)},$$

$$K_2^*(L, dL) = \left(\int_L^{L+dL} \frac{dx}{v(x)} \right)^{-1} \int_L^{L+dL} K_{IC}(x) \, \frac{dx}{v(x)}.$$

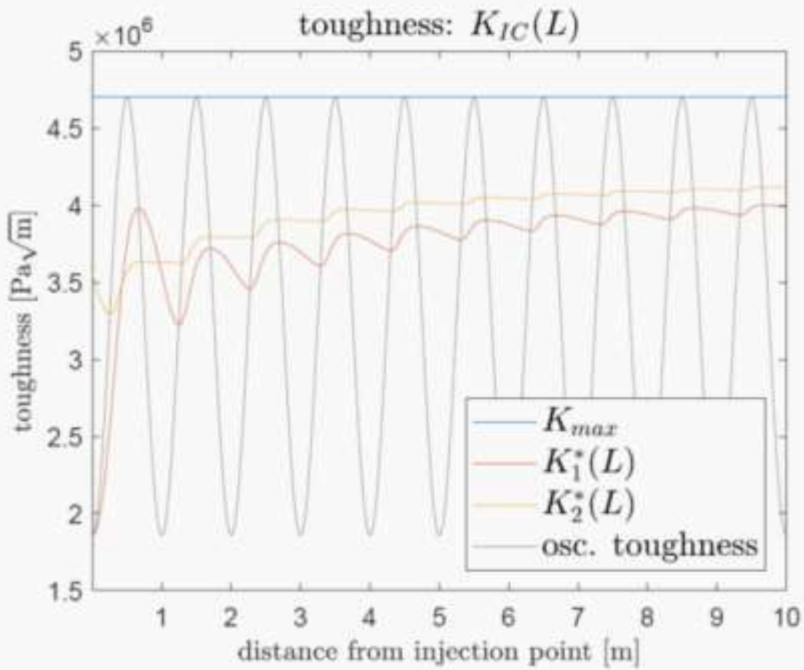
Still, the values are clearly process dependent !!!

<u>Good news</u>: natural moving frame can be now inked to the period!

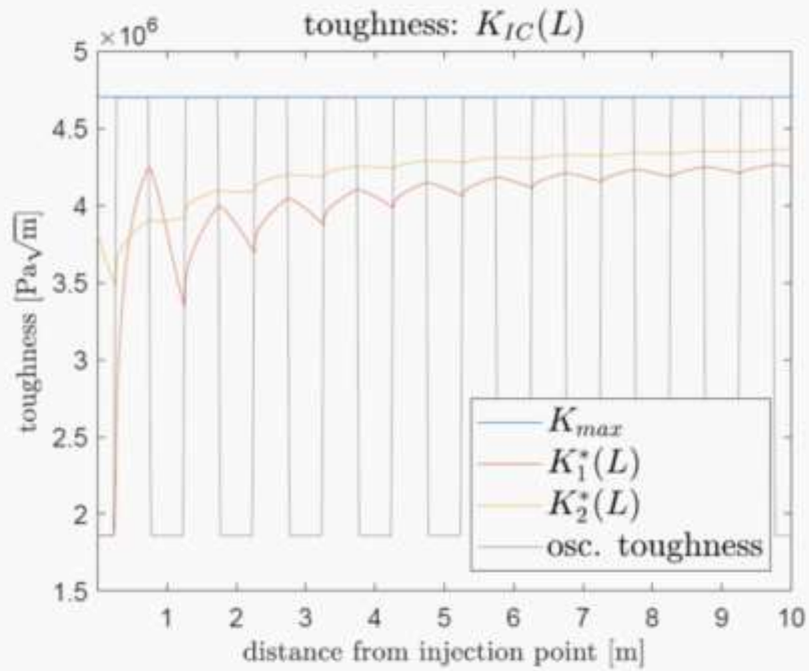
Proposed averaging strategies (in space)
Initial part of the crack propagation path

Regime indicator (max and min): $\delta_{max} = 10 \text{ and } \delta_{min} = 1$

Sinusoidal



Step-wise

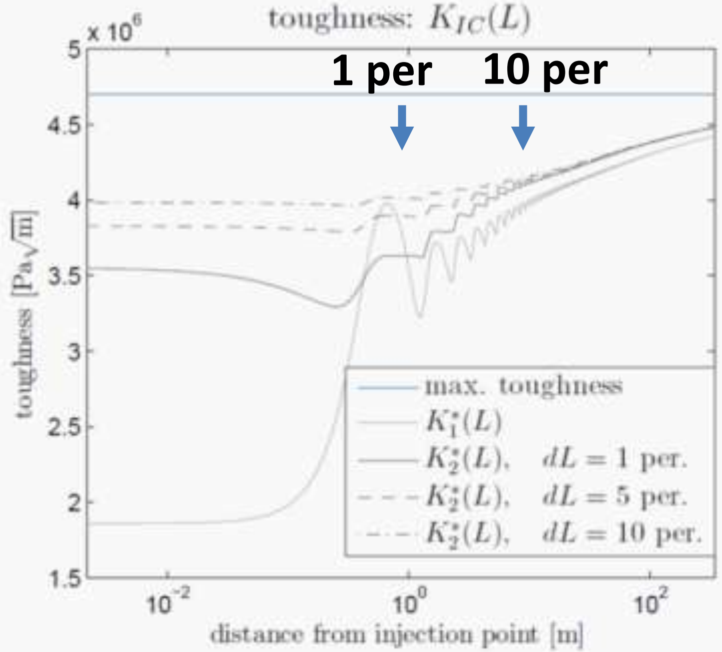


How do they behave for long crack (converge to the maximum toughness)?

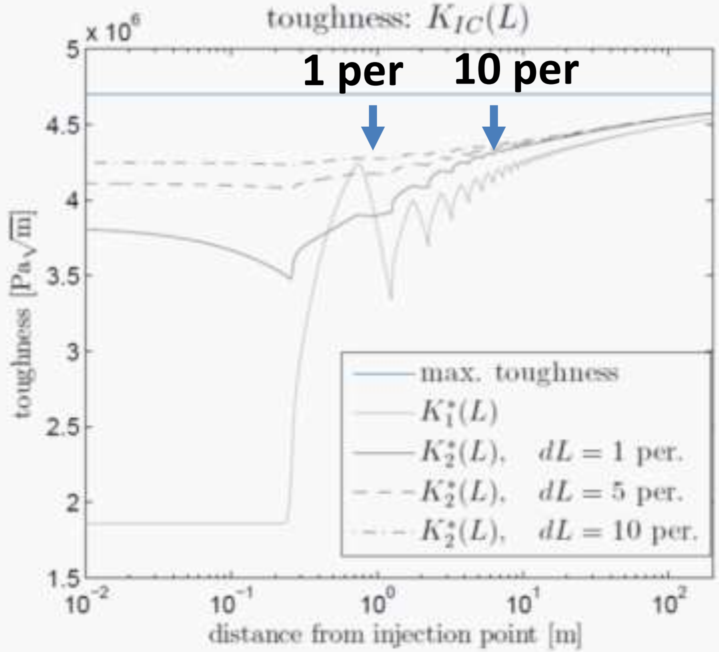
Proposed averaging strategies (in space)

Regime indicator (max and min): $\delta_{max} = 10 \text{ and } \delta_{min} = 1$

Sinusoidal



Step-wise



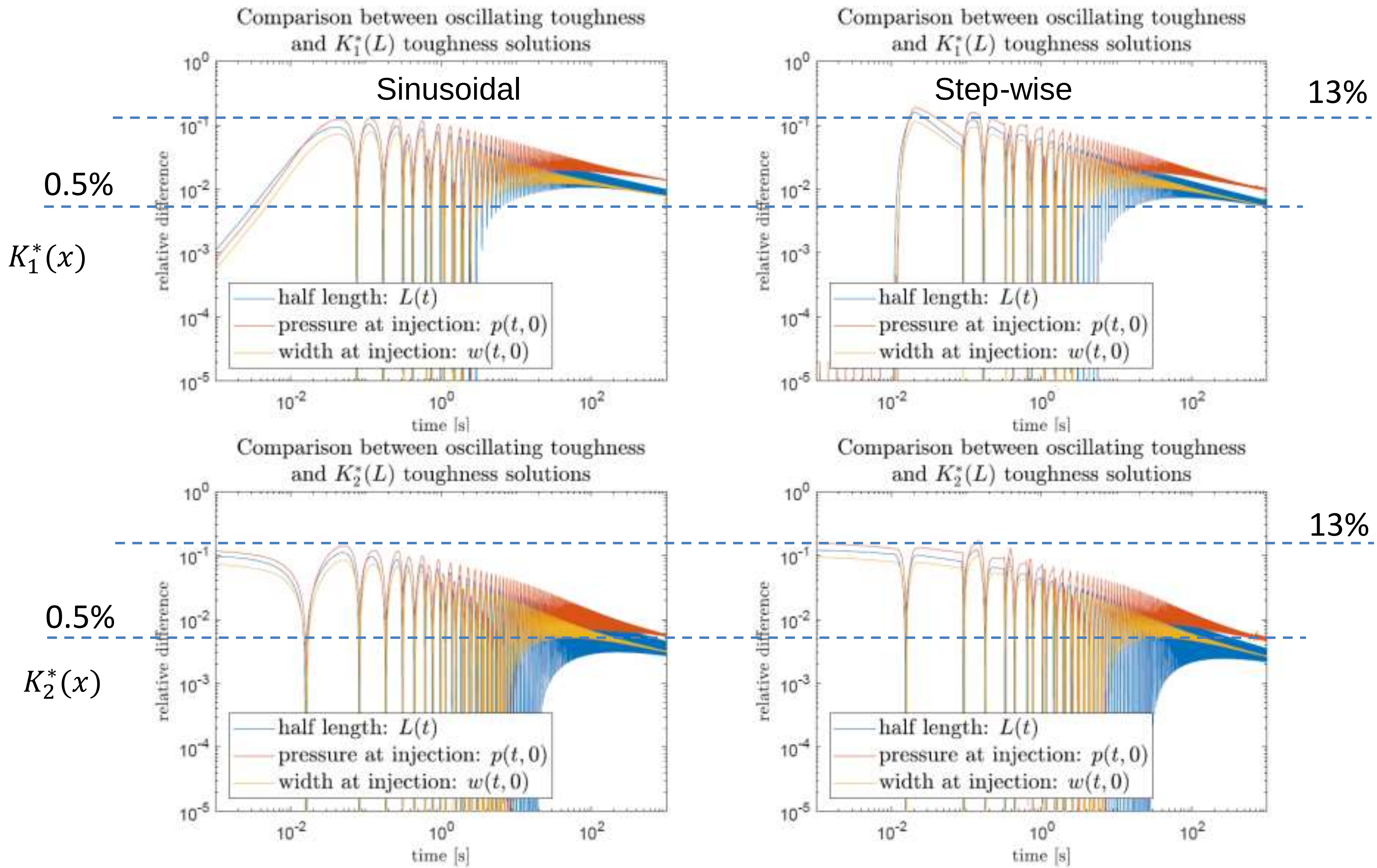
Both averages are promising (converge to the maximum toughness)!

<u>Do those averages produce better results than MTC?</u>

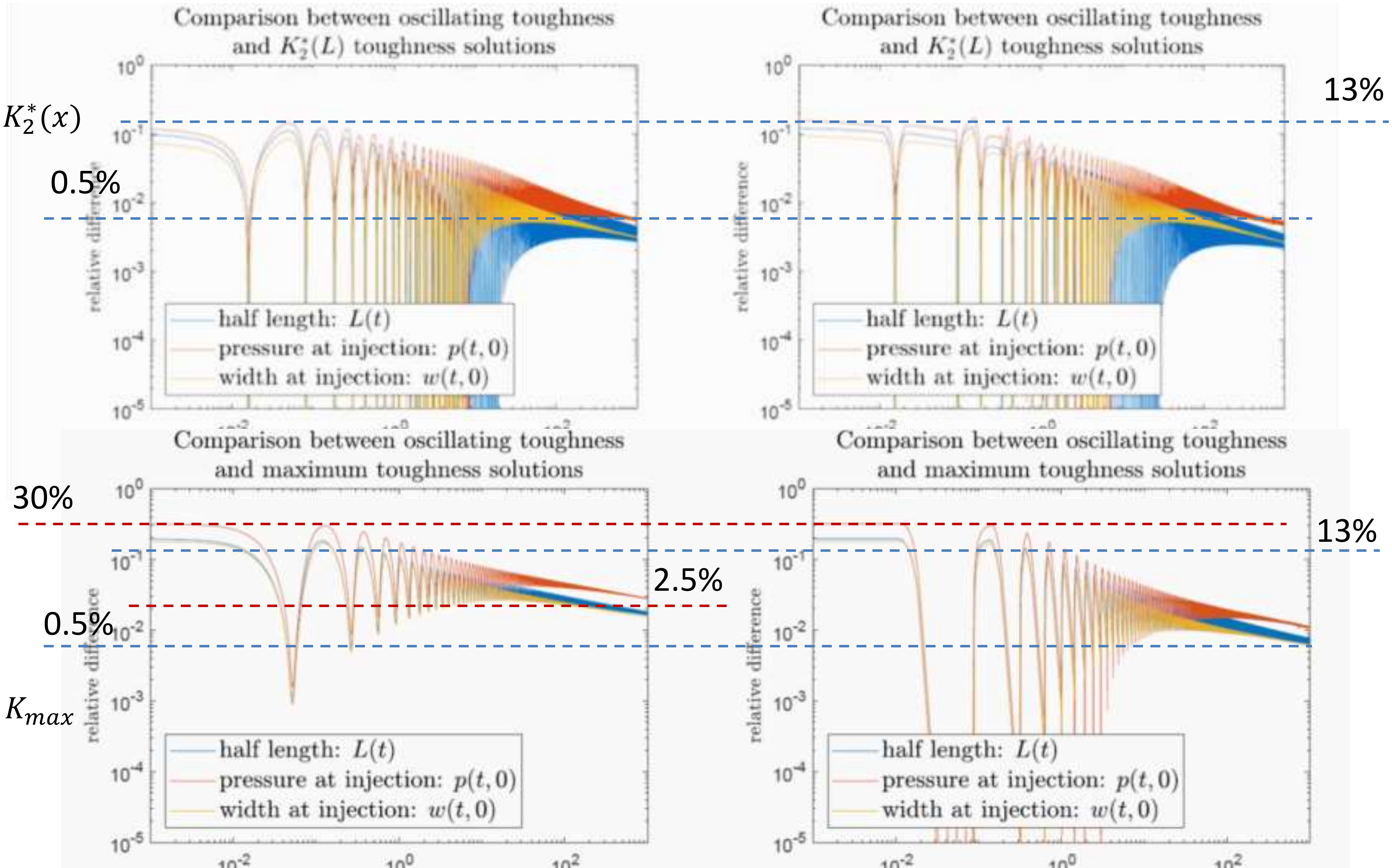
We perform new computations with those average toughness…

Do those averages produce “good” results?

Answer – YES! $\delta_{max} = 10$ and $\delta_{min} = 1$



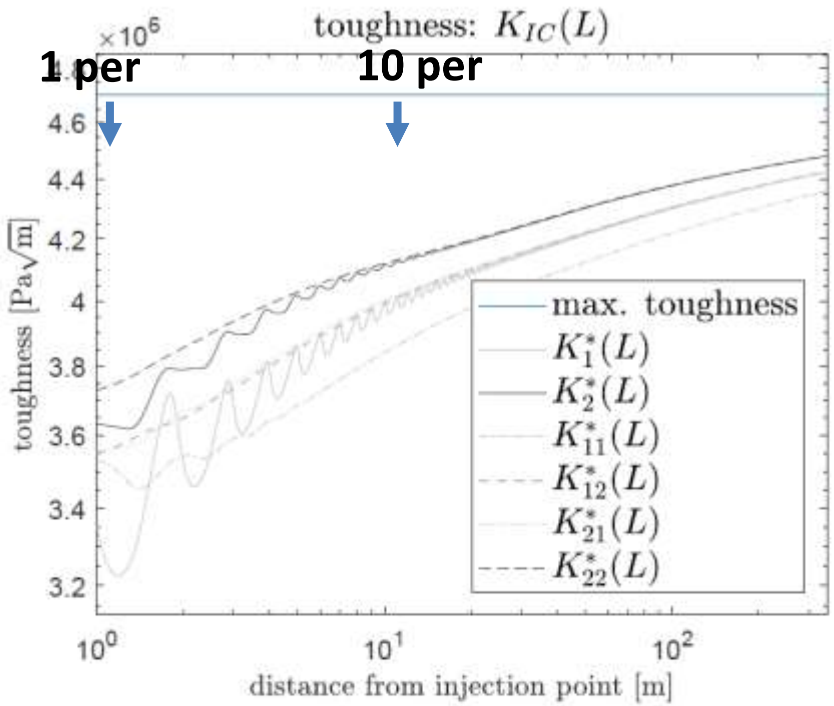
Do those averages produce better results?

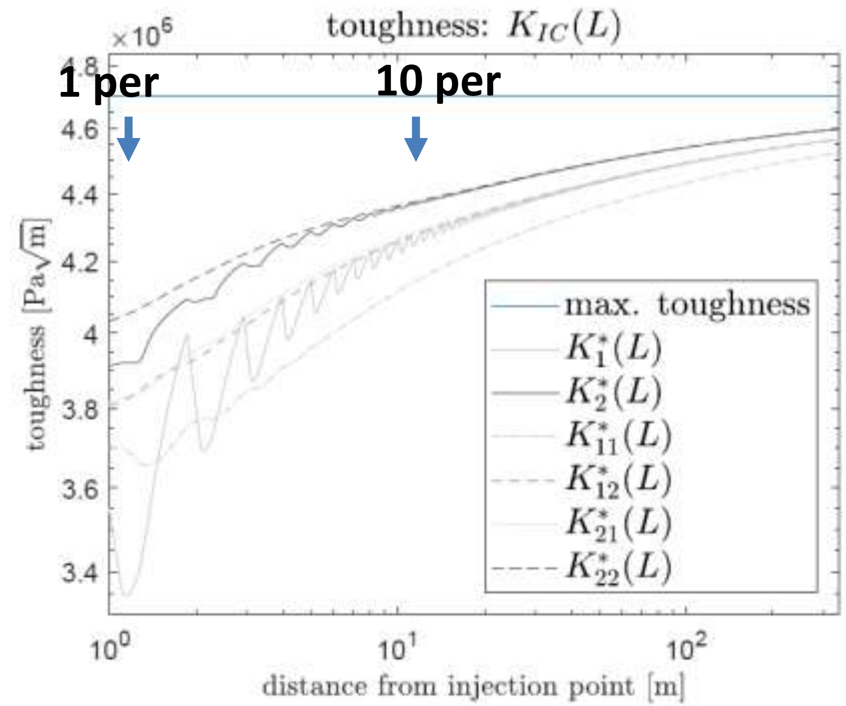


"Consistency-check 1": what happens with the "procedure iterations"?

$$K_{IC}(x) \Rightarrow w, p(x,t), v(x) \rightsquigarrow K_j^*(x) \Rightarrow w_j^* p_j^*(x,t), v_j^*(x) \rightsquigarrow K_{jk}^*(x) \Rightarrow \ldots.$$

Sinusoidal $\delta_{max} = 10$ and $\delta_{min} = 1$ Step-wise





Second type averaging is consistent (mainly reproduces itself after the iteration)

“Consistency-check 2”: Change in the layer positions

Another averaging basis (fracture energy)

$$K_3^*(L,dL) = \sqrt{\left(\int_L^{L+dL} \frac{dx}{v(x)}\right)^{-1} \int_L^{L+dL} K_{IC}^2(x)\frac{dx}{v(x)}}$$

$$K_2^*(L,dL) = \left(\int_L^{L+dL} \frac{dx}{v(x)}\right)^{-1} \int_L^{L+dL} K_{IC}(x)\frac{dx}{v(x)}$$

Energy arguments do work even “globally” better then the local one (even if the latter is identical to the former locally)

Some results have been reported

[1] Peck, D., Da Fies, G., Dutko, M., Mishuris, G. (2022). Periodic toughness distribution - Can an effective/average toughness concept be feasible? Case study: KGD fracture in an impermeable rock. *Mathematics and Mechanics of Solids* (submitted). https://arxiv.org/abs/2203.11985

[2] Da Fies, G., Dutko, M., Mishuris, G. (2021) Remarks on Dealing With Toughness Heterogeneity in Modelling of Hydraulic Fracture. 55th U.S. Rock Mechanics/Geomechanics Symposium, ARMA-2021-2010

Intermediate Conclusions

- *Both proposed averaging strategies are more accurate than the MTC prediction for the entire process time*
- *Progressive averaging gives a few orders better accuracy at small time than the moving average (for all process parameters)*

and vice versa

- *Moving average produces one order better prediction for a long crack than the progressive average (for all process parameters)*
- ***The question remains unanswered**: <u>How to deliver those averages without performing preliminary HF simulations? (parameters are process dependent!)</u>*
- *Not to forget: MTC (Dontsov, et al) is so simple that it can be recommended for utilisation for any long time prediction.*

Final Conclusions

(Take-away message from this talk)

- Hydraulic Fracture as a part of the Fracture Mechanics has influenced back to the fundamental elements of the FM.
- HF has inspired us to discover the fourth SIF (related to the action of the shear traction induced by the fluid on the crack surfaces)
- This, in turn, has allowed to determine complete (general) ERR formulation and the related Irwin's crack closure integral form
- Some kind of toughness averaging is indeed possible, but ...
  It is process dependent parameter!
- There are a few candidates for this measure, but it is difficult to imagine building a consistent theory like those in classic homogenisation.
- For practical applications, providing realistic (accepted by practitioners) measure can be delivered numerically/empirically (MTC the first from them)
- Further analysis is still required

Thank you all for listening

Profs. Stanislaw Stupkiewicz & Bogdan Kazmierczak for the invitation.

Special thanks to all my Colleagues,
to The Royal Society (WRMA) and to EU (current *EffectFact* project)
&
last but not least - to the *Welsh Government*
for the Ser Cymru Future Generations Industrial Fellowship with Rockfield that allows us continuing research in this (HF) direction

Questions?

Llywodraeth Cymru
Welsh Government

Rockfield

UNDEB EWROPEAIDD
EUROPEAN UNION
Llywodraeth Cymru
Welsh Government
Cronfa Datblygu Rhanbarthol Ewrop
European Regional Development Fund