Hydrodynamic friction of a polymer adsorbed on a planar surface

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Outline

- Preliminary information
- Model polymer
- Multipole method
- Force induced on polymer
- Long polymers
- Force averaged over polymer configurations
- Final remarks and summary

Preliminary Information

Significance/Processes:

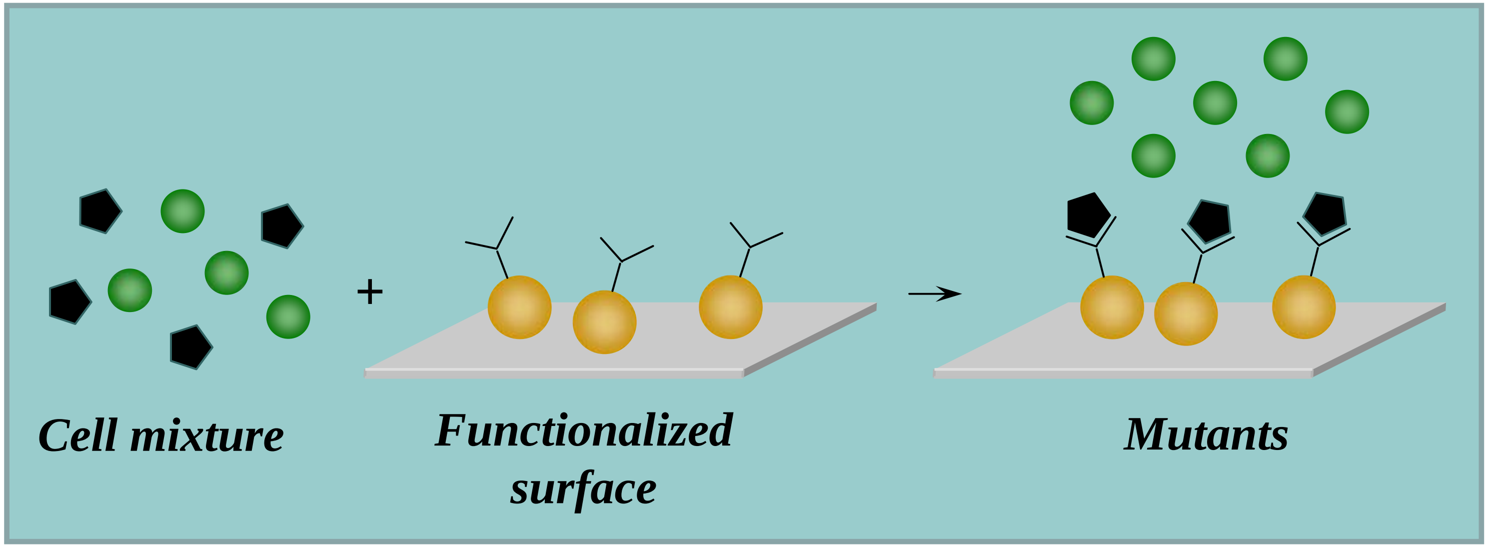
- biosensors
- separation of DNA, proteins, viruses, cells
- immunological assays
- filtration (water treatment)

Particles:

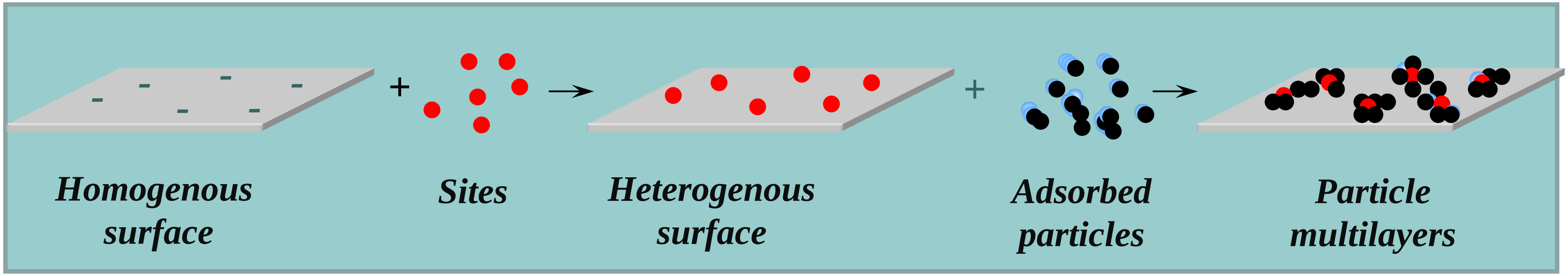
- DNA, proteins, viruses, cells
- polyelectrolytes
- Colloids, polymers

Significance

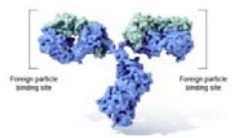
Separation of proteins, cells:



Formation of particle multilayers of desired architecture:

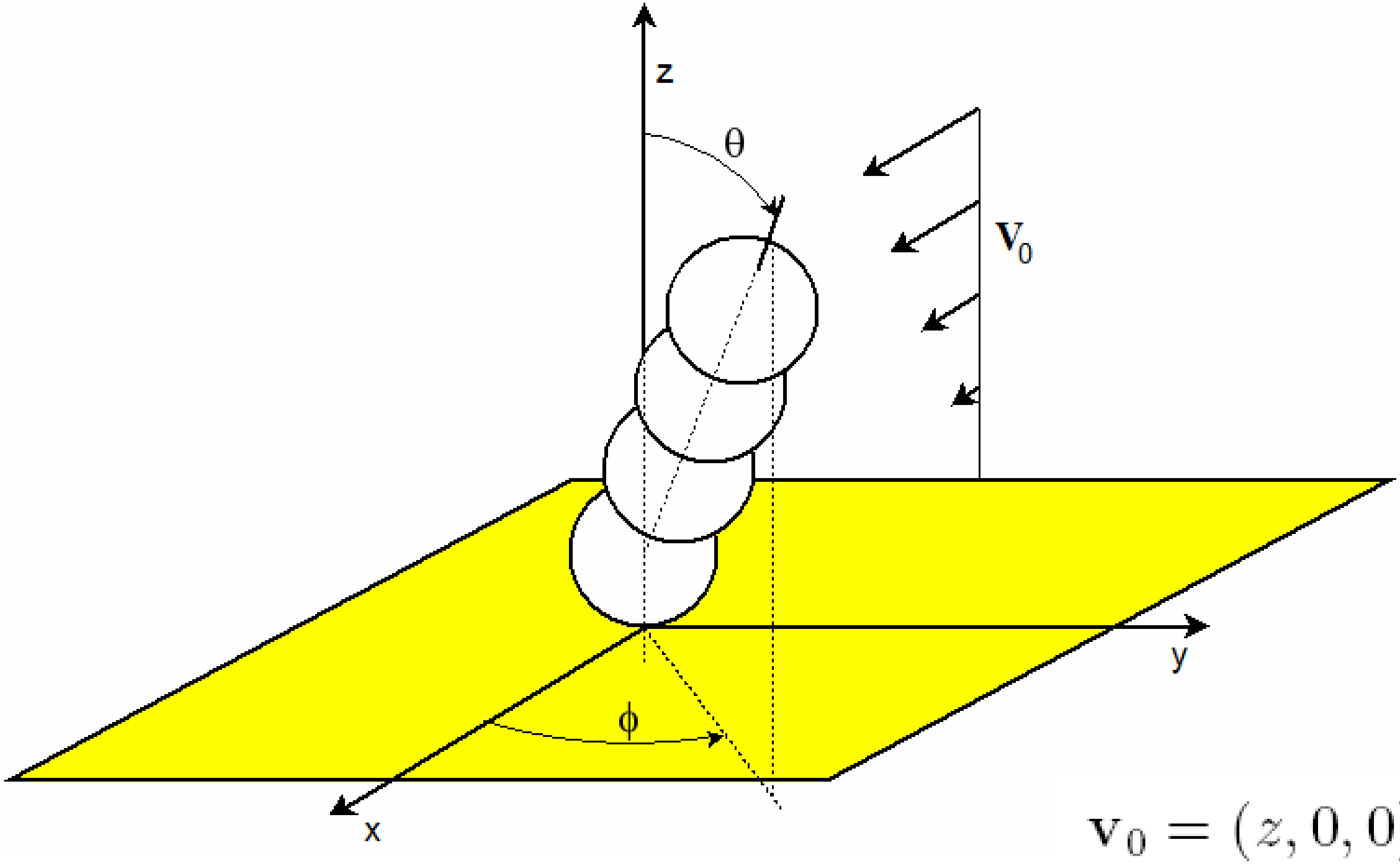


Particle Size

Particle	Effective size
	Lizosym (M_w=14.000), 4x3x3 nm
	BSA (M_w=67.000), 14x4x4 nm
	IgG (M_w=170.000), 24x4.4x4.4 nm
	Fibrynogen (M_w=420.000), 45x5x5 nm
	Colloidal particle (latex polystyrene d= 40 nm)

0 10 20 30 40 50 nm

Model System



$$\mathbf{v}_0 = (z, 0, 0).$$

Multipole method

Stokes equation:

$$\eta\nabla^2\mathbf{v} - \nabla p = 0,$$
$$\nabla\cdot\mathbf{v} = 0,$$

Boundary conditions:

$$\mathbf{v}(\boldsymbol{r}) = \mathbf{v}_0(\boldsymbol{r}) \quad \text{for} \quad r\to\infty,$$
$$0 = \mathbf{v}(\boldsymbol{r}) = \boldsymbol{u}_i(\boldsymbol{r}) \equiv \boldsymbol{U}_0 + \boldsymbol{\Omega}\times\boldsymbol{r} \quad \text{for} \quad \boldsymbol{r}\in S.$$

+ stick boundary conditions on walls

Integral form:

$$\mathbf{v}(\boldsymbol{r}) - \mathbf{v}_0(\boldsymbol{r}) = \sum_j \oint d\boldsymbol{r}'\boldsymbol{T}(\boldsymbol{r}-\boldsymbol{r}')\cdot\mathrm{f}_j(\boldsymbol{r}').$$

$$[-\mathbf{v}_0(\boldsymbol{r})]_{\boldsymbol{r}\in S_i} = \sum_j \int d\boldsymbol{r}'\boldsymbol{T}(\boldsymbol{r},\boldsymbol{r}')\mathrm{f}_j(\boldsymbol{r}'), \qquad i=1,\ldots,N.$$

Blake tensor:

Induced force densities

… Multipole method

$$\begin{pmatrix} \mathcal{F} \\ \mathcal{T} \end{pmatrix} = \begin{pmatrix} \zeta^{tt} & \zeta^{tr} & \zeta^{td} \\ \zeta^{rt} & \zeta^{rr} & \zeta^{rd} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{v}_0 \\ \boldsymbol{\omega}_0 \\ \boldsymbol{g}_0 \end{pmatrix},$$

Friction matrix

$$\mathcal{F} = \begin{pmatrix} \boldsymbol{F}_1 \\ \vdots \\ \boldsymbol{F}_N \end{pmatrix}, \qquad \mathcal{T} = \begin{pmatrix} \boldsymbol{T}_1 \\ \vdots \\ \boldsymbol{T}_N \end{pmatrix}.$$

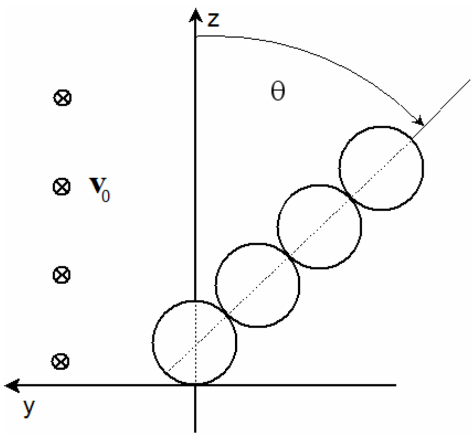
$$\mathbf{v}_{0i} = \mathbf{v}_0(\boldsymbol{R}_i),$$

$$\boldsymbol{\omega}_{0i} = \frac{1}{2}\, \boldsymbol{\nabla} \times \mathbf{v}_0(\boldsymbol{r})\big|_{\boldsymbol{r}=\boldsymbol{R}_i},$$

$$g_{0i,\alpha\beta} = \frac{1}{2}\left[\nabla_\alpha \mathrm{v}_{0\beta}(\boldsymbol{r}) + \nabla_\beta \mathrm{v}_{0\alpha}(\boldsymbol{r})\right]_{\boldsymbol{r}=\boldsymbol{R}_i},$$

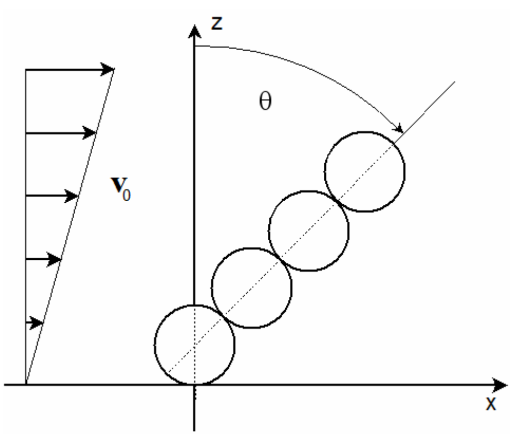
$$\mathbf{v}_0 = \begin{pmatrix} \mathbf{v}_{01} \\ \vdots \\ \mathbf{v}_{0N} \end{pmatrix}, \quad \boldsymbol{\omega}_0 = \begin{pmatrix} \boldsymbol{\omega}_{01} \\ \vdots \\ \boldsymbol{\omega}_{0N} \end{pmatrix}, \quad \boldsymbol{g}_0 = \begin{pmatrix} \boldsymbol{g}_{01} \\ \vdots \\ \boldsymbol{g}_{0N} \end{pmatrix}.$$

Force induced on polymer



$$\boldsymbol{F} = \begin{pmatrix} \alpha(\theta, N) \\ 0 \\ 0 \end{pmatrix}.$$

$$\boldsymbol{F}(\theta, \phi) = \begin{pmatrix} \alpha(\theta)\sin^2\phi + \beta(\theta)\cos^2\phi \\ (\beta(\theta) - \alpha(\theta))\sin\phi\cos\phi \\ \gamma(\theta)\cos\phi \end{pmatrix}.$$

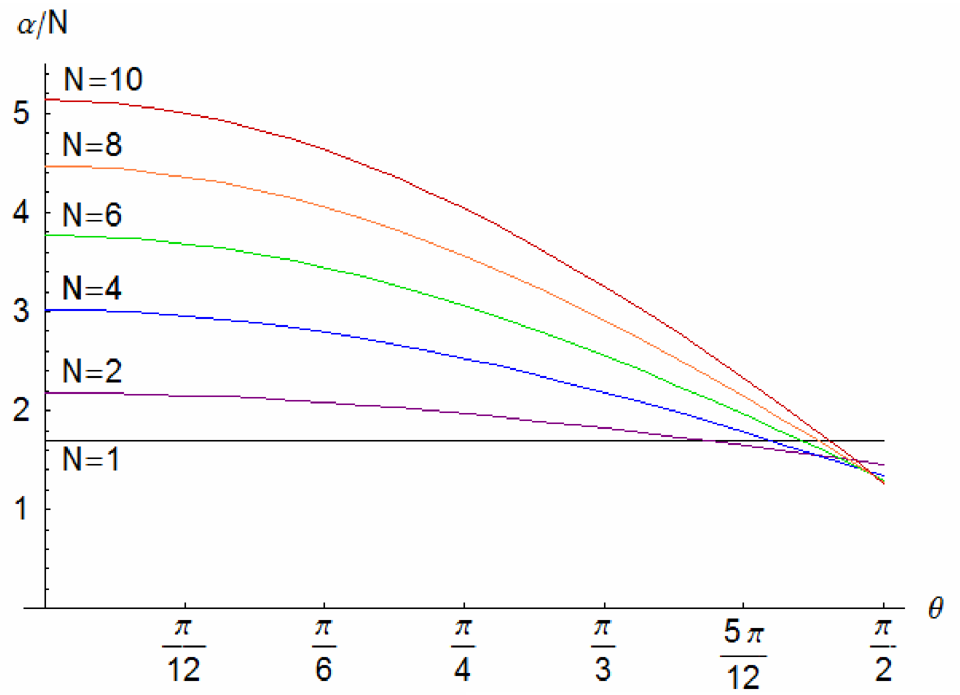


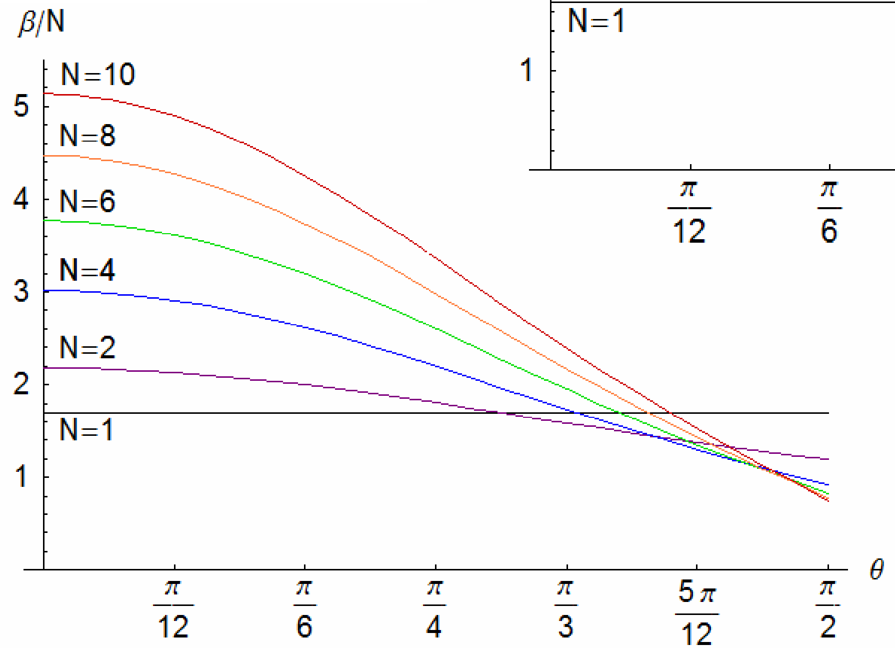
$$\boldsymbol{F} = \begin{pmatrix} \beta(\theta, N) \\ 0 \\ \gamma(\theta, N) \end{pmatrix},$$

… Force induced on polymer

Normalization:

$$6\pi\eta a^2\dot{\gamma}$$





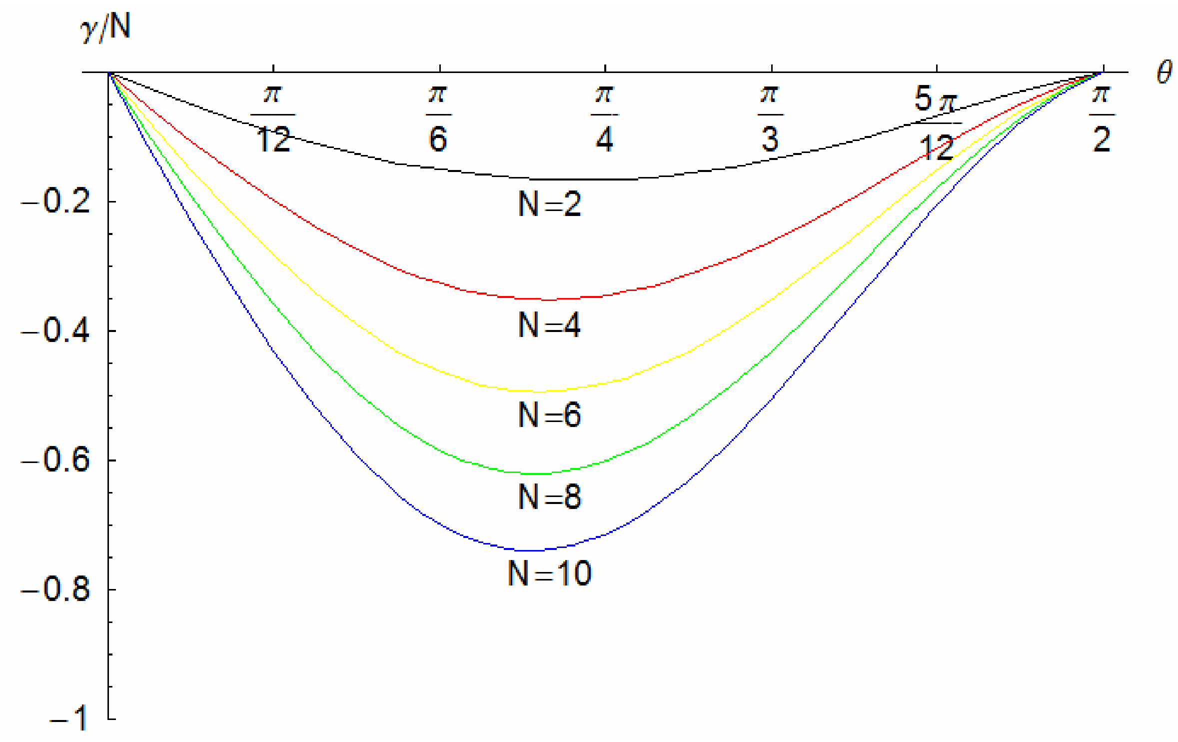
Total friction force acting on polymer N=2:

$$2.39 \leq F(\phi = 0) \leq 4.36$$

$$2.92 \leq F(\phi = \pi/2) \leq 4.36$$

Single sphere $F = 1.7009$

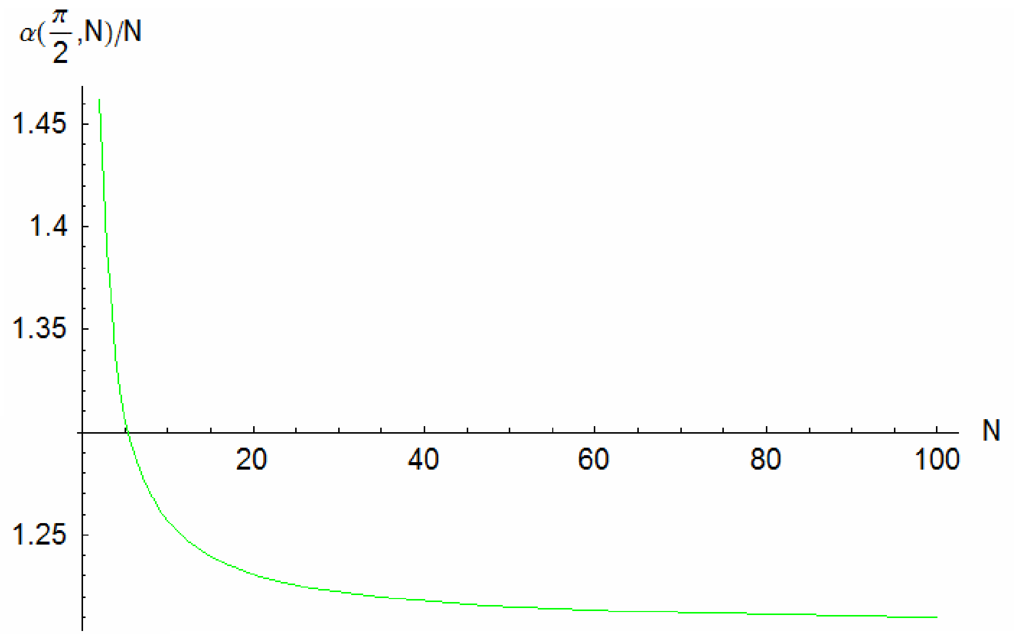
… Force induced on polymer

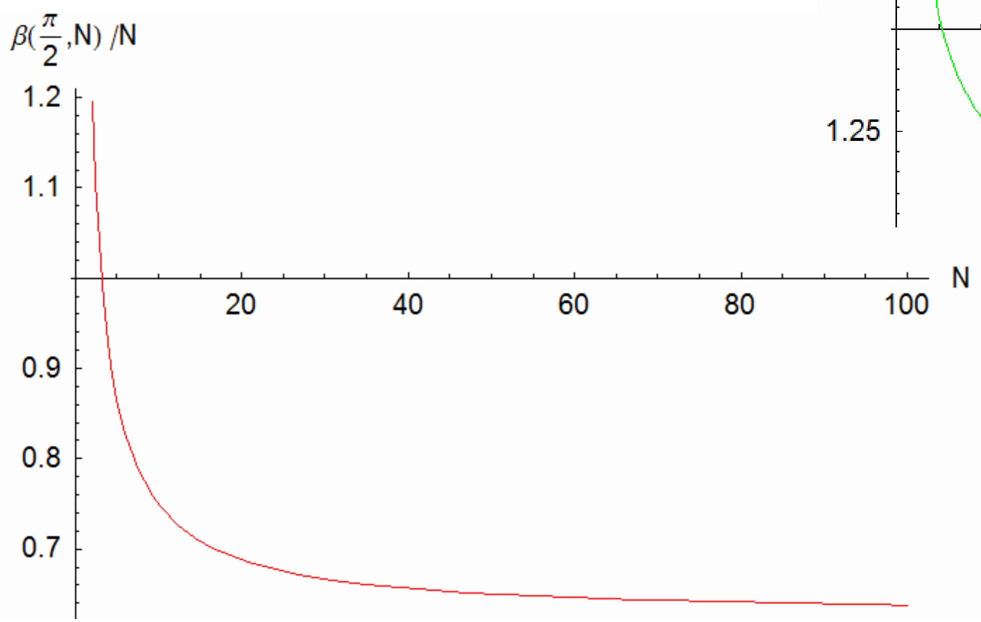


Normalization:

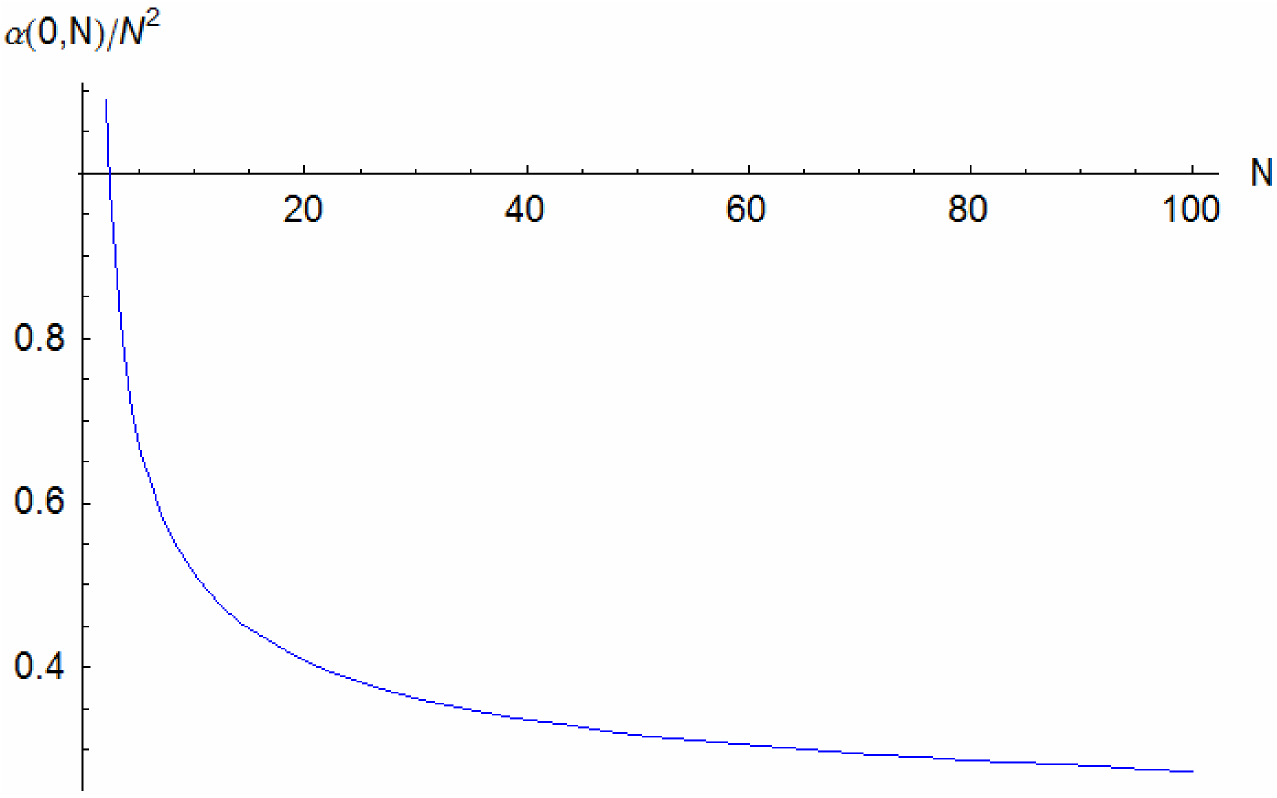
$$6\pi\eta a^2\dot{\gamma}$$

Long polymers

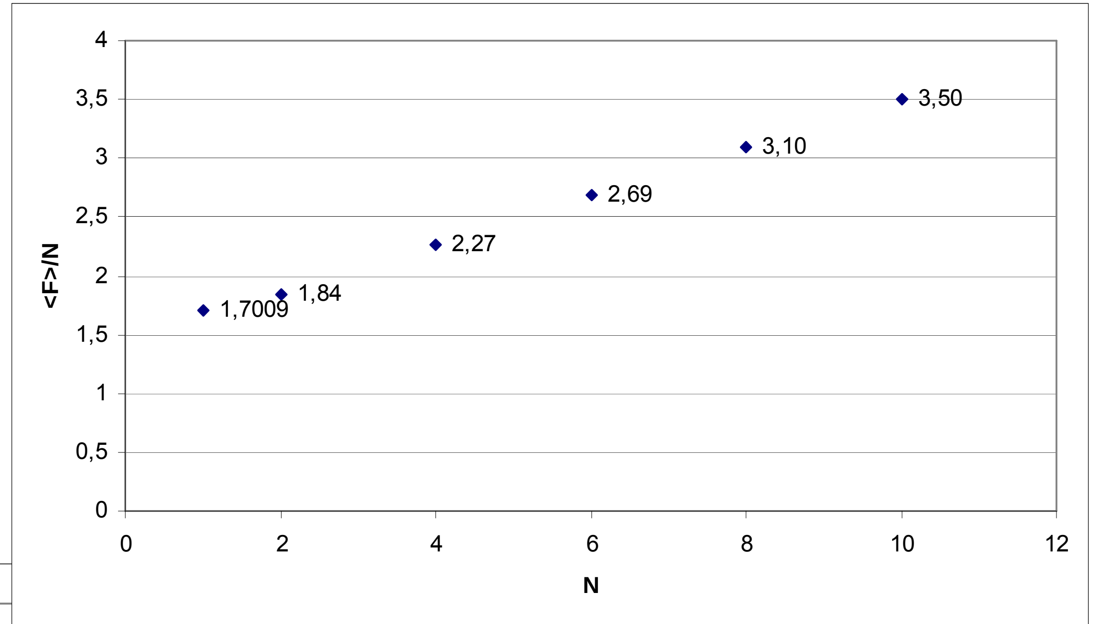


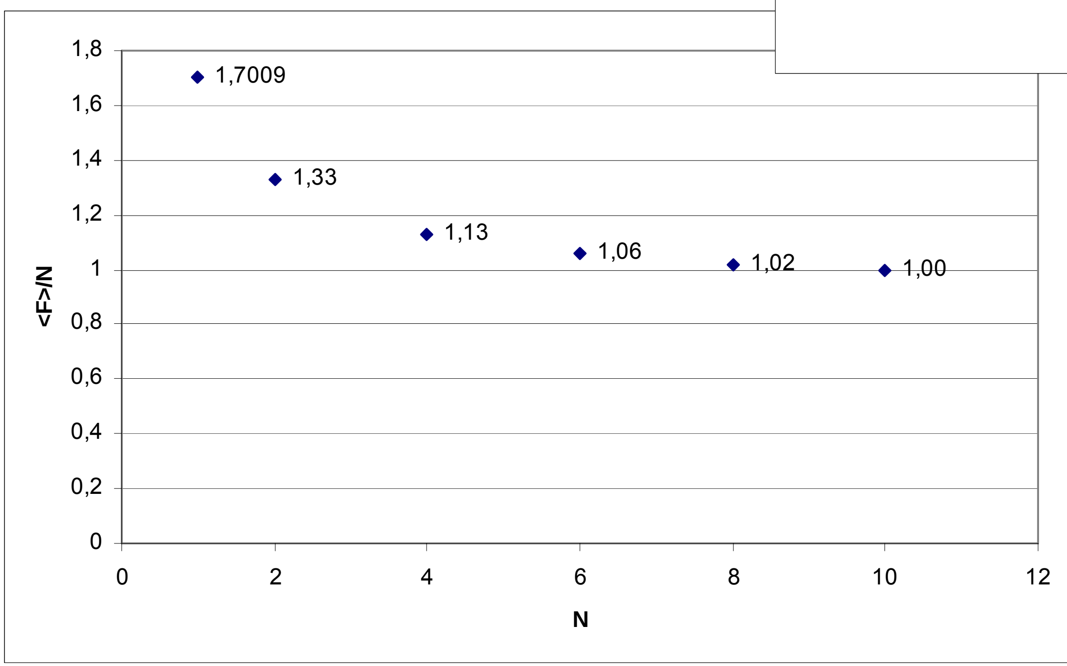


…Long polymers



Average force acting on a polymer in arbitrary configuration





Average force acting on a polymer parallel to the wall

Final remarks and summary

- Averaged total force exerted on the particles → ς-potential / streaming potential
- Results: friction force exerted by the fluid on a polymer of arbitrary length in arbitrary configuration.
- Wall effects important