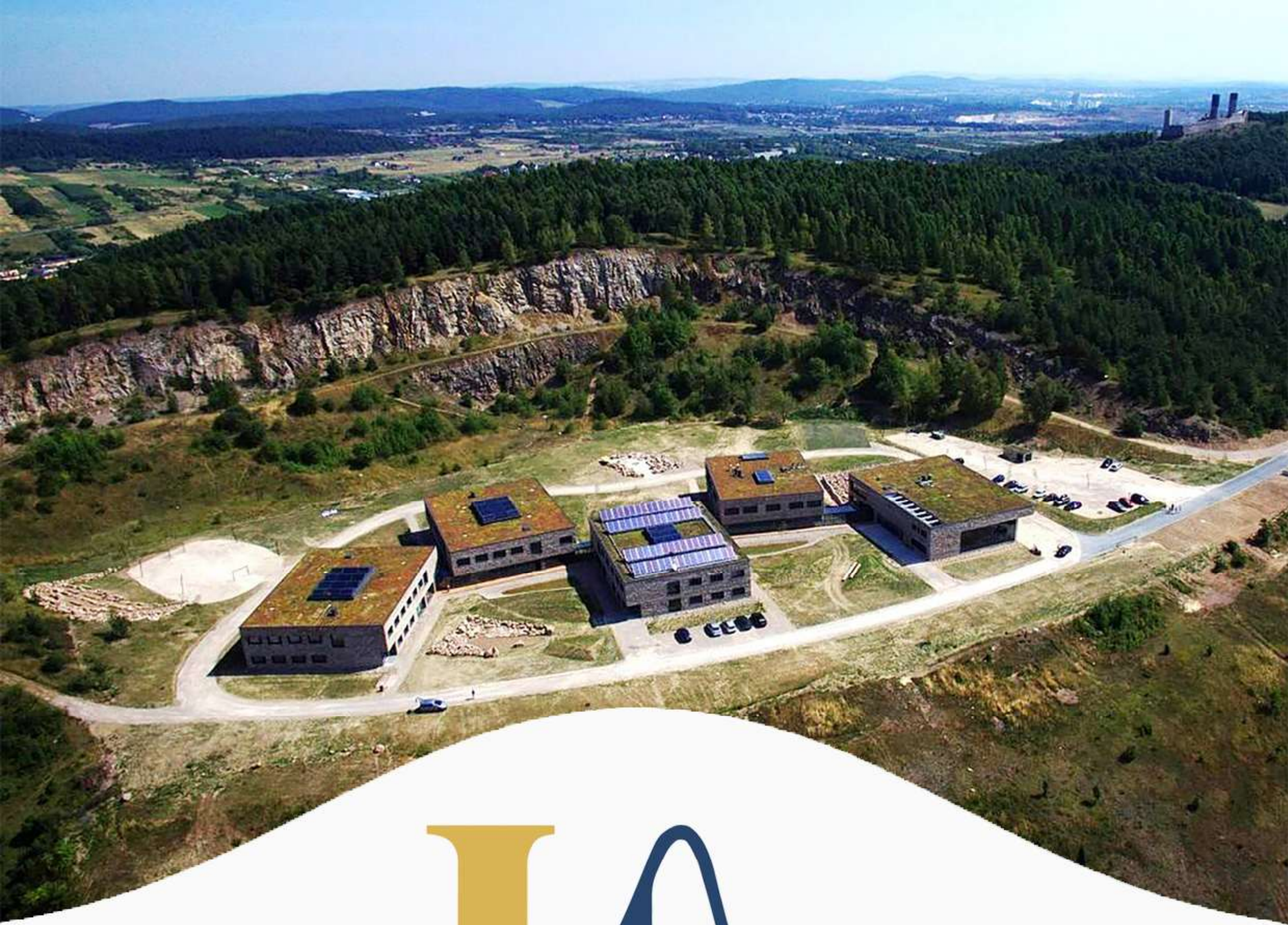

# L Konferencja

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# Deformed sphere as a new reference model for the geoid

**Wasyl Kowalczuk**

Department for Theory of Continuous Media and Nanostructures
Institute of Fundamental Technological Research, Polish Academy of Sciences

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In our research we concentrate on some applications of differential geometry to geodesy and cartography. In particular, we are interested in development of a new reference model for the geoid (originally proposed by Faridi and Schucking back in 1987 in their paper [1]) that turns out to be a valid alternative to the classical ellipsoidal reference model (used, e.g., in World Geodetic System 1984). The considered new reference model is based on the concept of deformed spheres with the parameter of deformation $\lambda < 1/3$ (where $\lambda = 0$ corresponds to the usual, non-deformed sphere).

In our paper [2] we have compared the main geometric characteristics of the deformed spheres and the standard spheroids (ellipsoids of revolution), such as semi-major (equatorial) axis $a$ and semi-minor (polar) axis $b$, quarter-meridian length $m_P$, surface area $S$, volume $V$, sphericity index $\Psi$, and tipping (bifurcation) point $T^{1/2}$ for geodesics. Using the minimization process of the RMS error for individual pairs of the discussed geometric characteristics, we obtain a proposition of the optimized value of the deformation parameter $\lambda_{\mathrm{RMS}} \approx 0.003\ 349\ 672\ \ldots$ for the deformed sphere's reference model of the geoid.

Quite interestingly, it turns out that the value of the standard spheroid's (Earth's) flattening factor $f = 1 - b/a \approx 0.003\ 352\ 811\ \ldots$ is quite a good approximation for the obtained optimized value of the deformation parameter $\lambda_{\mathrm{RMS}}$ with the relative error $\delta f = (f - \lambda_{\mathrm{RMS}})/f \approx 9.362 \ldots \times 10^{-4}$.

In papers [3, 4] we have also found the explicit solutions of the direct and inverse problems for loxodromes (rhumb lines) and orthodromes (geodesics) for navigation purposes on the deformed spheres and compared the obtained results for the selected important air routes on the Earth's surface with the corresponding results obtained from the standard ellipsoidal reference model WGS 84. It has turned out that there is a good agreement in the predictions of both reference models wit relative errors of the order $10^{-6}$–$10^{-7}$ in distances and reversed azimuths and of the order $10^{-5}$–$10^{-7}$ in azimuths.

## References

[1] Faridi A., Schucking E., *Geodesics and deformed spheres*, Proc. Am. Math. Soc., 100 (1987), 522–525.

[2] Kowalczuk W., Mladenov I., *Comparison of main geometric characteristics of deformed sphere and standard spheroid*, Bulletin of the Polish Academy of Sciences: Technical Sciences, 71 (2023), e147058, 1–9.

[3] Kowalczuk W., Mladenov I., *Explicit solutions for geodetic problems on the deformed sphere as a reference model for the geoid*, Geom. Integr. Quant., 25 (2023), 73–94.

[4] Kowalczuk W., Mladenov I., *$\lambda$-spheres as a new reference model for geoid: explicit solutions of the direct and inverse problems for loxodromes (rhumb lines)*, Mathematics, 10 (2022), 3356-1–10.