Reliability based limit design of steel frames with limited residual strain energy capacity

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The aim of this paper is to take into consideration the influence of the limited load carrying capacity of the connections on the plastic limit state of elasto-plastic steel (or composite) framed structures under multi-parameter stochastic loading and probabilistically given conditions. In addition to the plastic limit design to control the plastic behaviour of the structure, bound on the complementary strain energy of the residual forces is also applied. This bound has significant effect for the load parameter. If the design uncertainties (manufacturing, strength, geometrical) are expressed by the calculation of the complementary strain energy of the residual forces a reliability based extended limit design problem is formed. The formulations of the problems yield to nonlinear mathematical programming which are solved by the use of sequential quadratic algorithm. The bi-level optimization procedure governed by the reliability index calculation.

1 Introduction

A general approach is presented for the reliability-based analysis and the optimum designs of steel frames under multi-parameter stochastic loading and probabilistically given residual energy conditions with take into consideration the influence of the limited load carrinng capacity of the connections. In addition to the limit analysis and design to control the plastic behaviour of the structure, bound on the complementary strain energy of the residual forces is also applied. This bound has significant effect for the load parameter [2].

2 Notations

In the extended limit design the following notations are used: $\mathbf{M}_h^e$, $\mathbf{M}_d^e$: fictitious elastic moments calculated from the live and dead loads assuming that the structure is purely elastic; $\mathbf{M}^r$: residual internal moments; $\mathbf{M}_h^p$, $\mathbf{M}_d^p$: plastic moments; $\overline{M}^p$, $\overline{M}_d^p$, $\overline{M}_h^p$: plastic moments of the Semi Rigid Connections (SRC) ; W_{p0}^h , $\overline{W}_{p0}^h$, σ_W : bound of the complementary strain energy of the residual forces which is assumed to be a random variable with Gaussian distribution, its mean value and the standard deviation, respectively; P_1, P_2: two random variables with Gumbel distribution; V_0 : Volume limit; σ_y, E: yield stress and Young's modulus; A_i, I_i, S_{0i} and ℓ_i: area, moments of inertia of the cross-section and length of the finite elements $(i = 1, 2, \ldots, n)$, respectively; $\overline{S}_j$:stiffness of the SRC; **F**, **K**, **G**, $\mathbf{G}^*$: flexibility, stiffness, geometrical and equilibrium matrices; β: reliability index; m_{adm}, m_{ph}: plastic limit load multipliers, admissible and calculated, respectively.

3 Reliability based plastic limit design using the crude Monte Carlo method

The crude Monte Carlo method (sometimes called conventional, standard or plain Monte Carlo method) consists in generating realizations $\mathbf{x}$, here $\mathbf{x} = [p_1, p_2, w_{p0}^h]$, of the random vector $\mathbf{X}$, here $\mathbf{X} = [P_1, P_2, W_{p0}^h]$, from their joint probability density function $f_{\mathbf{X}}(\mathbf{x})$, here $f_{\mathbf{X}}(\mathbf{x}) = f_{W_{p0}^h}(w_{p0}^h) f_{\mathbf{P}}(p_1, p_2)$ and checking if a given realization leads to failure, $g(\mathbf{X}) \leq 0$, or not, $g(\mathbf{X}) > 0$, where $g(w_{p0}^h, p_1, p_2) = m_{adm} - m_{ph}(w_{p0}^h, p_1, p_2)$ is the limit state function. The number of points in the failure domain with respect to the total number of generated points is an estimator of the probability of failure. Instead of direct calculation of the following integral

$$P_f = \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \,, \quad \text{it can be easily estimated} \quad \widehat{P}_f = \frac{1}{N} \sum_{k=1}^{N} \chi_{D_f}(\mathbf{X}_k) \,. \tag{1}$$

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This idea can be expressed by introducing the following indicator function of failure domain $D_f = \{\mathbf{x} : g(\mathbf{x}) \leq 0\}$

$$\chi_{D_f}(x) = \begin{cases} 1 \; if \mathbf{x} \in D_f; \\ 0 \; if \mathbf{x} \notin D_f. \end{cases} \tag{2}$$

Let assumed that due to the (manufacturing, strength, geometrical) uncertainties the bound for the magnitude of the complementary strain energy of the residual forces is given randomly and for sake of simplicity it follows the Gaussian distribution with given mean value $\overline{W}_{p0}^{h}$ and standard deviation σ_W . Limit loads P_1 and P_2 describe variability of maximum realizations of loads in considered period of time (e.g. lifetime of structure), therefore Gumbel distribution is assumed.

The solution method based on the static theorem of extended limit analysis is formulated as below:

$$\max m_{ph} \tag{3}$$

Subjected to

$$\mathbf{G}^*\mathbf{M}_d^p + \mathbf{P}_d = \mathbf{0}; \;\; \mathbf{G}^*\mathbf{M}_h^p + m_{ph}\mathbf{Q}_h = \mathbf{0};$$
$$\mathbf{M}_d^e = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1}\mathbf{P}_d; \;\; \mathbf{M}_h^e = \mathbf{F}^{-1}\mathbf{G}\mathbf{K}^{-1}m_{ph}\mathbf{Q}_h;$$
$$M_{hi}^r = [maxM_{hi}^e + M_{di}^e] - [maxM_{hi}^p + M_{di}^p], \text{ (i=1,2,...,n) };$$
$$-2S_{0i}\sigma_y \leq (M_{di} + M_{hi}^p) \leq 2S_{0i}\sigma_y, \text{ (i=1,2,...,n) },$$
$$-\overline{M}_j^p \leq (\overline{M}_{dj}^p + \overline{M}_{hj}^p) \leq \overline{M}_j^p, \text{ (j=1,2,...,k) };$$
$$V - V_0 \leq 0; \quad V = \sum A_i\, L_i; \qquad \beta_{target} - \beta_{calc} \leq 0.$$

The formulations of the problems yield to nonlinear mathematical programming which is solved by the use of sequential quadratic algorithm by applying direct integration or Monte Carlo simulation.The optimization procedure governed by the reliability index calculation.

In Fig. 1 the investigation on the cross section of the structure can be seen. In Fig. 2 one can see the evaluation of the P_f in function of different connection rigidities by considering Monte Carlo simulation.

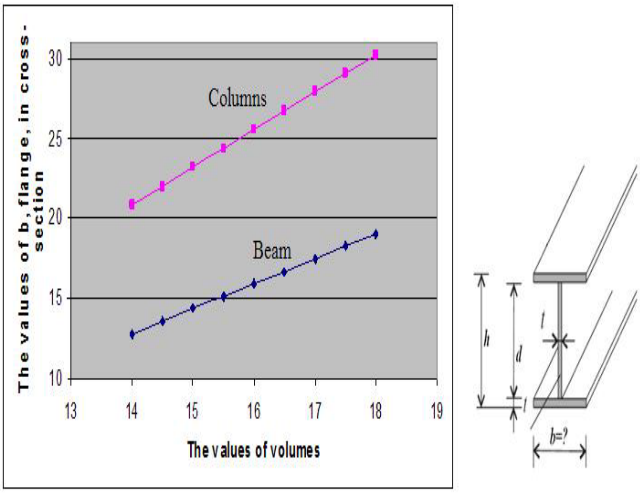

Fig. 1 Optimal volumes

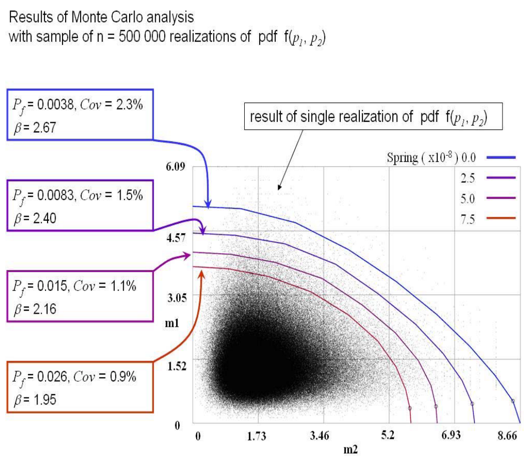

Fig. 2 Plastic limit curves

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References

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