EVOLUTION OF SUSPENSION DROPS SETTLING UNDER GRAVITY IN A VISCOUS FLUID NEAR A VERTICAL WALL

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Summary Using the point-force model, we analyze how evolution of a suspension drop settling under gravity in a viscous fluid is influenced by the presence of a vertical wall near by. In particular, we show that a close drop moves away from the wall while settling along.

INTRODUCTION

In the unbounded fluid, dynamics of settling suspension drops has been studied by many authors [1, 2, 3, 4]. In our previous publications [5, 6], we have analyzed, experimentally and numerically, destabilization of a suspension drop of diameter D settling gravitationally in a viscous fluid close to a vertical planar solid wall. We have shown that the destabilization time and length are linear functions of the inverse interparticle distance h between the drop center and the wall, for $h \geq 1.5D$. In this presentation, we use the point-particle model to analyze earlier stages of the drop evolution. We determine as functions of time such quantities as the number of particles, mean velocity, horizontal and vertical size, and the distance from the wall.

POINT-PARTICLE MODEL OF A SUSPENSION DROP

In a viscous fluid of viscosity η, bounded by a solid wall at $z = 0$, there are N_0 identical point-forces, $\mathbf{F}_\alpha$=$\mathbf{F} = (-F, 0, 0)$ (with $F > 0$), at positions $\mathbf{r}_\alpha$, distributed with the uniform probability inside a spherical volume of diameter D. At a low-Reynolds-number, the particle dynamics is governed by the following system of ODEs,

$$\frac{d\mathbf{r}_\alpha}{dt} = \sum_{\beta\neq\alpha}^{N_0} [\mathbf{T}_0(\mathbf{r}_{\alpha\beta}) + \tilde{\mathbf{T}}(\mathbf{r}_{\alpha\beta'})] \cdot \mathbf{F} + \tilde{\mathbf{T}}(\mathbf{r}_{\alpha\alpha'}) \cdot \mathbf{F}, \qquad \alpha = 1, ..., N_0. \tag{1}$$

Here, $\mathbf{r}_{\alpha\gamma} = \mathbf{r}_\alpha - \mathbf{r}_\gamma$, and β' is the mirror image of the particle β, obtained with the reflection operator $\mathbf{P} = \mathbf{1} - 2\hat{\mathbf{z}}\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ the unit vector perpendicular to the wall. The Oseen tensor, $\mathbf{T}_0(\mathbf{r}_{\alpha\beta})$, describes the interaction of a particle α with the particle β in an unbounded fluid,

$$\mathbf{T}_0(\mathbf{r}_{\alpha\beta}) = \frac{1}{8\pi\eta r_{\alpha\beta}} \left(\mathbf{I} + \frac{\mathbf{r}_{\alpha\beta}\mathbf{r}_{\alpha\beta}}{r_{\alpha\beta}^2} \right), \tag{2}$$

where $r = |\mathbf{r}|$ and $\mathbf{I}$ is the unit tensor. The difference $\tilde{\mathbf{T}}(\mathbf{r}_{\alpha\beta'})$ between the Green tensors for the fluid bounded by the wall and unbounded (i.e. the Blake tensor [7] and the Oseen tensor shown in Eq. (2)),

$$\tilde{\mathbf{T}}(\mathbf{r}_{\alpha\beta'}) \cdot \mathbf{F} = -\mathbf{T}_0(\mathbf{r}_{\alpha\beta'}) \cdot \mathbf{F} - 2h_\beta \mathbf{F} \cdot \mathbf{P} \cdot \boldsymbol{\nabla}_{\mathbf{r}_{\alpha\beta'}} \mathbf{T}_0(\mathbf{r}_{\alpha\beta'}) \cdot \hat{\mathbf{z}} + \frac{2h_\beta^2}{8\pi\eta} \mathbf{F} \cdot \mathbf{P} \cdot \boldsymbol{\nabla}_{\mathbf{r}_{\alpha\beta'}} \left(\frac{\mathbf{r}_{\alpha\beta'}}{r_{\alpha\beta'}^3} \right), \tag{3}$$

describes the interaction of a particle α with the mirror image β' of the particle β. Here h_β is the distance between the particle β and the wall.

Velocity of a point-particle α, specified at the right-hand-side of Eq. (1), consists of two terms. The first one is the sum of the fluid velocity fields generated by all the other particles and their images at the position $\mathbf{r}_\alpha$ where the particle α is located. The second term is the self contribution: it specifies the velocity field generated by the particle's α own image α'. The wall is parallel to the force $\mathbf{F}$, therefore

$$\tilde{\mathbf{T}}(\mathbf{r}_{\alpha\alpha'}) \cdot \mathbf{F} = -\frac{3\mathbf{F}}{32h_\alpha\pi\eta}. \tag{4}$$

We have chosen the frame of reference moving with the Stokes velocity V_0 of a single point-particle in an unbounded fluid. In this way, the dynamics of particles is independent of the particle radius.

We computed evolution of 30 different random initial configurations of particles inside a spherical volume of diameter D, centered at 13 different distances h from the wall, for the initial number of particles inside the drop $N_0 = 700$ and 1000. For a given h, the quantities of interest were averaged over the initial configurations.

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DROP EVOLUTION

Examples of our results are shown in Fig. 1. In the plots, time evolution of two characteristic parameters of the drop is shown: the instantaneous number N of the particles inside the drop, normalized by the initial value, $N_0 = 1000$, and the increase κ of the drop distance from the wall, normalized by its diameter D,

$$\kappa/D \quad = \quad \left[\frac{1}{N}\sum_{\alpha=1}^{N} x_\alpha - h\right]/D. \tag{5}$$

Our simulations are shown until the moment of the first destabilization. The time unit

$$\tau \quad = \quad \frac{5\pi\eta D^2}{4N_0 F}. \tag{6}$$

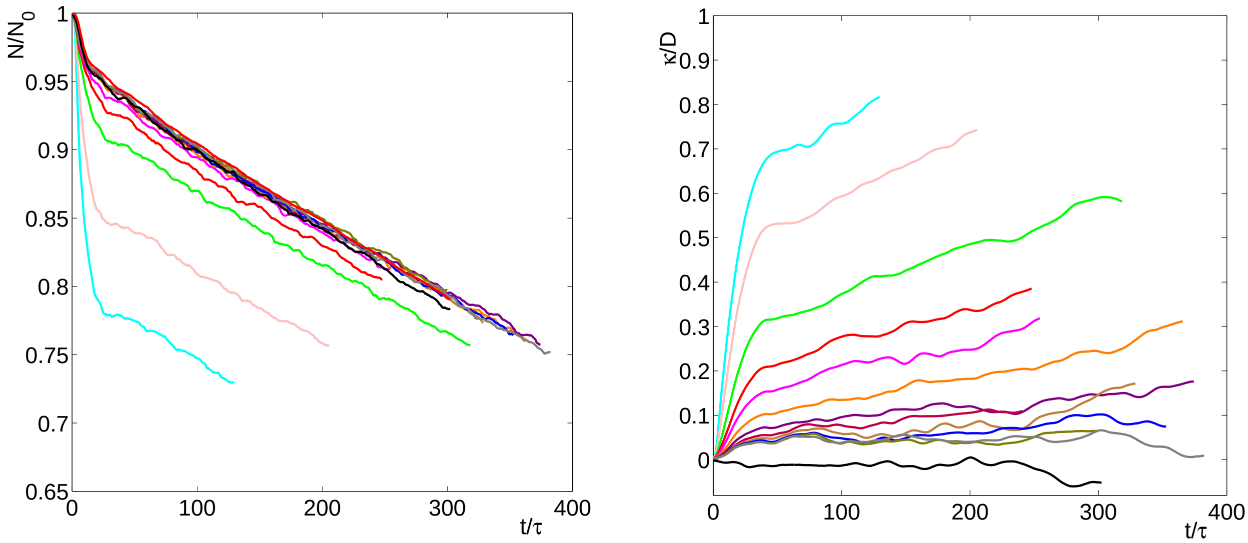

Figure 1. Evolution of a settling suspension drop. Left: the instantaneous number N of particles inside the drop, normalized by the initial value $N_0 = 1000$. Right: the instantaneous shift away from the wall, κ/D, see Eq. (5). Colors denote the initial distance from the wall, $h/D = 0.75$, 1, 1.5, 2, 2.5, 3.5, 5, 7.5, 10, 15, 30, 70, ∞.

CONCLUSIONS

Point-particle model close to a vertical wall has been used to model evolution of a suspension drop settling under gravity in a viscous fluid near a vertical wall. Before destabilization, evolution of a drop consists of two stages. The first one is shorter and characterized by a much larger rate of the particle loss and a faster drop migration away from the wall than the second one. At the first stage, drops at a smaller distance from the wall faster loose particles and faster migrate. At the second stage of the evolution, dN/dt is constant in time and independent of the drop distance from the wall. Finally, in the presence of a vertical wall, suspension drops destabilize faster and after traveling a shorter path than in an unbounded fluid, in consistence with our earlier findings [5].

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