ON SHELL ELEMENTS DERIVED FROM HU-WASHIZU FUNCTIONAL

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1. Introduction

The purpose of the presentation is to summarize our recent results regarding the formulation of shell elements based on the Hu-Washizu (HW) functional with rotational degrees of freedom, [1, 2, 3, 4]. Several numerical examples will illustrate such aspects of their performance as: accuracy, radius of convergence, required number of iterations of the Newton method or the arc-length method and time of computations. Some examples enabling comparisons with the 'solid-shell' element based on the HW functional will be provided.

2. HW shell element with rotational degrees of freedom

The formulation of our four-node shell element with rotational degrees of freedom based on the Hu-Washizu (HW) functional is described in detail in [1]. It is an enhanced element with six dofs/node, enabling finite (unrestricted) rotations, and developed for Green strain. The drilling rotation is included through the drilling Rotation Constraint equation. The key features of the approach are as follows.

1. The shell HW functional is derived from the shell potential energy functional, which is an alternative to the derivation from the three-dimensional HW functional. This method is more versatile as it enables the derivation of the so-called partial HW functionals, with different treatment of the bending/twisting part and the transverse shear part of strain energy.

2. For the membrane part of HW shell elements, a 7-parameter stress, a 9-parameter strain and a 2-parameter EADG enhancement are selected as optimal. The assumed representations of stress and strain are defined in skew coordinates in the natural basis at the element's center. This improves accuracy and has positive theoretical consequences.

3. The drilling Rotation Constraint equation is treated by the Perturbed Lagrange method. The faulty term resulting from the equal-order approximations of displacements and the drilling rotation is eliminated and one spurious mode is stabilized using the gamma method. The proposed formulation is insensitive to the element's distortions and yields a large radius of convergence in the examples involving in-plane bending.

The performance of 4 four-node shell HW elements, differing in formulation of the bending/twisting and the transverse shear parts, is analyzed on several numerical examples. The element with 29 parameters (HW29) is selected as the best performer.

3. Example: quarter of orthotropic hemisphere

A quarter of the hemispherical shell with an 18^o hole is loaded by two external forces, see Fig.1a. The same boundary conditions are used as for an isotropic material, which for an orthotropic material do not preserve symmetry of deformation. The mesh consists of 16×16 elements, and the 4-node element HW29 with rotational dofs and the 8-node 'solid-shell' element SS HW47 are used.

The material is the carbon T300/epoxy composite, and the orthotropic material constants are as follows: $E_{11} = 58.9$, $E_{22} = 52.1$, $E_{33} = 11.2$, $\nu_{12} = 0.048$, $\nu_{13} = 0.442$, $\nu_{23} = 0.46$, $G_{12} = 4.01$,

$G_{13} = 3.87$, $G_{23} = 3.71$. The material orientation is defined using the spherical coordinate system and the material direction vector 1 is tangent to the parallels of latitude.

The nonlinear analyzes are performed using the arc-length method for $P = 1/10^5$ and the results are shown in Fig.1b. The element HW29 crashes at the inward displacement equal to 13.5, while the 'solid-shell' elements SS HW47 performs up to 15, for which a very distorted shape of is obtained. Up to the inward displacement equal to 10, both these classes of elements yield an almost identical deflection, despite the fact that they use different types of orthotropy; the first one uses full orthotropy (9 constants) while the other uses the orthotropy modified by the Zero-Normal-Stress condition (6 constants).

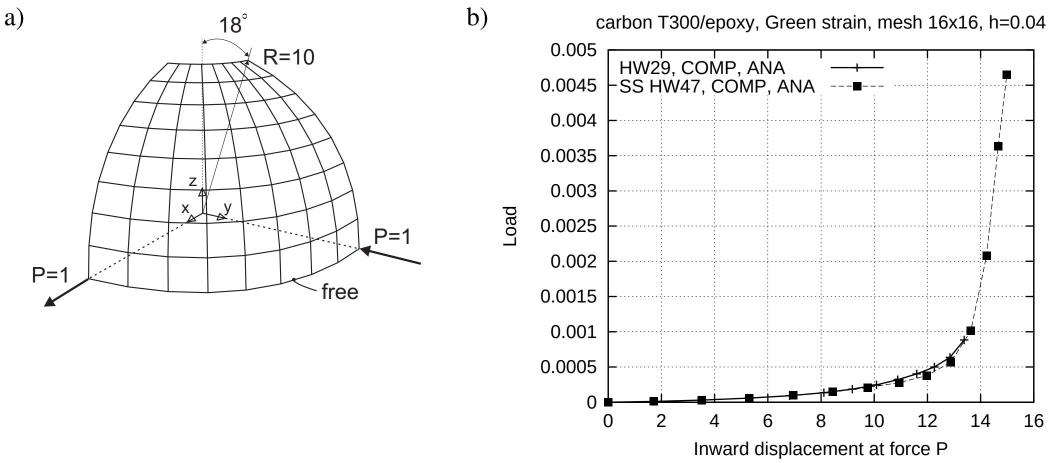

Figure 1. Quarter of hemispherical shell. a) Geometry, b) Nonlinear displacements for various HW elements.

4. References

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