Sixth World Conference on Structural Control and Monitoring

Proceedings of the 6th edition of the World Conference of the International Association for Structural Control and Monitoring (IACSM), held in Barcelona, Spain

15-17 July 2014

Edited by:

J. Rodellar
Universitat Politècnica de Catalunya-BarcelonaTech, Spain

A. Güemes
Universidad Politécnica de Madrid, Spain

F. Pozo
Universitat Politècnica de Catalunya-BarcelonaTech, Spain

A publication of:

International Center for Numerical Methods in Engineering (CIMNE)
Barcelona, Spain

CIMNE

Construction of Virtual Structure for Damage Identification *

Jilin Hou, Łukasz Jankowski, and Jinping Ou

Abstract—**This paper presents a damage identification method using virtual structure. The main concept is based on Virtual Distortion method (VDM), which belongs to a fast structural reanalysis method and employs the virtual distortions or virtual forces to simulate the structural modifications. In this paper, the structure with virtual mass, damping or stiffness is defined as virtual structure. Firstly, the frequency response of the virtual structure is constructed by VDM method; Secondly, the natural frequencies of virtual structure with additional masses or stiffness are estimated; At last, the estimated natural frequencies of the virtual structure are used for damage optimization of the structure. A numerical beam model is used to describe and verify the proposed method.**

I. INTRODUCTION

Structural health monitoring (SHM) is an important researched field in civil engineering. Thereinto structure damage are usually identified using the information such as structure mode (frequency and mode shape) [1], flexibility matrix [2], time domain response [3] and frequency response [4]. However due to the complexity of structures, limitation of the sensors and the insensitivity to the local damage, damage identification for large structures is not easy to be performed accurately in civil engineering.

Adding control parameters [5], such as mass [6], stiffness[7], support, load or thermal loading, provide efficient way for damage identification. However, in real application sometimes it is not easy to add mass or stiffness on structures perfectly. Therefore this paper proposes a method for structural damage identification by the construction of virtual structure using Virtual Distortion Method (VDM). VDM [8,9] belongs to a fast structural reanalysis method which employs virtual distortion or virtual force to simulate structural changes. The damage or modified parameters can be structural stiffness, mass or damping. Via VDM, the responses of modified structure are constructed using the responses of the original structure and certain virtual distortions (virtual forces), and so the whole analysis of the modified structure model is avoided. VDM is used versatilely on structural parameter identification, like stiffness [8,9], mass[10], moving mass[11], and damping identification[12].

This paper employs VDM to construct virtual structure for damage identification. The paper is structured as follows. The next section describes and derives the theory of the proposed approach. While a numerical example of a plane beam is utilized to test the proposed method. The effectiveness and availability of the approach is demonstrated at the assumed Gaussian measurement noise level of 5 % rms.

II. THE CONSTRUCTION OF VICTUAL STRUCTURE BY VDM

Assume that a structure contains n_d degrees of freedom (Dofs). Denote by $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ the mass, stiffness, and damping matrix of its original real structure respectively, equation of the motion of the structure in frequency domain can be written as:

$$\boldsymbol{M}\ddot{\boldsymbol{X}}(\omega) + \boldsymbol{C}\dot{\boldsymbol{X}}(\omega) + \boldsymbol{K}\boldsymbol{X}(\omega) = \boldsymbol{B}\boldsymbol{F}(\omega) \tag{1}$$

$$\boldsymbol{Y}(\omega) = \boldsymbol{H}(\omega)\boldsymbol{B}\boldsymbol{F}(\omega) \tag{2}$$

where $F(\omega)$ is input excitation with dimension of $n_f \times 1$, n_f is the number of the excitation; $Y(\omega)$ is the measured response of structure with the dimension $n_y \times 1$; n_y is the number of the sensors, then sensors could be acceleration, velocity, displacement, strain and so on; $\boldsymbol{B}$ is the matrix of load displacement, with dimension of $n_d \times n_f$; $\boldsymbol{H}(\omega)$ is the impulse response matrix with dimension of $n_d \times n_f$, of which $\boldsymbol{H}_{ij}(\omega)$ is the response of the ith sensor to excitation applied on the jth Dof of the structure.

Denoted by $\Delta\boldsymbol{M}, \Delta\boldsymbol{C}, \Delta\boldsymbol{K}$ the modification of the mass, damping and stiffness matrix of the original real structure, then equation of motion of the modified structure in frequency domain is as follows:

$$(\boldsymbol{M}+\Delta\boldsymbol{M})\ddot{\boldsymbol{X}}^V(\omega) + (\boldsymbol{C}+\Delta\boldsymbol{C})\dot{\boldsymbol{X}}^V(\omega) + (\boldsymbol{K}+\Delta\boldsymbol{K})\boldsymbol{X}^V(\omega) = \boldsymbol{B}\boldsymbol{F}(\omega) \tag{3}$$

where $\boldsymbol{X}_V(\omega)$ is the response of the modified structure. By moving the modification terms to the right-hand side of (3), which is expressed as the equivalent form:

*Resrach supported by the support of the National Basic Research Program of China (973 Program) (2013CB036305), of National Science Foundation of China (NSFC) (51108057, 51108066), of the Fundamental Research Funds for the Central Universities (China) (DUT13LK13), of Special Financial Grant from the China Postdoctoral Science Foundation (2012T50255), and of the Project of National Key Technology R&D Program (China) (2011BAK02B01, 2011BAK02B03,2006BAJ03B05). Financial support of the Polish National Science Centre Project "AIA" (DEC-2012/05/B/ST8/02971) and of Structural Funds in the Operational Programme-Innovative Economy (IE OP) financed from the European Regional Development Fund, Project "Modern material technologies in aerospace industry" (POIG.01.01.02-00-015/08), is gratefully acknowledged.

Jilin Hou is with the Dalian University of Technology, Dalian, 116024, P. R. of China (corresponding author to provide phone: +8641184706432; e-mail: hou.jilin@hotmail.com).

Łukasz Jankowski is with Institute of Fundamental Technological Research, Polish Academy of Sciences, 02-106, Warsaw, Poland

Jinping Ou is with the Dalian University of Technology, Dalian, 116024, P. R. of China, and Harbin Institute of Technology, Harbin 150090, P. R. of China

$$\boldsymbol{M}\ddot{\boldsymbol{X}}^V(\omega)+\boldsymbol{C}\dot{\boldsymbol{X}}^V(\omega)+\boldsymbol{K}\boldsymbol{X}^V(\omega)=\boldsymbol{B}\boldsymbol{F}(\omega)-\Delta\boldsymbol{M}\ddot{\boldsymbol{X}}^V(\omega)-\Delta\boldsymbol{C}\dot{\boldsymbol{X}}^V(\omega)-\Delta\boldsymbol{K}\boldsymbol{X}^V(\omega) \tag{4}$$

In this method, only local structure is modified. Denote by n_m, n_c, n_k respectively the number of Dofs of the modified mass, damping, and stiffness respectively, and let $\Delta\boldsymbol{m}, \Delta\boldsymbol{c}, \Delta\boldsymbol{k}$ be the corresponding physical matrix of local structure, which are square matrices with respective dimension of n_m, n_c, n_k. Then $\Delta\boldsymbol{M}=\boldsymbol{T}_m^T\Delta\boldsymbol{m}\boldsymbol{T}_m, \Delta\boldsymbol{C}=\boldsymbol{T}_c^T\Delta\boldsymbol{c}\boldsymbol{T}_c, \Delta\boldsymbol{K}=\boldsymbol{T}_k^T\Delta\boldsymbol{k}\boldsymbol{T}_k$, where $\boldsymbol{T}_m^T$, $\boldsymbol{T}_c^T$ and $\boldsymbol{T}_k^T$ are the relative transfer matrices from local coordinate to global coordinate. They have the respective dimension of $n_m\times n_d$, $n_c\times n_d$, $n_k\times n_d$.

Set, $\ddot{z}^V=\boldsymbol{T}_m\ddot{\boldsymbol{X}}^V, \dot{z}^V=\boldsymbol{T}_c\dot{\boldsymbol{X}}^V, z^V=\boldsymbol{T}_k\boldsymbol{X}^V, Z^V=\begin{Bmatrix}\ddot{z}^V(\omega)\\ \dot{z}^V(\omega)\\ z^V(\omega)\end{Bmatrix}$, which are the responses along modified Dofs .The dimension of column vectors $\ddot{z}^V, \dot{z}^V, z^V$ respectively are n_m, n_c, n_k. Then (4) can be written as:

$$Y^V(\omega,\Delta)=Y(\omega)-\left[\boldsymbol{H}(\omega)\boldsymbol{T}_m^T\Delta\boldsymbol{m}\quad \boldsymbol{H}(\omega)\boldsymbol{T}_c^T\Delta\boldsymbol{c}\quad \boldsymbol{H}(\omega)\boldsymbol{T}_k^T\Delta\boldsymbol{k}\right]Z^V \tag{5}$$

Let $\boldsymbol{h}_m(\omega)=\boldsymbol{H}(\omega)\boldsymbol{T}_m^T$, $\boldsymbol{h}_c(\omega)=\boldsymbol{H}(\omega)\boldsymbol{T}_c^T$ and $\boldsymbol{h}_k(\omega)=\boldsymbol{H}(\omega)\boldsymbol{T}_k^T$, which are the impulse response matrices corresponding to the Dofs of modified mass, damping and stiffness respectively, with respective dimension of $n_y\times n_m$, $n_y\times n_c$, $n_y\times n_k$. Let

$\boldsymbol{h}_v=[\boldsymbol{h}_m\quad \boldsymbol{h}_c\quad \boldsymbol{h}_k]$, $\boldsymbol{\Delta}_v=\begin{bmatrix}\Delta\boldsymbol{m} & & \\ & \Delta\boldsymbol{c} & \\ & & \Delta\boldsymbol{k}\end{bmatrix}$, then (5) can be written as:

$$Y^V(\omega,\Delta)=Y(\omega)-\boldsymbol{h}_v\boldsymbol{\Delta}_v Z^V \tag{6}$$

Here measured response $Y(\omega)$ is divided into two parts $Y_\Delta(\omega)$ and $Y_s(\omega)$, i.e. $Y(\omega)$=[$Y_\Delta(\omega)$; $Y_s(\omega)$], where $Y_\Delta(\omega)$ are the response related to modified dofs (or response Z^V) with dimension of $(n_m+n_c+n_k)\times 1$, and $Y_s(\omega)$ are the response of the rest sensors with of dimension of $(n_y-(n_m+n_c+n_k))\times 1$. $Y_\Delta(\omega)$ can be expressed by (7).

$$Y_\Delta^V=PZ^V \tag{7}$$

where P is the observe matrix of local modified structure with dimension of $(n_m+n_c+n_k)\times(n_m+n_c+n_k)$, and it is required to be reversible matrix in this method.

The frequency response is also divided into two parts corresponding to the measured response: $\boldsymbol{h}_v=\begin{bmatrix}\boldsymbol{h}_\Delta(\omega)\\ \boldsymbol{h}_s(\omega)\end{bmatrix}$. It can be computed using (8) ,

$$U_\Delta(\omega)=\boldsymbol{h}_\Delta(\omega)Q(\omega);U_s(\omega)=\boldsymbol{h}_s(\omega)Q(\omega) \tag{8}$$

where $Q(\omega)$ is measured excitation matrix where the excitation is applied on the modified Dofs. The dimension of $Q(\omega)$ is $(n_m+n_c+n_k)\times(n_m+n_c+n_k)$. $Q_{ij}(\omega)$ is the excitation applied on the i-th modified Dof of the j-th group excitations.

Substitute (7) and (8) into (6), there is

$$\begin{cases}Y_\Delta^v(\omega,\Delta)=Y_\Delta(\omega)-U_\Delta(\omega)Q^{-1}(\omega)\boldsymbol{\Delta}_v P^{-1}Y_\Delta^v\\ Y_s^v(\omega,\Delta)=Y_s(\omega)-U_s(\omega)Q^{-1}(\omega)\boldsymbol{\Delta}_v P^{-1}Y_\Delta^v\end{cases} \tag{9}$$

The response $Y_\Delta^v(\omega,\Delta)$ is computed using the first equation of (9)，see (10)，then response $Y_s^v(\omega,\Delta)$ can be computed by substituting (10) into the second equation of (9), see (11):

$$Y_\Delta^v(\omega,\Delta)=\left(I+U_\Delta(\omega)Q^{-1}(\omega)\boldsymbol{\Delta}_v P^{-1}\right)^{-1}Y_\Delta(\omega) \tag{10}$$

$$Y_s^v(\omega,\Delta)=Y_s(\omega)-U_s(\omega)Q^{-1}\boldsymbol{\Delta}_v P^{-1}\left(I+U_\Delta(\omega)Q^{-1}\boldsymbol{\Delta}_v P^{-1}\right)^{-1}Y_\Delta(\omega) \tag{11}$$

III. Numerical Example

A. Finite Element Model (FEM) of Beam

A simply supported beam is used for numerical verification, see Fig.1. The length is 100 cm; the cross-section is 4 cm × 0.4 cm. Young's modulus of the beam is 206 GPa, and the density is 8950kg/m^3. The FEM model of the beam is divided into 20 elements. The beam is divided to 5 substructures, and each substructure contains 4 elements, see Fig.1.The mode shape and natural frequency are respectively shown in Fig. 2 and Table I.

The assumed damage extents of the 5 substructure are shown in Fig.3, and the corresponding natural frequencies of damaged beam are shown in Table I.

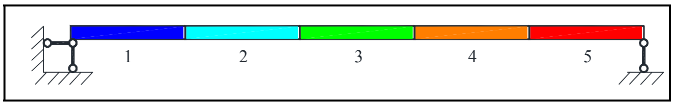


Figure 1. The FEM of beam

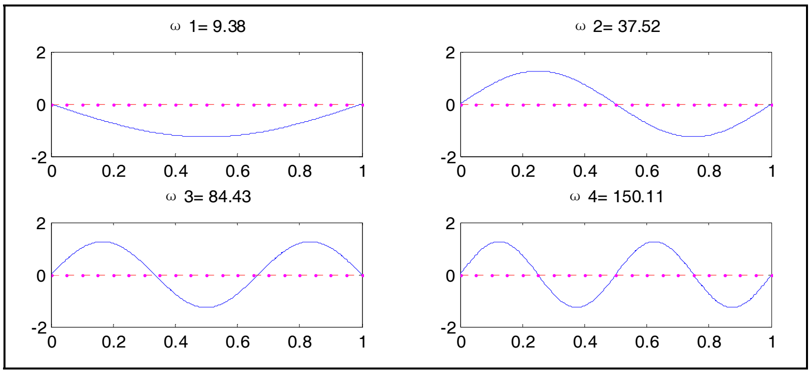


Figure 2. The first 4 orders of structural modes

TABLE I THE NATURAL FREQUENCIES (HZ)

Order	1	2	3	4
Intact	9.38	37.53	84.43	150.12
Damaged	8.62	33.84	79.31	138.83

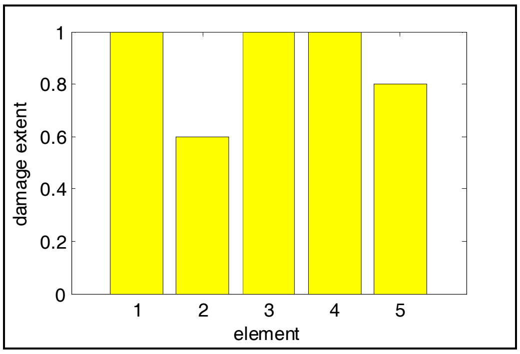


Figure 3 The damage extent

B. The construction of Victual structure by VDM

Added virtual mass

In this section, virtual mass is added on the middle of substructure 2, see Fig.4. Two acceleration sensors are placed on the middle of substructure 2 and 3. Hammer excitation is applied on the middle of substructure 2 and 3 respectively, see Fig. 5, and the corresponding responses are respectively shown in Fig.6 and Fig.7.

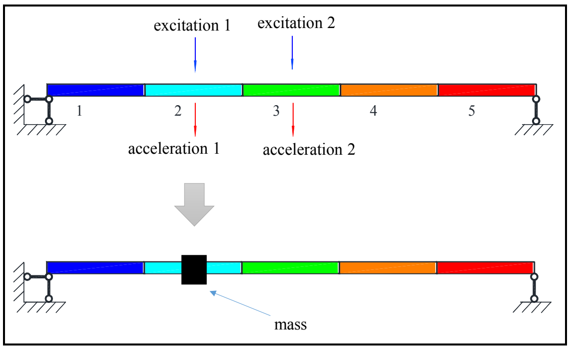


Figure 4. The senor placements and added virtul mass

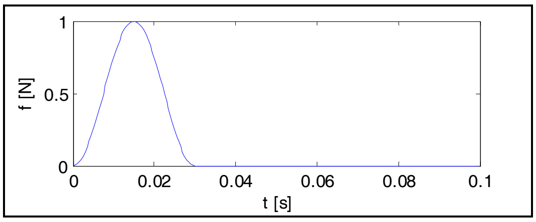


Figure 5. Hammer exciation

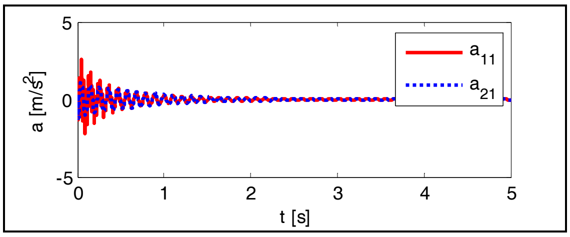


Figure 6. The acceratation responses to hammer exciation on substruture 2

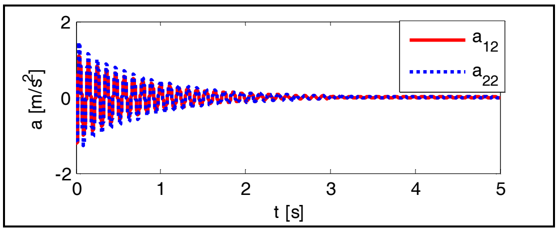
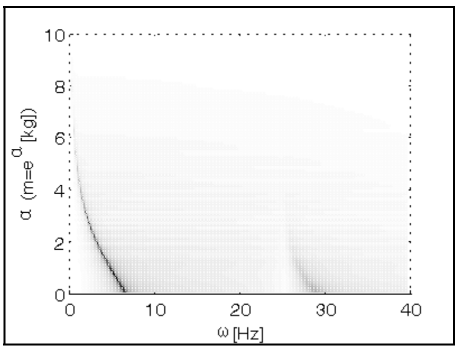
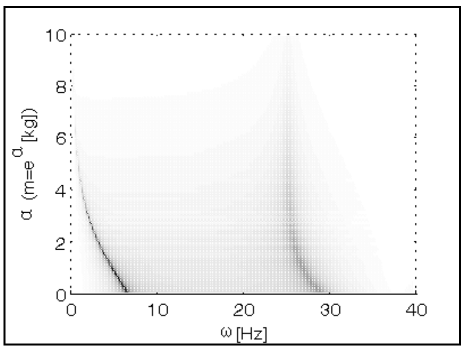


Figure 7. The acceratation responses to hammer exciation on substruture 3

Perform FFT on the responses in Fig.6 and Fig.7 and substitute them into (10), then the nephogram of the constructed frequency response of virtual structural with attached mass are shown in Fig.8(a) and Fig.8(b).There the depth of the colour reflects the amplitude of the frequency response, so that the darkest points correspond to its peaks, that is to the natural frequencies of the virtual structure. It can be seen that as added mass value increases, the natural frequencies decrease. When mass value arrives at a certain degree, the frequencies of virtual structure become stable and unchanged with the mass value increasing.



(a) Hammer excitation applied on subtruutrue 2



(b) Hammer excitation applied on subtruutrue 3

Figure 8. The nephogram of the constructed frequency response of virtual structural with added mass

Added virtual stiffness

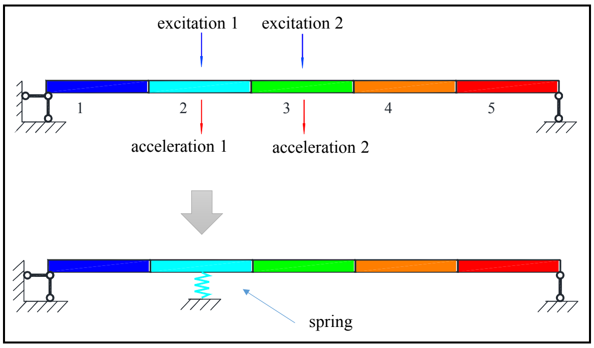
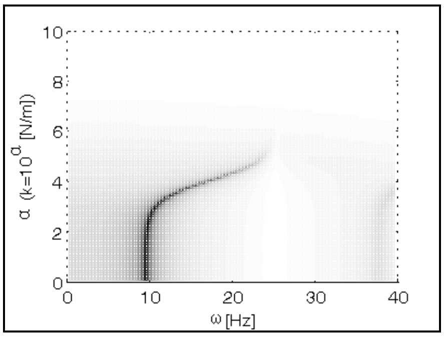
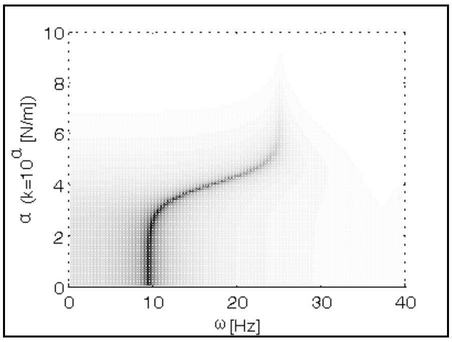


Figure 9. The senor placement and added spring stiffness

In this section, virtual spring stiffness is added on the middle of substructure 2, see Fig.9. Two displacement sensors are placed on the middle of substructure 2 and 3, and hammer excitation is applied on the middle of substructure 2 and 3 respectively, see Fig. 9. Similar to added virtual mass, the nephogram of constructed frequency responses of virtual structure with added spring stiffness are computed and they are shown in Fig.10(a) and Fig.10(b). The depth of the colour reflects the amplitude of the frequency response, so that the darkest points correspond to the natural frequencies of the virtual structure. It can be seen that as the value of added stiffness increases, the natural frequencies decrease. When added stiffness reaches a certain value, the frequencies of virtual structure become stable and unchanged with the stiffness value increasing.



(a) Hammer excitation applied on substructure 2



(b) Hammer excitation applied on substructure 3

Figure 10. The nephogram of the constructed frequency response of virtual structural with adding sping stiffness

Added virtual damping

In this section, virtual damping is added on the middle of substructure 2, see Fig.11. Two velocity sensors are placed on middle of substructure 2 and 3. Hammer excitation is applied on the middle of substructure 2 and 3 respectively, see Fig. 11. Similar to the case of added virtual mass, the nephogram of the constructed frequency response of virtual structural with added damping are computed and are shown in Fig.12(a) and Fig.12(b). From colour depth of the nephogram , it tells that there is no obvious change of the structural frequencies with the damping value increasing because there is no relation between structural frequency and damping. But phase step happens on frequencies when the added damping reaches a certain value.

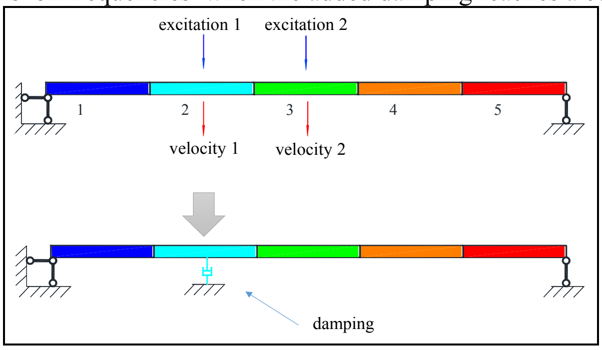
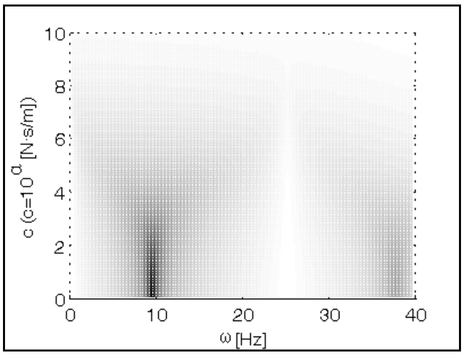
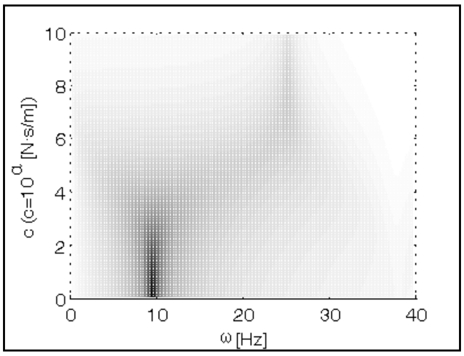


Figure 11. The senor placement and added damping





(a) **Hammer excitation applied on substructure 2** (b) **Hammer excitation applied on substructure 3**

Figure 12. The nephogram of the constructed frequency responses of virtual structure with added virtul damping

C. Damage identification using constructed virtual structure

To perform damage identification, first, a sensor is located on the middle of each substructure successively, and correspondingly model force harmer is successively used to apply the excitation along the measure senor, see Fig.13. The measured excitation and response are used to construct the virtual added mass, stiffness and damping which are added on each substructure. At last damage extents are identified using the virtual structural mode.

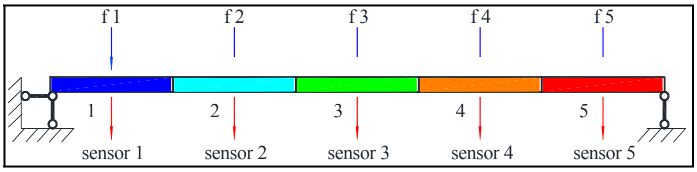


Figure 13. The placement of senor and hammer exciatiation

Added virtual mass

In oreder to simulate actual case, 5% Gaussian noise is added to the simulated measured responses. Hammer excitation and acceleration resposne are used to constructe added virtul mass which is added in the middle of the relative substructure, and the corresponding nephogram of the constructed frequency responses of virtual structure are shown in Fig. 14. Five virtual added mass wih values of [0.500 1.125 1.750 2.375 3.000]kg, are applied on each substructure, see Table II. Fig. 15 shows the frequency responses of substruture 1 with the five vitual added masses. The first order of natrual frequency of the virtual structure is identified using Peak Pick mehtod through the constructed frequency responses correspodning to each virtual added mass. Table II lists the identified frequencies, which are used to identify damage extents. The identified resultes are shown in Fig. 16, which has nice accuracy.

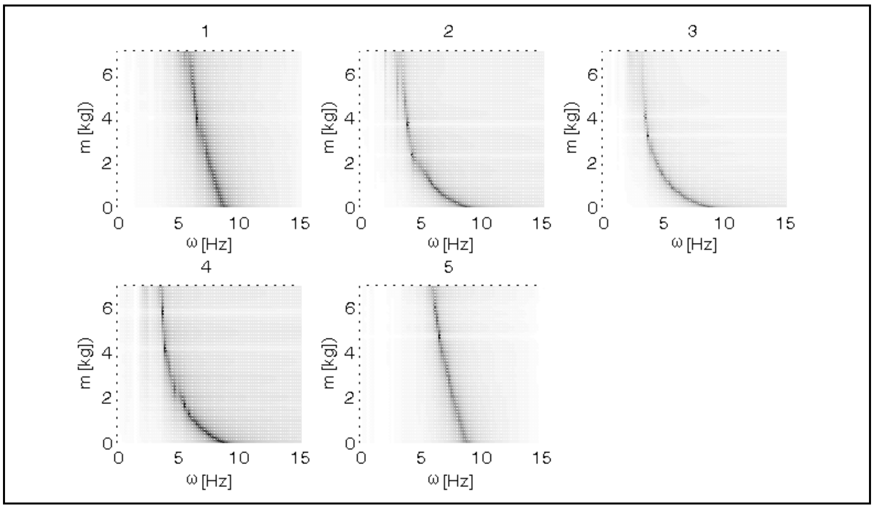


Figure 14. The nephogram of constructed frequency responses of virtual structural with added virtual mass

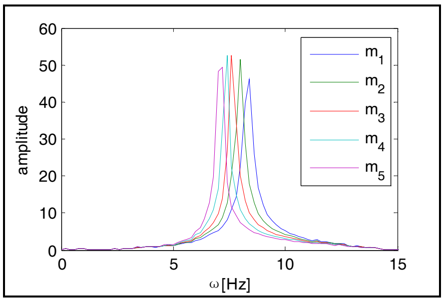


Figure 15. The constructed frequency response with 5 virtual added mass on substruture 1

TABLE II THE NATURAL FREQUENCY (HZ)

	Substructure	Mass	Identified natural frequency
1	1	0.500	8.198
2	1	1.125	7.798
3	1	1.750	7.598
4	1	2.375	7.199
5	1	3.000	6.999
6	2	0.500	6.799
7	2	1.125	5.799
8	2	1.750	4.999
9	2	2.375	4.199
10	2	3.000	3.999
11	3	0.500	6.399
12	3	1.125	5.199
13	3	1.750	4.599
14	3	2.375	3.999
15	3	3.000	3.599
16	4	0.500	7.199
17	4	1.125	5.999
18	4	1.750	5.399
19	4	2.375	4.599
20	4	3.000	4.121
21	5	0.500	8.398
22	5	1.125	7.998
23	5	1.750	7.598
24	5	2.375	7.399
25	5	3.000	7.199

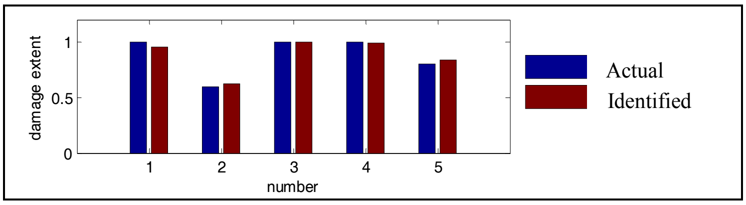


Figure 16. The identified damage extents

Added virtual stiffness

Similarly to performance of virtual added mass value, if velocity sensor is used instead of displacement sensor, then virtual stiffness can be constructed, which are added respective in the middle of the substructures. The corresponding nephogram of the constructed frequency responses of virtual structure with added stiffness are shown in Fig. 17. From Fig. 17, 40 added stiffness are selected and the corresponding frequencies are obtained, which are used to identify damage extents. The identifed results are shown in Fig. 18. It proves that damage extents can be identified accurately with both virtual added mass and virtual added spring stifness.

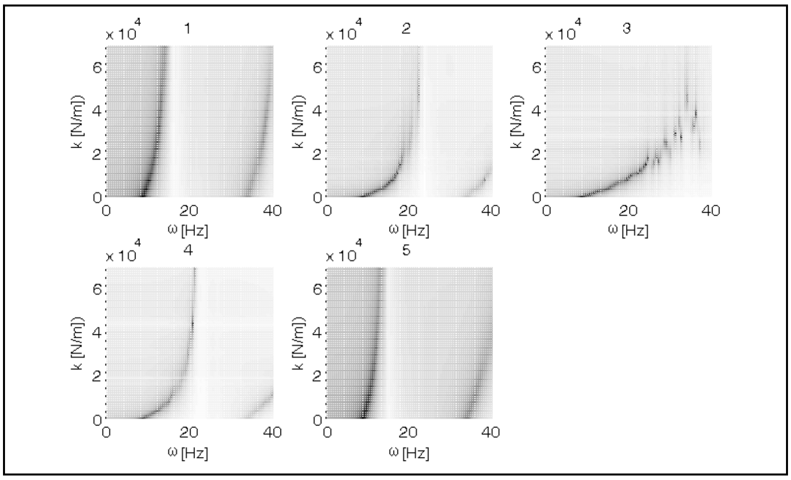


Figure 17. The nephogram of the constructed frequency response of virtual structural with added spring stiffness

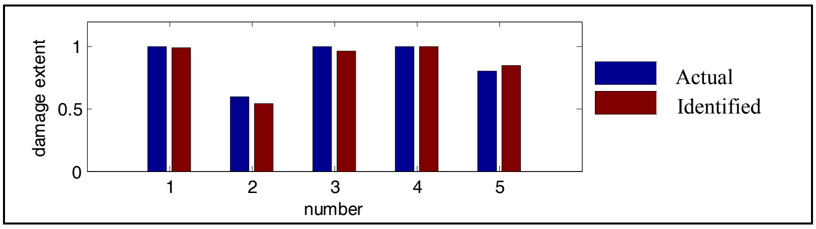


Figure 18. The identified damage extents

IV. Conclusion

This paper proposed a damage identification method by constructing virtual structure. The added virtual mass and stiffness provide more mode data for damage identification, and thus the identification accuracy is increased.

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