Dynamic substructuring approach for human induced vibration of a suspension footbridge

Bartłomiej Błachowski[1], **Witold Gutkowski**[2], **Piotr Wiśniewski**[3]*

[1,3] *Institute of Fundamental Technological Research, Polish Academy of Sciences*
Pawińskiego 5b, 02-106 Warsaw, Poland
e-mail: bblach@ippt.gov.pl [1], *pwisn@ippt.gov.pl* [3]

[2] *Institute of Mechanized Construction and Rock Mining*
Racjonalizacji 6/8, 02-673 Warsaw, Poland
e-mail: wgutkow@ippt.gov.pl

Abstract

A substructuring method for the prediction of the dynamic response of footbridges subjected to loadings induced by a pedestrian is presented. The dynamic system is composed of two independent subsystems. The first one is the model of the suspension footbridge, and the second one is the model of a pedestrian. The former is obtained using the standard finite element method, while the latter is created by applying a two-step identification approach. The first step is an inverse kinematics based on an experimental human motion analysis. The second one relies on Proper Orthogonal Decomposition extracting a number of most important modes, describing the motion of the pedestrian. The presented methodology will be demonstrated by a numerical example of the pedestrian-footbridge interaction.

Keywords: dynamic substructuring, human-structure interaction, suspension footbridges

1. Introduction

Synchronized activity of people can cause a number of dynamical structural problems. One example is a group of humans jumping or dancing on steel-concrete composite floors [1]. Another example is the dynamic interaction of a walking pedestrian with a flexible footbridge. Recently, a number of papers have been devoted to this topic [2,3]. The authors of these papers are using different techniques for analysing the problem. Some of them are using the discrete element method to model crowd motion. However, it seems that a better approach should be based on dynamic substructuring, for instance proposed by Biondi et al. in the case of a train-track-bridge system [4].

In the paper we are following Biondi's approach however, instead of modelling the pedestrian motion, we are using experimental data to identify it. Then, FE model of the suspension footbridge and the identified model of the pedestrian will be coupled using the dynamic substructuring technique.

2. Substructures of the dynamical model

2.1. Substructure 1 – suspension footbridge

Dynamic data of an existing a 67,1m long suspension footbridge are used for the numerical simulation. A general view of the bridge is shown in Figure 1. The main structural components of the footbridge are: two towers, two cables, 42 hangers, and a suspended deck. The towers are built of two I-beam columns; the tower columns are braced by I-beams. The suspended deck consists of two C-shaped channel stringers, rectangular hollow section floor beams (transoms) and L-shaped bracings (diagonals). The main cable is a stranded wire, and the vertical hangers are placed at 3 m intervals.

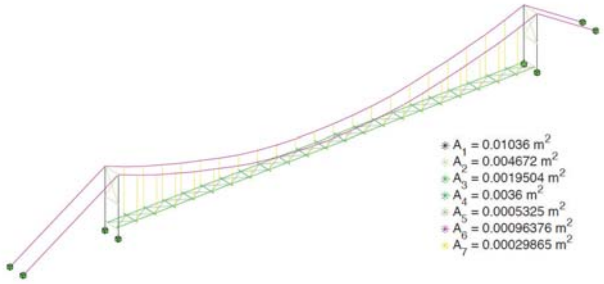

Figure 1: FE model of the suspension footbridge under investigation.

Equations of motion of the suspension footbridge take the form:

$$\mathbf{M}_b \ddot{\mathbf{u}}_b(t) + \mathbf{C}_b \dot{\mathbf{u}}_b(t) + \mathbf{K}_b \mathbf{u}_b(t) = \mathbf{f}_b(t) + \mathbf{g}_b(t) \tag{1}$$

where

$\mathbf{M}_b, \mathbf{C}_b, \mathbf{K}_b$ are mass, damping and stiffness matrices, respectively,

$\ddot{\mathbf{u}}_b(t), \dot{\mathbf{u}}_b(t), \mathbf{u}_b(t)$ - acceleration, velocity and displacement vectors, respectively

$\mathbf{f}_b(t)$ - external force vector (such as wind pressure)

$\mathbf{g}_b(t)$ - interaction force caused by the moving human

2.2. Substructure 2 - pedestrian

The second substructure is a walking human, whose motion is modelled with the aid of a two-step identification procedure. In the first step, experimental data of the human walking provided by the OpenSim project [5], are used to perform inverse kinematics (Figure 2). Then, using MATLAB, Proper

---

*Acknowledgements. The authors wish to express their gratitude for financial support provided by the National Science Centre, grant N N501 0494 40.

Orthogonal Decomposition of the motion is performed to identify the most dominant modes of the walking human.

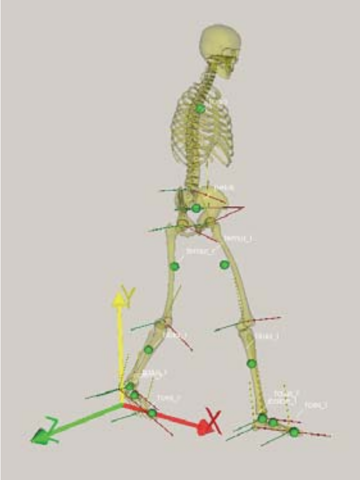

Figure 2: Mass distribution within a pedestrian body

Having the modes and knowing the mass distribution of human body, we linearize the dynamic model of the pedestrian:

$$\mathbf{M}_p\ddot{\mathbf{u}}_p(t)+\mathbf{C}_p\dot{\mathbf{u}}_p(t)+\mathbf{K}_p\mathbf{u}_p(t)=\mathbf{f}_p(t)+\mathbf{g}_p(t) \tag{2}$$

where left hand side components of the equations (2) are analogous to equations (1), $\mathbf{f}_p(t)$ denotes an external (independent of the footbridge movement) force acting on the pedestrian, and $\mathbf{g}_p(t)$ is the interaction force, which is in equilibrium with $\mathbf{g}_b(t)$.

2.3. *Pedestrian-footbridge combined model*

Finally, dynamic model of both substructures, may be combined into one dynamic system

$$\begin{bmatrix}\mathbf{M}_b & 0\\ 0 & \mathbf{M}_p\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{u}}_b(t)\\ \ddot{\mathbf{u}}_p(t)\end{bmatrix}+\begin{bmatrix}\mathbf{C}_b & 0\\ 0 & \mathbf{C}_p\end{bmatrix}\begin{bmatrix}\dot{\mathbf{u}}_b(t)\\ \dot{\mathbf{u}}_p(t)\end{bmatrix}+ \\ +\begin{bmatrix}\mathbf{K}_b & 0\\ 0 & \mathbf{K}_p\end{bmatrix}\begin{bmatrix}\mathbf{u}_b(t)\\ \mathbf{u}_p(t)\end{bmatrix}=\begin{bmatrix}\mathbf{f}_b(t)\\ \mathbf{f}_p(t)\end{bmatrix}+\begin{bmatrix}\mathbf{g}_b(t)\\ \mathbf{g}_p(t)\end{bmatrix} \tag{3}$$

or in a more compact form

$$\hat{\mathbf{M}}\ddot{\mathbf{u}}(t)+\hat{\mathbf{C}}\dot{\mathbf{u}}(t)+\hat{\mathbf{K}}\mathbf{u}(t)=\hat{\mathbf{f}}(t)+\mathbf{g}(t) \tag{4}$$

The above equations have to be considered together with the compatibility condition and interface force equilibrium.

$$\begin{cases}\mathbf{B}(t)\mathbf{u}(t)=\mathbf{0}\\ \mathbf{L}^T(t)\mathbf{g}(t)=\mathbf{0}\end{cases} \tag{5}$$

where $\mathbf{B}(t)$ is a matrix containing elements dependent on the current position of the pedestrian on the footbridge and the assumed shape functions of the FE model of the footbridge, and $\mathbf{L}(t)$ matrix satisfies the following condition

$$\mathbf{L}(t)=null(\mathbf{B}(t)) \tag{6}$$

Then assuming a set of independent generalized coordinates we can write

$$\mathbf{u}(t)=\mathbf{L}(t)\mathbf{q}(t) \tag{7}$$

Finally, substituting (6) into (4) and (5) we get

$$\mathbf{M}(t)\ddot{\mathbf{q}}(t)+\mathbf{C}(t)\dot{\mathbf{q}}(t)+\mathbf{K}(t)\mathbf{q}(t)=\mathbf{f}(t) \tag{8}$$

The resulting dynamical system is then a linear system containing mass, damping and stiffness matrices varying with time.

3. Numerical simulation

The theoretical considerations, presented in the paper, are illustrated with a numerical simulation of the dynamic interaction between the pedestrian and suspension footbridge. The results of numerical simulations are verified using experimental data in the form of acceleration signals from sensors mounted on the cables of the footbridge and on the pedestrian's back (Figure 3).

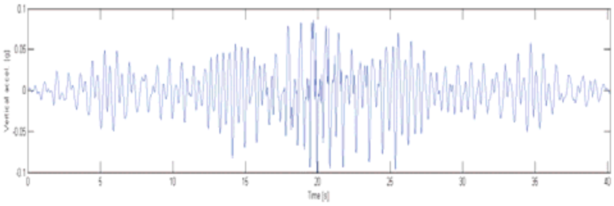

Figure 3: Accelerations of the middle point of the suspension cable.

References

[1] Blachowski, B., Gutkowski, W., Minimum weight design of composite floors under human induced vibrations, *Journal of Civil Engineering, Environment and Architecture JCEEA*, t. XXXI, 61 (2/14), 2014.

[2] Maca, J., Valasek, M., Interaction of Human Gait and Footbridges, *Proceedings of the 8th International Conference on Structural Dynamics*, EURODYN 2011, Leuven, Belgium, 4-6 July 2011.

[3] Carroll, S.P., Owen, J.S., Hussein, M.F.M., A coupled biomechanical/discrete element crowd model of crowd-bridge dynamic interaction & application to the Clifton Suspension Bridge. *Engineering Structures*, 49, pp. 58-75, 2013.

[4] Biondi, B., Muscolinio, G., Sofi, A., A substructure approach for the dynamic analysis of the train-track-bridge system, *Computers and Structures,* 83, pp. 2271-2281, 2003.

[5] Delp, S.L., Anderson, F.C., Arnold, A.S., Loan, P., Habib, A., John, C.T., Guendelman, E., Thelen, D.G., OpenSim: Open-source Software to Create and Analyze Dynamic Simulations of Movement, *IEEE Transactions on Biomedical Engineering,* 2007.