Optimization of thermomechanical structures using PSO

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Abstract

The paper is devoted to the application of particle swarm optimizer of elastic bodies under thermomechanical loading. The Particle Swarm Optimiser presented by Kennedy and Eberhart [5] is proposed as an optimization tool. The optimization problem is formulated as minimization of the volume, the maximal value of the equivalent stress, the maximal value of the temperature or maximization of the total dissipated heat flux with respect to specific dimensions of a structure. The direct problem is computed by means of the finite element method (FEM). Numerical examples for some shape optimization are also included.

Keywords: particle swarm optimiser, optimization, finite element method, thermoelasticity, computational intelligence

1. Introduction

The paper deals with an application of particle swarm optimiser (PSO) and the finite element method to the optimization problems of a heat radiators used to dissipate heat from electrical devices. Recently, swarm methods have found various applications in mechanics, and also in structural optimization. The swarm algorithms are based on the models of the animals social behaviours: moving and living in the groups. PSO algorithm realizes directed motion of the particles in n-dimensional space to search for solution for n-variable optimisation problem. PSO works in an iterative way. The location of one individual (particle) is determined on the basis of its earlier experience and experience of whole group (swarm). Moreover, the ability to memorize and, in consequence, returning to the areas with convenient properties, known earlier, enables adaptation of the particles to the life environment. The optimisation process using PSO is based on finding the better and better locations in the search-space (in the natural environment that are for example hatching or feeding grounds). The main advantage of the bio-inspired method is the fact that these approach do not need any information about the gradient of the fitness function and give a strong probability of finding the global optimum. The main drawback of these approaches is the long time of calculations. The fitness function is calculated for each swarm particle in each iteration by solving the boundary-value problem by means of the finite element method (FEM).

2. The Particle Swarm Optimiser

The particle swarm algorithms [5], similarly to the evolutionary and immune algorithms, are developed on the basis of the mechanisms discovered in the nature. The swarm algorithms are based on the models of the animals social behaviours: moving and living in the groups. The animals relocate in the three-dimensional space in order to change their stay place, the feeding ground, to find the good place for reproduction or to evading predators.

We can distinguish many species of the insects living in swarms, fishes swimming in the shoals, birds flying in flocks or animals living in herds (Fig. 1).

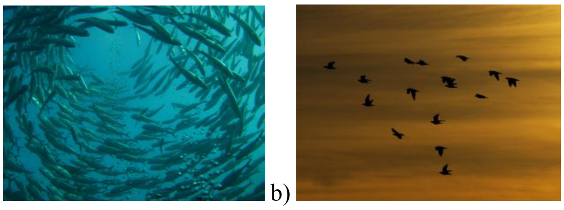
a) b)

Figure 1: Particles swarms:
a) fish shoal (http://www.sxc.hu/photo/1187373),
b) bird flock (http://www.sxc.hu/photo/1095384).

A simulation of the bird flocking was published in [7]. They assumed that this kind of the coordinated motion is possible only when three basic rules are fulfilled: collision avoidance, velocity matching of the neighbours and flock centring. The computer implementation of these three rules showed very realistic flocking behaviour flaying in the three dimensional space, splitting before obstacle and rejoining again after missing it. The similar observations concerned the fish shoals. Further observations and simulations of the birds and fishes behaviour gave in effect more accurate and more precise formulated conclusions [3]. The results of this biological examination where used by Kennedy and Eberhart [4], who proposed Particle Swarm Optimiser – PSO. This algorithm realizes directed motion of the particles in n-dimensional space to search for solution for n-variable optimisation problem. PSO works in an iterative way. The location of one individual (particle) is determined on the basis of its earlier experience and experience of whole group (swarm). Moreover, the ability to memorize and, in consequence, returning to the areas with convenient properties, known earlier, enables adaptation of the particles to the life environment. The optimisation process using PSO is based on finding the better and better locations in the search-space (in the natural environment that are for example hatching or feeding grounds).

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The algorithm with continuous representation of design variables and constant constriction coefficient (constricted continuous PSO) has been used in presented research. In this approach each particle oscillates in the search space between its previous best position and the best position of its neighbours, with expectation to find new best locations on its trajectory. When the swarm is rather small (swarm consists of several or tens particles) it can be assumed that all the particles stay in neighbourhood with currently considered one. In this case we can assume the global neighbourhood version and the best location found by swarm so far is taken into account – current position of the swarm leader (Fig. 2).

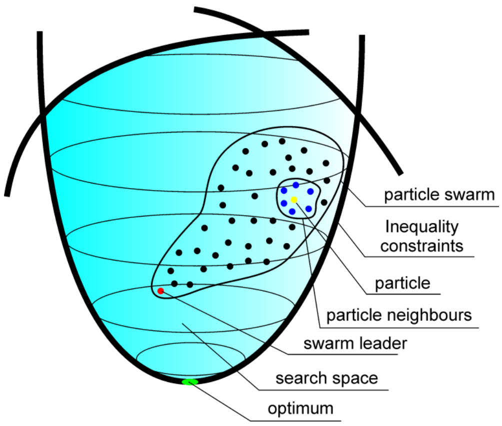


Figure 2: The idea of the particle swarm

The position of the i-th particle is changed by stochastic velocity vi, which is dependent on the particle distance from its earlier best position and position of the swarm leader. This approach is given by the following equations:

$$v_{ij}(k+1) = wv_{ij}(k) + \phi_{1j}(k)\left[q_{ij}(k) - d_{ij}(k)\right] + \phi_{2j}(k)\left[\hat{q}_{ij}(k) - d_{ij}(k)\right] \quad (1)$$

$$d_{ij}(k+1) = d_{ij}(k) + v_{ij}(k+1), \quad i = 1,2,...,m \; ; j = 1,2,...,n \quad (2)$$

where:

$\phi_{1j}(k) = c_1 r_{1j}(k);\ \phi_{2j}(k) = c_2 r_{2j}(k)$,

m – number of the particles,

n – number of design variables (problem dimension),

w – inertia weight,

c_1, c_2 – acceleration coefficients,

r_1, r_2 – random numbers with uniform distribution [0,1],

$d_i(k)$ – position of the i-th particle in k-th iteration step,

$v_i(k)$ – velocity of the i-th particle in k-th iteration step,

$q_i(k)$ – the best found position of the i-th particle found so far,

$\hat{q}_i(k)$ – the best position found so far by swarm – the position of the swarm leader,

k – iteration step.

The velocity of i-th particle is determine by three components of the sum in Equation (1). The first component $wv_i(k)$ plays the role of the constraint to avoid excessive oscillation in the search space. The inertia weight w controls the influence of particle velocity from the previous step on the current one. In this way this factor controls the exploration and exploitation. Higher value of inertia weight facilitates the global searching, and lower – the local searching. The inertia weight plays the role of the constraint applied for the velocities to avoid particles dispersion and guaranteeing convergence of the optimisation process. The second component $\phi_1(k)\left[q_i(k) - d_i(k)\right]$ realizes the cognitive aspect. This component represents the particle distance from its best position found earlier. It is related to the natural inclination of the individuals (particles) to the environments where they had the best experiences (the best value of the fitness function). The third component $\phi_2(k)\left[\hat{q}_i(k) - d_i(k)\right]$ represents the particle distance from the position of the swarm leader. It refers to the natural inclination of the individuals to follow the other which achieved a success.

The flowchart of the particle swarm optimiser is presented in Fig. 11. At the beginning of the algorithm the particle swarm of assumed size is created randomly. Starting positions and velocities of the particles are created randomly. The objective function values are evaluated for each particle. In the next step the best positions of the particles are updated and the swarm leader is chosen. Then the particles velocities are modified by means of the Equation (1) and particles positions are modified according to the Equation (2). The process is iteratively repeated until the stop condition is fulfilled. The stop condition is typically expressed as the maximum number of iterations.

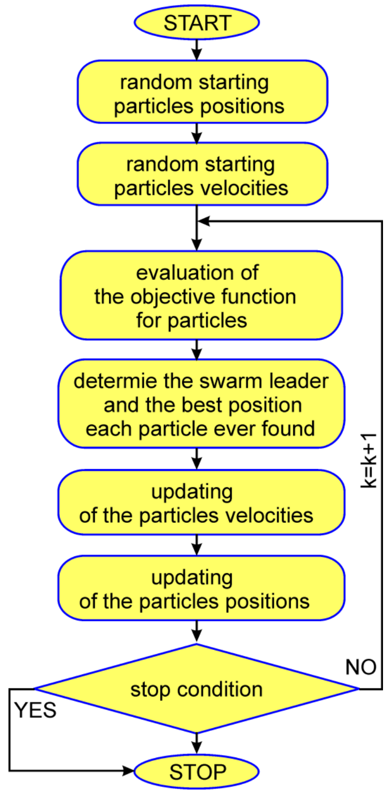


Figure 3: Particle swarm optimiser – block diagram.

The general effect is that each particle oscillates in the search space between its previous best position (position with the best fitness function value) and the best position of its best neighbour (relatively swarm leader), hopefully finding new best positions (solutions) on its trajectory, what in whole swarm sense leads to the optimal solution.

3. Evaluation of the fitness function

The fitness function is computed with the use of the steady-state thermoelsticity. Elastic body occupied the domain Ω bounded by the boundary Γ is considered (Figure 3)

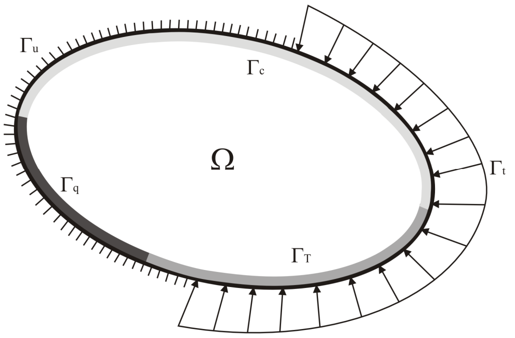


Figure 4: Elastic structure subjected to thermomechanical boundary conditions.

The governing equations of the linear elasticity and steady-state heat conduction problem is expressed by the following equations:

$$G\, u_{i,jj} + \frac{G}{1-2\nu} u_{j,ji} + \frac{2G(1-\nu)}{1-2\nu}\alpha T_{,i} = 0 \tag{3}$$

$$\lambda T_{,ii} + Q = 0 \tag{4}$$

where G is a shear modulus and ν is a Poisson ratio, u_i is a field of displacements, α is heat conduction coefficient, λ is a thermal conductivity, T is a temperature and Q is an internal heat source.

The mechanical and thermal boundary conditions for the equations (3) and (4) take the form:

$$\begin{gathered} \Gamma_t : t_i = \bar{t}_i \;\; ; \Gamma_u : u_i = \bar{u}_i \\ \Gamma_T : T_i = \bar{T}_i \;\; ; \Gamma_q : q_i = \bar{q}_i \;\; ; \Gamma_c : q_i = \alpha(T_i - T^{\infty}) \end{gathered} \tag{5}$$

where $\bar{u}_i, \bar{t}_i, \bar{T}_i, \bar{q}_i, \alpha, T^{\infty}$ is known displacements, tractions, temperatures, heat fluxes heat conduction coefficient and ambient temperature respectively.

Separate parts of the boundaries must fulfill the following relations:

$$\begin{gathered} \Gamma = \Gamma_t \cup \Gamma_u = \Gamma_T \cup \Gamma_q \cup \Gamma_c \\ \Gamma_t \cap \Gamma_u = \varnothing \\ \Gamma_T \cap \Gamma_q \cap \Gamma_c = \varnothing \end{gathered} \tag{6}$$

In order to solve numerically thermoelasticity problem finite element method is proposed. After discretization taking into account boundary conditions following system of linear equations can be obtained:

$$\begin{gathered} \mathbf{KU = F} \\ \mathbf{ST = R} \end{gathered} \tag{7}$$

where **K** denotes stiffness matrix, **S** denotes conductivity matrix, **U**, **F**, **T**, **R** contain discretized values of the boundary displacements, forces, temperatures and heat fluxes.

This problem is solved by the FEM software – MENTAT/MARC [13]. The preprocessor MENTAT enables the production of the geometry, mesh, material properties and settings of the analysis. In order to evaluate the fitness function for each particle following four steps must be performed:

Step 1 *(generated using MENTAT)*

Create geometry and mesh on the base of the particles

Step 2 *(generated using MENTAT)*

Create the boundary conditions, material properties, settings of the analysis

Step 3 *(solved using MARC)*

Solves thermoelasticity problem

Step4

Calculate the fitness functions values on the base of the output MARC file

4. Formulation of the optimization problem

The problem of the optimal shape of a heat radiator used to dissipate heat from electrical devices is considered [2]. The exemplary heat exchangers are presented in Fig. 5.

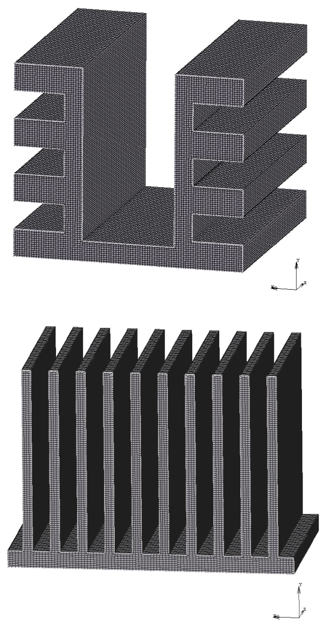


Figure 5: Proposed geometry of considered heat radiators

The shape optimization problem is solved by the minimization of appropriate functionals. In the present paper following functionals are proposed:

• The volume of the structure defined as:

$$\min_{\mathrm{X}} V(\mathrm{X}) \tag{8}$$

with imposed constrains on the maximal value of temperature ($T - T^{ad} \le 0$) and the maximal value of equivalent stress ($\sigma_{eq} - \sigma_{eq}^{ad} \le 0$).

• The minimization of the maximal value of the equivalent stress defined as:

$$\min_{\mathrm{X}} \sigma_{eq}^{\max}(\mathrm{X}) \tag{9}$$

• The minimization of the maximal value of the temperature in the structure defined as:

$$\min_{\mathrm{X}} T^{\max}(\mathrm{X}) \tag{10}$$

with imposed constrains on the maximal value of volume of the structure ($V - V^{ad} \le 0$).

X is the vector of design parameters which is represented by a particle with the floating point representation. The heat radiator is modelled as a two dimensional (2D) plain stress problem. The fitness function is computed with the use of the steady-state thermoelsticity. The governing equations of the linear elasticity and steady-state heat conduction problem are expressed by the following equations:

$$G\, u_{i,jj} + \frac{G}{1-2\nu} u_{j,ji} + \frac{2G(1-\nu)}{1-2\nu} \alpha T_{,i} = 0 \tag{11}$$

$$\lambda T_{,ii} + Q = 0 \tag{12}$$

where G is a shear modulus and ν is a Poisson ratio, u_i is a field of displacements, α is heat conduction coefficient, λ is a thermal conductivity, T is a temperature and Q is an internal heat source.

The mechanical and thermal boundary conditions for the equations (11) and (12) take the form:

$$\begin{gathered} \Gamma_t : t_i = \bar{t}_i \;;\; \Gamma_u : u_i = \bar{u}_i \\ \Gamma_T : T_i = \bar{T}_i \;;\; \Gamma_q : q_i = \bar{q}_i \;;\; \Gamma_c : q_i = \alpha(T_i - T^{\infty}) \end{gathered} \tag{13}$$

where $\bar{u}_i, \bar{t}_i, \bar{T}_i, \bar{q}_i, \alpha, T^{\infty}$ is known displacements, tractions, temperatures, heat fluxes heat conduction coefficient and ambient temperature respectively.

In order to solve numerically thermoelasticity problem finite element method (FEM) is used [1,8]. After discretization taking into account boundary conditions the following system of linear equations can be obtained:

$$\mathbf{KU} = \mathbf{F} \qquad \mathbf{ST} = \mathbf{R} \tag{14}$$

where **K** denotes stiffness matrix, **S** denotes conductivity matrix, **U, F, T, R** contain discretized values of the boundary displacements, forces, temperatures and heat fluxes. The commercial FEM software – Mentat/Marc [13] is used.

5. Geometry modeling

The choice of the geometry modeling method and the design variables has a great influence on the final solution of the optimization process. There is a lot of methods for geometry modeling. In the proposed approach Bezier curves are used to model the geometry of the structures. This type of the curve is a superset of the more commonly known NURBS (Non-Uniform Rational B-Spline). Using these curves in optimization makes the reduction of the number of design parameters possible. By manipulating the control points it provides the flexibility to design a large variety of shapes.

An nth-degree Bezier curve is defined by:

$$C(u) = \sum_{i=0}^{n} B_{i,n}(u) P_i \tag{15}$$

where u is a coordinate with changes range <0,1>, P_i are control points.

The basis functions $B_{i,n}$ are given by:

$$B_{i,n}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-1} \tag{16}$$

The 4-th degree Bezier curve is described by the following equation:

$$\begin{gathered} C(u) = (1-u)^4 P_0 + 4u(1-u)^3 P_1 + \\ 6u^2(1-u)^2 P_2 + 4u^3(1-u) P_3 + u^4 P_4 \end{gathered} \tag{17}$$

An example of the 4-th Bezier curves is shown in Figure 7. By manipulating the control points, it provides the flexibility to design a large variety of shapes.

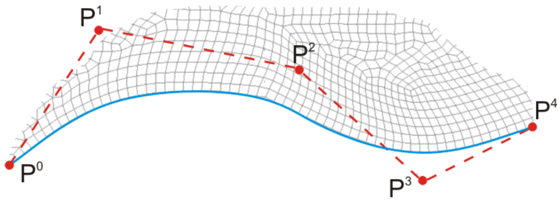


Figure 6: The example modeling of the shape of the structure by 4th-degree Bezier curve

By changing the value of u between 0 and 1 successive points of the curve are obtained. For $u=0$ $C(u)=P^0$ and for $u=1$ $C(u)=P^4$. The shapes of Bezier curve depend on the position of control points. In order to obtain more complicated shapes, it is necessary to raise up the degree of the Bezier curve and introduce more control points.

6. Numerical examples

a) Example 1

The shape optimization problem is solved by the minimization of the volume of the structure with constrains imposed on the temperature and equivalent stress ($\sigma_{eq}^{ad} = 40 MPa$). Three cases of constraints of the temperature were considered ($T^{ad} = 90,\ 100,\ 110\,°C$). Geometry, scheme of loading and the distribution of design parameters are presented in Fig. 7. Parameters of particle swarm optimiser and boundary conditions values are presented in Tab. 1.

Table 1: Parameters of PSO and boundary conditions values

Parameters of PSO	Numbers of particles	15	Boundary condition values	Dissipated heat	80W
	Inertia weight w	0.73		P	10N
	Acceleration coefficient c_1, c_2	1.47		Ambient temperature	25 °C
	The number of design variables	5		Heat convection coefficient	2W/m^{2}K

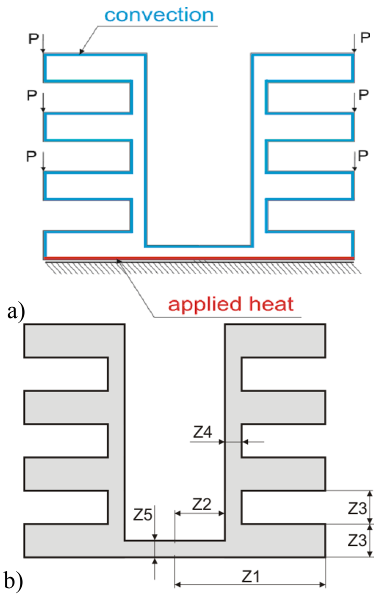


Figure 7: a) Geometry and scheme of loading b) Design parameters

Tab. 2 includes the admissible values of the design parameters and results of optimization. Geometry after optimization process in the figure 8 is presented.

Table 2: The admissible values of the design parameters and results of optimization

The admissible values of the design parameters						
Design variable	Z1 [mm]	Z2 [mm]	Z3 [mm]	Z4 [mm]	Z5 [mm]	
Range	20÷100	2÷10	4÷10	4÷10	4÷10	
Results of optimization						
	Z1 [mm]	Z2 [mm]	Z3 [mm]	Z4 [mm]	Z5 [mm]	Volume [mm^3]
T^{ad}=90°C	43.62	2.86	4	4.29	4	22884
T^{ad}=100°C	39.42	4.99	4	4	4.81	19935
T^{ad}=110°C	32.09	2	4	4	4	17199

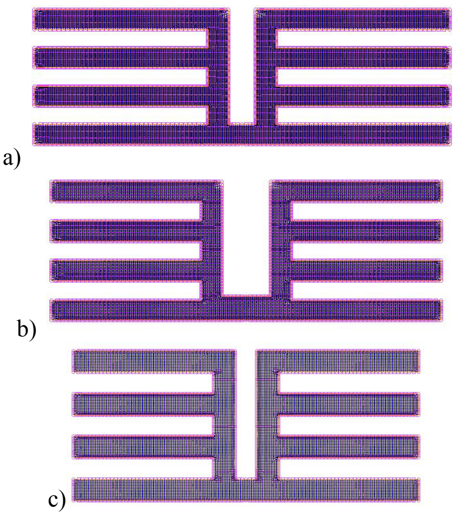


Figure 8: Geometry after optimization process

a) Constraint: $T^{ad} = 90\,°C$

b) Constraint: $T^{ad} = 100\,°C$

c) Constraint: $T^{ad} = 110\,°C$

b) Example 2

The shape optimization problem modelled using Bezier curve is solved by the minimization of the three fitness functions: volume of the structure with constrains imposed on the temperature and equivalent stress ($\sigma_{eq}^{ad} = 15MPa$), temperature and equivalent stresses with constraints imposed on the volume of the structures. Geometry, scheme of loading and the distribution of design parameters are presented in Fig. 9. Parameters of particle swarm optimiser and boundary conditions values are presented in Tab. 3.

Table 3: Parameters of PSO and boundary conditions values

Parameters of PSO	Numbers of particles	15	Boundary condition values	Pressure	5000Pa
	Inertia weight w	0.73		Heat flux	1000W/m^2
	Acceleration coefficient c_1, c_2	1.47		Ambient temperature	25 °C
	The number of design variables	7		Heat convection coefficient	2W/m^{2}K

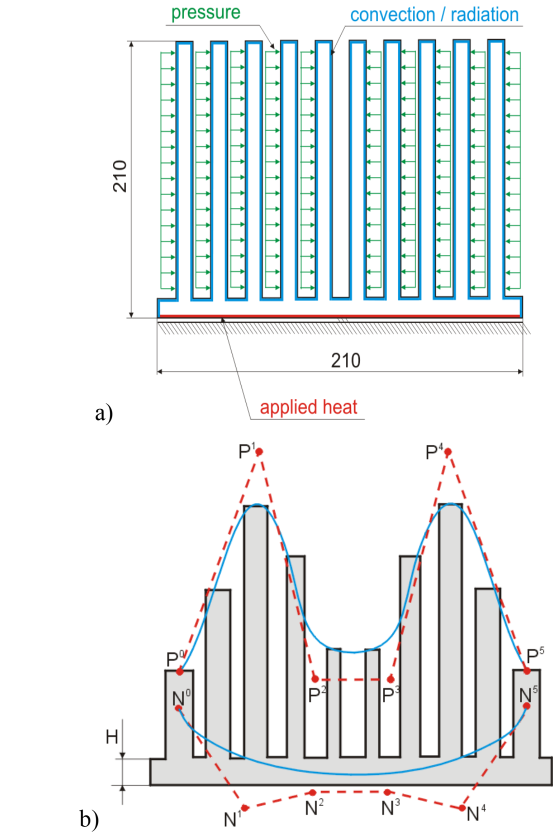


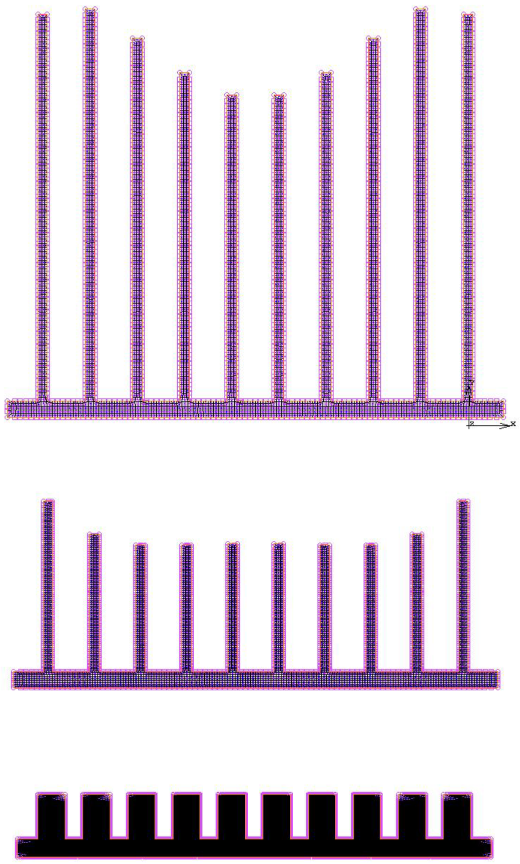
Figure 9: a) Geometry and scheme of loading b) Design parameters

Tab. 4 includes the admissible values of the design parameters and results of optimization.

Table 4: The admissible values of the design parameters and results of optimization

The admissible values of the design parameters						
Design variable	P^0 [mm]	P^1 [mm]	P^2 [mm]	P^3 [mm]	P^4 [mm]	P^5 [mm]
Range	30 ÷ 200	30 ÷ 200	30 ÷ 200	30 ÷ 200	30 ÷ 200	30 ÷ 200
Design variable	N^0 [mm]	N^1 [mm]	N^2 [mm]	N^3 [mm]	N^4 [mm]	N^5 [mm]
Range	4÷12	4÷12	4÷12	4÷12	4÷12	4÷12
Design variable	H [mm]					
Range	7÷15					
Results of optimization (minimization of temperature)						
$P^0= P^5$	$P^1= P^4$	$P^2= P^3$	$N^0= N^5$	$N^1= N^4$	$N^2= N^3$	H
174.1	200	104.5	4	4	4	7
Fitness function evaluation			58.22°C			
Results of optimization (minimization of volume)						
$P^0= P^5$	$P^1= P^4$	$P^2= P^3$	$N^0= N^5$	$N^1= N^4$	$N^2= N^3$	H
83.9	45.8	73.7	4	4	4	7
Fitness function evaluation			0.0007719 m^3			
Results of optimization (minimization of equivalent stresses)						
$P^0= P^5$	$P^1= P^4$	$P^2= P^3$	$N^0= N^5$	$N^1= N^4$	$N^2= N^3$	H
30	30	30	12	11.98	11.94	8.19
Fitness function evaluation			0.13 MPa			

Geometry after optimization process in the figure 10 is presented.



a)

b)

c)

Figure 10: Geometry after optimization process
a) minimization of temperature
b) minimization of volume
c) minimization of equivalent stresses

c) Comparison between PSO and AIS

Additional comparison between two optimization tools (particle swarm optimiser and artificial immune system – AIS [9]) is presented in the Table 5 and 6. Fitness function values, iteration numbers and numbers of fitness function evaluations are compared. The parameters of artificial immune system are included in the Table 5.

Table 5: Parameters of artificial immune system AIS

The number of memory cells	6
The number of the clones	6
Probability of Gaussian mutation	50%
Crowding factor	0.5

Table 6: Comparison between PSO and AIS for example 1

PSO						
Results of optimization						
T^{ad} [°C]	Z1 [mm]	Z2 [mm]	Z3 [mm]	Z4 [mm]	Z5 [mm]	Vol. [mm^3]
90	43.62	2.86	4	4.29	4	22884
100	39.42	4.99	4	4	4.81	19935
110	32.09	2	4	4	4	17199
AIS						
Results of optimization						
T^{ad} [°C]	Z1 [mm]	Z2 [mm]	Z3 [mm]	Z4 [mm]	Z5 [mm]	Vol. [mm^3]
90	47.87	7.83	4	4.17	4	23102
100	44.80	10	4	4	4	20631
110	34.69	5.03	4	4	4	17361

Comparison of the iteration number and fitness function evaluations			
PSO			
T^{ad} [°C]	F.f. value Vol. [mm^3]	Iter. no.	F.f. eval.
90	22884	124	1860
100	19935	9	135
110	17199	32	480
AIS			
T^{ad} [°C]	F.f. value Vol. [mm^3]	Iter. no.	F.f. eval.
90	23102	89	1869
100	20631	46	966
110	17361	76	1596

Table 7: Comparison between PSO and AIS for example 2

PSO						
Results of optimization (minimization of temperature)						
$P^0 = P^5$	$P^1 = P^4$	$P^2 = P^3$	$N^0 = N^5$	$N^1 = N^4$	$N^2 = N^3$	H
174.1	200	104.5	4	4	4	7
Fitness function evaluation			58.22°C			
Results of optimization (minimization of volume)						
$P^0 = P^5$	$P^1 = P^4$	$P^2 = P^3$	$N^0 = N^5$	$N^1 = N^4$	$N^2 = N^3$	H
83.9	45.8	73.7	4	4	4	7
Fitness function evaluation			0.0007719 m^3			
Results of optimization (minimization of equivalent stresses)						
$P^0 = P^5$	$P^1 = P^4$	$P^2 = P^3$	$N^0 = N^5$	$N^1 = N^4$	$N^2 = N^3$	H
30	30	30	12	11.98	11.94	8.19
Fitness function evaluation			0.13 MPa			

AIS						
Results of optimization (minimization of temperature)						
$P^0 = P^5$	$P^1 = P^4$	$P^2 = P^3$	$N^0 = N^5$	$N^1 = N^4$	$N^2 = N^3$	H
186.3	144	141	4	4	4.01	7
Fitness function evaluation			58.29°C			
Results of optimization (minimization of volume)						
$P^0 = P^5$	$P^1 = P^4$	$P^2 = P^3$	$N^0 = N^5$	$N^1 = N^4$	$N^2 = N^3$	H
95.5	75	30	4	4	4	7
Fitness function evaluation			0.0007731 m^3			
Results of optimization (minimization of equivalent stresses)						
$P^0 = P^5$	$P^1 = P^4$	$P^2 = P^3$	$N^0 = N^5$	$N^1 = N^4$	$N^2 = N^3$	H
30	30	35.1	9.01	10.03	11.1	9.42
Fitness function evaluation			0.15 MPa			

Comparison of the iteration number and fitness function evaluations

PSO						
Min.	F.f. value	Iter. no.	F.f. eval.	F.f. value	Iter. no.	F.f. eval.
temp.	58.25 °C	54	1080	58.22°C	147	2940
vol.	0.0007726 m^3	12	240	0.0007719 m^3	144	2880
eq. stress	0.14 MPa	5	100	0.13 MPa	147	2940

AIS			
Min.	F.f. value	Iter. no.	F.f. eval.
temp.	58.29 °C	36	756
vol.	0.0007731 m^3	100	2100
eq. stress	0.15 MPa	54	1134

7. Conclusions

An effective tool of swarm optimization of elastic bodies under thermomechanical loading is presented. Using this approach the optimal shape of a heat radiators used to dissipate heat from electrical devices is obtained. Implementing of the swarm algorithms to this approach gives a strong probability of finding the global optimal solutions. Described approach is free from limitations connected with classic gradient optimization methods referring to the continuity of the objective function, the gradient or hessian of the objective function and the substantial probability of getting a local optimum. Besides in the case of using gradient methods finding the global solution depends on the starting point. The swarm algorithm performs multidirectional optimum searching by exchanging information between particles and finding better and better particles positions. Comparison between PSO and AIS proves good effectiveness of particle swarm optimization method. The results of the numerical examples confirm the efficiency of the proposed optimization method and demonstrate that the method based on particle swarm computation is an effective technique for solving computer aided optimal design problems.

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