

## The sequential self-consistent scheme for modelling elastic-viscoplastic polycrystals

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**Key words:** Micromechanics, Sequential linearization, Self-Consistent Scheme, Polycrystals

The self-consistent (SC) scheme is the mean-field method widely used to estimate effective properties of polycrystalline materials. The method employs the Eshelby solution for an ellipsoidal inhomogeneity embedded in the infinite linear (elastic) medium, which for the SC scheme is identified with the homogenized medium of yet unknown properties. The Eshelby result can be concisely written in the form of the Hill interaction equation enabling to relate local response with the overall one. In the case of non-linear viscoplastic polycrystals, tangent or secant linearization of local response is performed in order to use the Eshelby result. The problem complicates when both elastic and viscous properties must be simultaneously taken into account. In the case of two-phase composites, methods based on the Laplace Transform technique or so-called approximate schemes can be used (for a literature review see [1] or [2]). For polycrystalline materials there are no established solutions so far, although some attempts to adopt the approximate additive scheme due to Molinari [3] can be found in [4].

In [1] the sequential linearization method has been formulated for elastic-viscoplastic materials. A sequence of interaction equations is solved for each strain increment, corresponding to its conceptual subdivision into instantaneous elastic response and creep-type viscous one. The proposed linearization method was applied first to solve the Eshelby problem for elastic-viscoplastic materials and then to estimate effective properties of two-phase materials. For the Mori-Tanaka scheme it was observed that the method yields the same results as the additive scheme [1,5]. However, in the case of the self-consistent scheme an additional accommodation step must be added to obtain satisfactory predictions, especially when the high contrast in phase properties is present.

The application of the sequential method for estimating the mechanical response of polycrystals is discussed in this paper. In Fig. 1a the preliminary result for random linear viscoelastic  $\gamma$ -TiAl polycrystal composed of 100 grains deforming by ordinary and super-dislocation modes is presented, where the initial critical shear stresses vary strongly ( $\tau_c^{\text{sup}}/\tau_c^{\text{ord}}=50$ ; more details concerning material data can be found in [2]). The result is compared with other averaging schemes. Since the anisotropy of viscous response is high the estimated overall response is dramatically different for different averaging schemes. The effect of adding the accommodation step is also visible. In Fig. 1b similar result is shown for random copper polycrystal (100 grains) studied in [6]. The non-linear viscoplasticity described by the power law is here considered ( $n=10$ ) without hardening. Since in this case the viscous anisotropy is not strong only the results without additional accommodation step are demonstrated. On the other hand the effect of different linearization procedure for the

viscous part is studied, denoted as secant, affine and tangent according to the nomenclature proposed in [7]. The results are compared to the recent FFT analysis performed in [6]. For the studied example the tangent variant provides the overall response that agrees best with the FFT predictions.

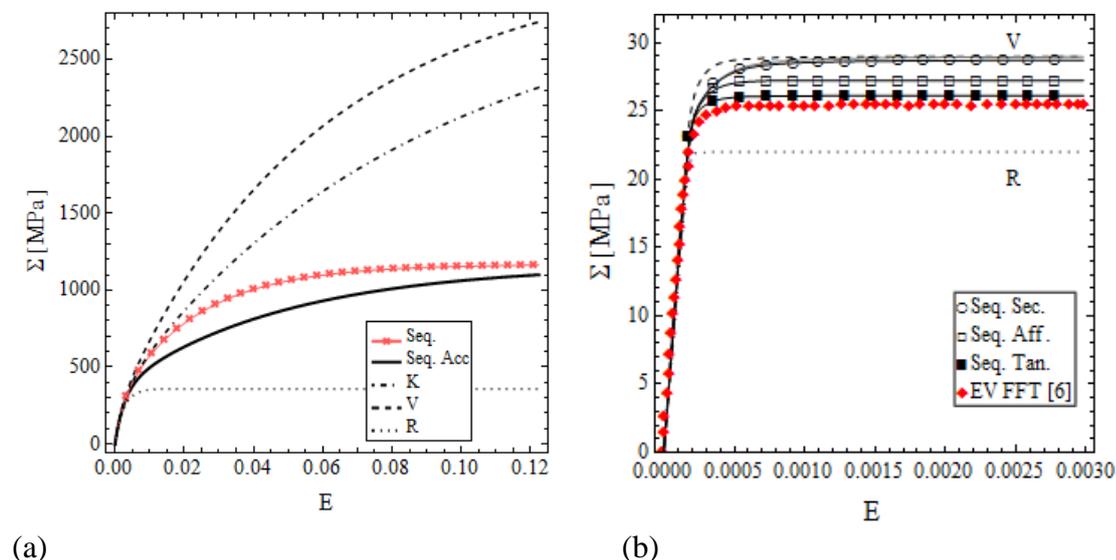


Figure 1: Comparison of predictions of overall response of (a) linear  $\gamma$ -TiAl polycrystal in isochoric extension (b) copper polycrystal ( $n=10$ ) in uniaxial tension, obtained by different averaging schemes (Seq. - sequential SC scheme without accommodation step, Seq. Acc. - sequential SC scheme with accommodation step, Sec., Aff., Tan. - linearization method for non-linear case, V, R – Voigt and Reuss schemes, K – Kroner-Weng scheme, EV FFT – Fast Fourier Transform result provided in [6]).

**Acknowledgement.** The research were partially supported by the project of the National Science Centre (NCN) granted by the decision No. DEC-2013/09/B/ST8/03320

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