Orthotropic remodelling of cancellous bone based on a parametric constitutive model - discussion of local microstructure constraints

P. Kowalczyk[1]

[1] Institute of Fundamental Technological Research, Polish Academy of Sciences, Warsaw, Poland

1. Introduction

Formation, maintenance and repair processes of bone tissues are only possible due to the internal ability of bones to remodel their microstructure. The mechanobiological factors of tissue formation, differentiation and regeneration in bone have been investigated by many authors. Here, the simplified yet well-justified Wolff's hypothesis will be recalled, postulating that processes of remodelling are mainly driven by mechanical factors. Its consequence is e.g. a very efficient distribution of stresses within the bone microstructure.

Attempts to numerically simulate the process of adaptive bone remodelling were reported in the literature since eighties of the last century. Local relations between bone density and stress corresponding to different load cases were formulated in [1,2]. Remodelling was understood as local density changes occuring in response to changed level of local stresses. Another approach was proposed in [3,4] in which remodelling problem was formulated as optimization of density distribution in bone to ensure minimum value of a certain global quality functional postulated as total strain energy. This approach appears in fact to be a generalized problem of structural topology optimization [5]. The approach was reformulated in [6] in terms of time rates, to include bone remodelling in response to loads variable in time.

Initially, publications in the subject were based on the assumption of bone isotropy, with only one parameter (apparent density) describing the bone microstructure. The models did not thus allow to include directionality in the bone microstructure and, in consequence, in its remodelling. Attempts to introduce anisotropy were made since late nineties by e.g. [7,8]. The main problem is definition of appropriate anisotropic microstructural parameters and relating them to elastic material constants.

In this study, a numerical model that allows to simulate the process of anisotropic remodelling of cancellous bone is presented. The bone is treated as continuum with linear elastic anisootropic mechanical properties that result from predefined microstructure topology and parameterized geometric proportions of trabeculae. Elastic constants and relative density are explicitly known functions of the geometric parameters [9]. The microstructure parameters (including orientation directions) are non-uniformly distributed in the bone volume and subject to changes in time according to a remodelling rule that takes the form of minimization of the elastic strain energy rate. Special attention is focused on constraints in the optimization procedure that allow to eliminate unrealistic microstructure instances.

2. Methods

2.1 Parametric constitutive model

The bone microstructure model proposed in [9] is adapted. Cancellous bone is modelled as repeatable cellular microstructure whose representative volume element (cell), presented in Fig. 1, is parameterized by three dimensionless geometric parameters t_c, t_h, t_v, defining proportions between trabecular plate widths and thickness in different directions. The microstructure is transversely isotropic. Different values of the parameters allow to mimic at a good approximation different types of cancellous bone microstructures encountered in real bones. Two Euler angles α_{prec}, α_{nut}, defining the local orientation are considered additional microstructure parameters. Thus, local microstructure is described by five parameter fields $\boldsymbol{\eta}(\mathbf{x})$, $\{\eta_i\} = \{t_c, t_h, t_v, \alpha_{prec}, \alpha_{nut}\}$.

Macroscopic density ρ, surface area density S, and five transversely isotropic elastic constants appearing in the constitutive stiffness tensor $\mathbf{D}$ have been determined numerically for various values of the microstructure parameters $\boldsymbol{\eta}$ [9] and expressed as their tabularized functions (see the website [10]).

2.2 Optimization problem - the remodelling law

Adaptive bone remodelling is postulated as evolution of its microstructure within the occupied domain Ω in a way ensuring the fastest improvement of bone quality at given loading conditions and at certain limitations resulting from bone physiology. In view of the introduced definitions and notation, this means evolution in time of the spatial parameter fields $\boldsymbol{\eta}(\mathbf{x},t)$ corresponding to the fastest decrease of a certain quality functional $G[\mathbf{u},\boldsymbol{\eta}]$ ($\mathbf{u}$ stands for the displacement field) at the above mentioned conditions and limitations. In other words, at each time instant t, the rate of the parameter fields $d\boldsymbol{\eta}(\mathbf{x},t)/dt$ is supposed to minimize the instantaneous rate dG/dt.

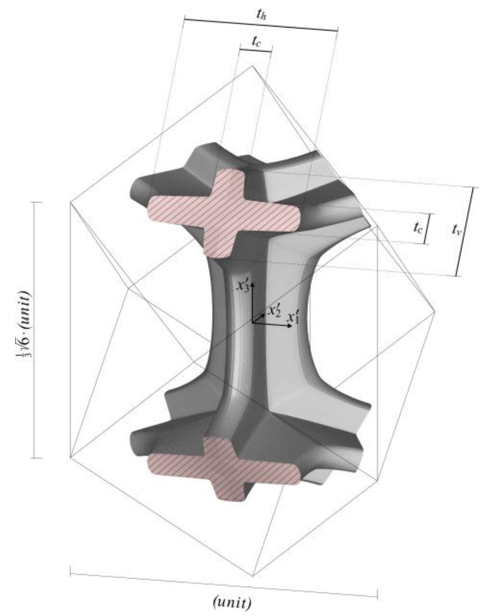

Fig. 1 Geometric model of the representative bone cell, based on a space-filling 14-walled polyhedron.

The quality functional G is postulated as the total strain energy, $G[\mathbf{u},\boldsymbol{\eta}] = \frac{1}{2}\int_{\Omega} u_{i,j}\, D_{ijkl}(\boldsymbol{\eta})\, u_{k,l}\, \mathrm{d}\Omega$. Constraints in this optimization problem are as follows:

- the equilibrium equation, $\forall \delta u_{k,l} \int_{\Omega} u_{i,j}\, D_{ijkl}(\boldsymbol{\eta})\, \delta u_{k,l}\, \mathrm{d}\Omega = \int_{\Omega} b_i\, \delta u_{i,j}\, \mathrm{d}\Omega + \int_{\Gamma} t_i\, \delta u_{i,j}\, \mathrm{d}\Gamma$
- the prescribed global mass constraint, $\int_{\Omega} \rho(\boldsymbol{\eta})\, \mathrm{d}\Omega = M(t)$
- prescribed parameter limits, $\eta_{i\ \min} \le \eta_i \le \eta_{i\ \max}$, $\dot{\eta}_{i\ \min} \le \dot{\eta}_i \le \dot{\eta}_{i\ \max}$
- physiological limitations on tissue aposition/resorption, $\rho_{\min} \le \rho(\boldsymbol{\eta}) \le \rho_{\max}$, $\dot{\rho}_{\min} \le \dot{\rho}(\boldsymbol{\eta}) \le \dot{\rho}_{\max}$
- constraints eliminating unrealistic microstructures

The last group of constraints are necessary in order to make sure that microstructures with extremely thin and wide plates do not appear in the numerical solution. Two such constraints are discussed: limitation on width-to-thickness ratio of trabecular plates and limitation on surface-to-volume ratio in the microstructure volume unit.

3. Results

Evolution of cancellous bone model will be shown on an example of human femur model during the conference presentation. Both density and anisotropy distribution will be discussed and compared to real bone distributions. Influence of the constraint type will be investigated.

4. Discussion

The analysis results appear to conform well to real bone microstructure observations. Details will be discussed during the conference presentation.

References

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