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## ACTIVE VIBROACOUSTIC CONTROL OF BEAMS AND PLATES WITH GENERAL BOUNDARY CONDITIONS

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### 1. Introduction

Active vibroacoustic control of beam and plate structures with arbitrary boundary conditions is considered. The goal is to develop a method of minimizing sound radiation efficiency of such structures. Primary sound field arise as a result of vibrations, due to external disturbances. It is assumed that the control system is compact - it does not contain any additional, ambient microphones. Piezoelectric transducers, mounted on the surface of the controlled object, are used as sensors and actuators. Accurate numerical model of the considered structure is needed to determine optimal parameters of the control system. Theoretical background and the results of numerical and experimental research are briefly introduced.

Due to the fact that it is not possible to give an analytical solution of such problem in general case, it is solved numerically. Eigenfrequencies and the corresponding mode shapes are found using the finite element method. Basing on the derived results and the actuator/sensor equations, the piezotransducers locations that ensure optimal sensing/actuating abilities for specific vibration modes of the structure are determined. The modes are selected taking into account fact that the main purpose of the described study is to minimise acoustic field generated by the vibrating element. It is assumed, that the piezotransducers are rectangle-shaped and their dimensions are given. The resultant radiation efficiency of controlled, vibrating structure is estimated using the Rayleigh integral, assuming that the element is placed in an infinite rigid baffle. Similar analysis is carried out for the beam structures, but, instead of using FEM for modal analysis, one dimensional analytical solutions are applied.

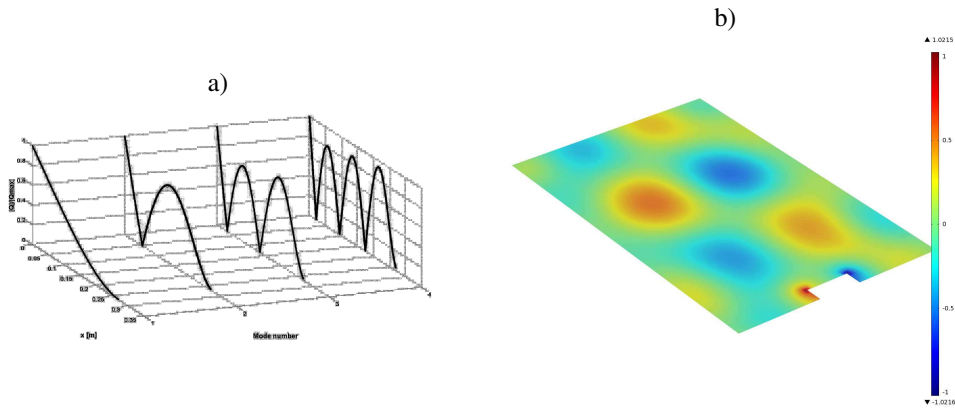
### 2. Acoustic radiation of vibrating beam and plate structures

Due to the undertaken assumptions, classical thin beam and thin plate theories are used for modelling. It is assumed, that vibrations of the structures are caused by external disturbances, that consist of finite number of harmonic forces with different spatial distribution. The response of the structure can be written as a sum of equivalent frequency components, each of which shape is modelled as a finite sum of the eigenmodes. Radiated sound power is calculated independently for each frequency. According to the initial assumptions described in the previous section, the Rayleigh integral is used to calculate the far-field acoustic pressure distribution. Taking into account decomposition of the spatial velocity distribution on the surface of the considered structure into the eigenmode vectors, following expression for the radiated sound power at frequency  $f = \frac{\omega}{2\pi}$  may be written:

$$(1) \quad \Pi = \frac{\rho_o \omega^2}{2\pi c_0} \left| \sum_{n=1}^N \left[ \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \hat{W}_n \left( \iint_S \phi_n e^{-jkx \sin \phi \cos \theta} e^{-jky \sin \phi \sin \theta} dS \right) \sin \phi d\phi d\theta \right] \right|^2,$$

where  $\rho_o$  and  $c_0$  are the density and the speed of sound of surrounding medium (air), respectively,  $S$  denotes the area of considered structure in  $x$  and  $y$  coordinates,  $N$  is the number of considered structural modes and the  $\hat{W}_n$  is complex amplitude of mode  $n$  whereas  $\phi_n$  denotes its normalized shape function.

In the considered low-frequency range and for plate dimensions much lower than the acoustic wavenelgh in air, radiation patterns of structural modes (considered separately) are quite regular,



**Figure 1.** Normalized amplitude of electric charge induced on a **a)** rectangle-shaped piezosensor, as a function of its location on the surface of cantilevered beam for the first four vibration modes **b)** point sensor on a plate surface for an example single vibration mode

close to monopole or dipole source patterns. The „dipole” modes are found to be very weak acoustic radiators. Those observations are important while developing the optimal strategy for the active control system.

### 3. Determining the optimal parameters of the control system and experimental verification

To minimize radiation efficiency of the controlled structure the following steps need to be executed. First, the parameters of the primary disturbance have to be estimated. Piezosensors are used to determine the frequency components and corresponding complex amplitudes of their decomposition into the structure eigenmodes. Then, the optimal feedback gain factors need to be computed for all piezoactuators. The goal is to minimize the total radiated sound power, given by the Eq. (1).

The location of the piezoelectric components on the surface of the controlled structure determines the ability of the active control system to sense and control specific structural modes. For that reason it is very important to properly choose positions of sensors and actuators, while developing system geometry. Results of an example analysis of sensitivity of different located piezoelectric sensors to specific vibration modes is presented on Figure 1.

Different beam and plate structures with piezoelectric elements mounted on the surfaces were used to verify experimentally conclusions obtained with theoretical and numerical investigations.

### 4. References

- [1] C.R. Fuller, S.J. Elliot and P.A. Nelson (1996). *Active Control of Vibration*, Academic Press, London.

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