Lagrangian dynamics based approach for 3D modelling of human gait

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Abstract

The purpose of this study is the mechanism of the spatial movement of a human during walking. The human is considered to be a rigid body with six degrees of freedom, and its mobility is reduced by two constraints. One of the constraints is related to an assumption of constant length of the leg, foot of which is moving along given straight line. The second constraint corresponds to an assumption that human's centre of mass can move only in vertical plane. The dynamic model of the mechanism is built under above assumptions connected to the human gait kinematics. Then, kinetic energy function is derived. In the next step of the study the muscle force in the stance leg, together with its potential, is discussed. It is assumed that this force is piecewise-linear, which can be reasonably approximated by a cubic polynomial. For that purpose a smoothing procedure is proposed and finally with the aid of the Lagrange function dynamic equations for the 3D human gait are formulated. The last part of the paper is devoted to the numerical solution of obtained nonlinear equations, arising from the Lagrange procedure.

Keywords: human walking, gait analysis, lagrangian dynamics

1. Introduction

Human gait analysis attracts researchers from many different fields such as medical sciences, biomechanics or even structural engineering. The first two mentioned areas of research concentrate on the human motion itself, while the latter is related to analysis of loading induced by humans during walking on civil structures among which are flexible floors or footbridges.

The excellent example of this first application is a book by Whittle [1] giving the state of the art of the methods and techniques used for human gait analysis. The simulation of human walking is however a complex problem especially when one tries to build multibody models of a human. Such an approach has been proposed by Garcia-Vallejo and Schiehlen in their work [2]. This complexity is the cause that the other researchers are still working on simpler conceptual models of the human walking such as presented by Yang et al. [3]. Recently, the influence of the human walking on structural response has been presented by Blachowski et al. [4,5].

The purpose of this study is to build a simple model of human walking consistent with Lagrangian mechanics. For that purpose kinematics of walking is proposed first. Then, on the basis of this kinematics fully coupled non-linear equations of motion are formulated.

2. The problem under consideration

The discussed problem consists of the human gait dynamics, described by the motion of mechanism having 2 DOFs describing uniquely position of the COM (Fig. 1). The system is composed of the body of the mass M, and massless legs. The masses of legs are included in the body mass.

The points at which legs contact with ground move along two parallel lines, forming the walking base. Additionally, legs along stride length are at stance phase and swing phase alternately. It is assumed that both phases are of the same length and last the same time.

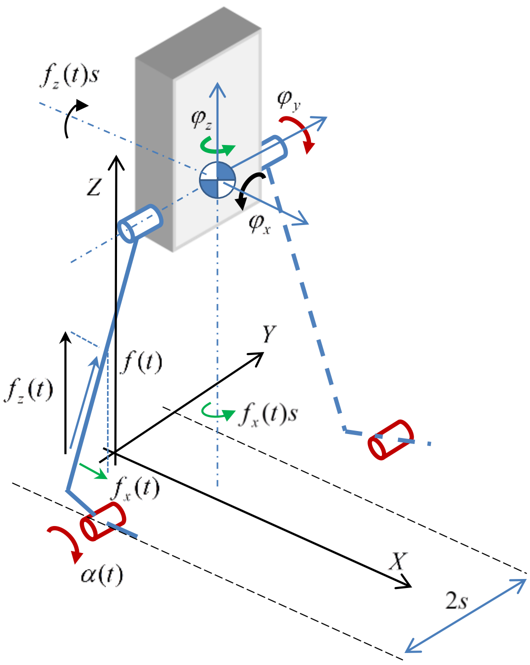

Figure 1: Three-dimensional model of human gait dynamics.

3. Assumptions for the human gait model

The described motion starts with the left leg, at the stance phase. At the same time t the right leg is in the swing phase and the toe-off position.

Muscles of the left limb, acting with a time varying force *f(t)*, lift the heel by its rotation around the horizontal axis, passing through point *A* (Fig. 3). The maximum of the force occurs at the moment when the vector of the gravity force *Mg* reaches the support point *A*, and angle α reaches α_0 value. After that, the force *f(t)* decreases rapidly, to its zero value (Fig. 2).

After passing point *A*, the gravity force is the main active force, driving the motion, causing a kind of a free fall of the body. The free fall ends when the right leg comes to its stance phase. At the same time the left leg enters its swing phase.

In the 5th section, equations of motion, separately for the left and right leg, are derived. It is assumed, that both phases - stance and swing last the same time t_0. The assumption is justified by the time measurements presented in the monograph [1].

After the end of the stance phase of the left leg, the stance phase of the right leg starts. This is the moment when the proposed algorithm switches from equations of motion for the left leg to the motion equations of the right leg. In other words, for each even number of t_0 the left leg motion equations hold, and for odd number of t_0 the right leg equations are valid. The velocity of COM $v_0 = dx/dt$, for all nt_0 (*n* natural number), is given.

Summarizing the above assumptions, the following kinematic equations describing motion of the COM can be written:

$$\begin{cases} x(t) = b(1-\cos\alpha(t)) + h\sin\alpha(t) \\ y(t) = 0 \\ z(t) = b\sin\alpha(t) + h\cos\alpha(t) \end{cases} \tag{1}$$

4. Smoothing procedure of muscle forces

Muscle forces of an individual limb have the following form

$$\begin{cases} f_1(t) = Mg\,x(t)/b, & 0 \le x(t) \le b \\ f_2(t) = Mg\dfrac{(x(t)-b)}{(\frac{4}{3}b - b)}, & b < x(t) \le \frac{4}{3}b \\ f_3(t) = 0, & \frac{4}{3}b < x(t) \le 2b \end{cases} \tag{2}$$

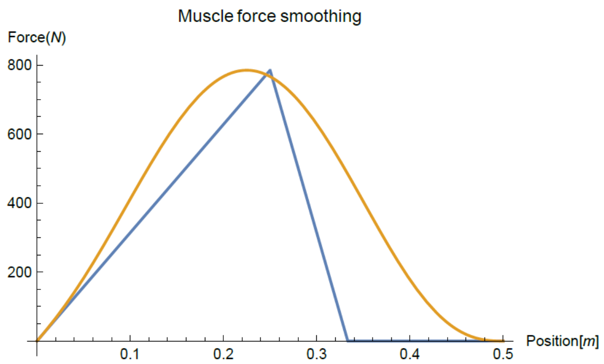

Figure 2: Piecewise-linear vs smooth muscle force.

For the sake of simplicity we assumed that the above muscle force function can be approximated by a cubic polynomial. The coefficients are chosen such that the square root difference between the piecewise-linear function and the cubic polynomial representation is minimal.

5. Lagrange equations for the proposed human gait model

In the final step of our study we followed the classical Lagrange formalism. Based on the kinematics represented by Eqs. (1) we determined the kinetic *T* and potential energy *V* of the system.

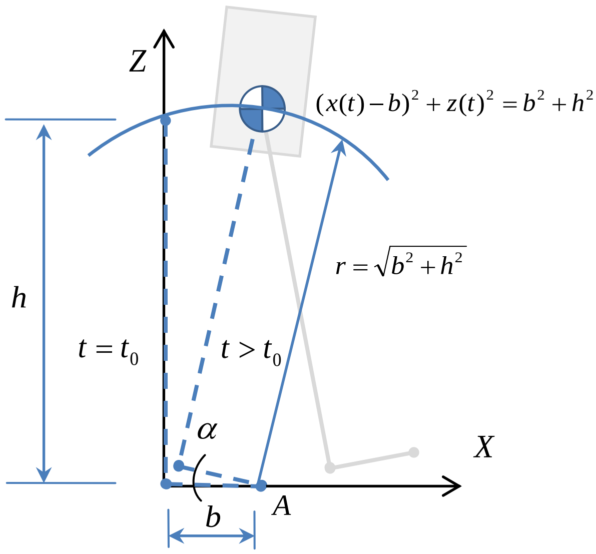

Figure 3: Centre of mass (COM) trajectory.

Next, we substituted the energies into the Lagrange function

$$L(\mathbf{q},\dot{\mathbf{q}}) = T(\mathbf{q},\dot{\mathbf{q}}) - V(\mathbf{q}) \tag{3}$$

where $\mathbf{q}(t)$, $\dot{\mathbf{q}}(t)$ denotes generalized coordinates and generalized velocities, respectively. Eventually, Lagrange's equations have the well-known form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right) = \frac{\partial L}{\partial \mathbf{q}} \tag{4}$$

Further investigations will be directed towards numerical time integration of the obtained equations of motion. For that purpose the computer method adjusted to the problem will be used.

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