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ABSTRACT BOOK

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RELIABILITY-BASED APPROCHES FOR TOPOLOGY OPTIMIZATION OF ELASTOPLASTIC STRUCTURES

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Summary Plasticity in topology optimization is almost forgotten direction in this popular research area. This paper presents a recently developed elasto-plastic topology optimization procedure extended with reliability constraint. It recalls fundamental concepts from reliability analysis and introduces an algorithm for topology optimization of elasto-plastic structures. In addition to the elasto-plastic constitutive law of the applied material the optimization algorithm includes stress constraints, as well. The presented numerical examples show dependence of the volume fraction on probability of failure.

INTRODUCTION

An important aspect of any optimization process is robustness to variability of structural parameters, either structural (geometry, material) or boundary condition (loading, support location) dependent. The more the structure becomes optimal, the lower the resistance to its parameter changes. One of the possible ways to tackle this issue is to add to the optimization formulation an additional constraint for the probability of failure [2, 5]. The designer will then assure that the optimized structure does not go below the assumed safety level. Since the probability of failure of engineering structures must be small (approximately 0.0001 or smaller), it is possible to obtain a relatively fast estimation of reliability by using first or second order methods [4, 9] or different simulation procedures. Most often, several iterations (finite element solutions) are enough to obtain convergence. Recent advances in probability/reliability based topology problems have been presented in papers by Guest et al. [3] Luo et al. [7], Xia et al. [11], da Silva et al. [8]. The authors of this paper have also several papers in this topic for more than a decade (Logo et al. [5, 6], Blachowski et al. [2]).

Topology optimization is a rather time consuming and discretization dependent computational problem. Due to the complexity of the task it is fundamental to use appropriate topology optimization procedure (Antonietti et al. [1]). In the present paper an iterative topology optimization algorithm together with elasto-plastic material formulation and reliability approach will be shortly described. The formulation based on a recently developed functor-oriented optimization procedure (Tauzowski et al. [10)]). Analysis of elasto-plastic structure will be presented on numerical example. All aspects of numerical analysis, finite element formulation, topology optimization as well as reliability analysis library are performed by our own software implemented in MATLAB and C++.

RELIABILITY BASED TOPOLOGY OPTIMIZATION APPROACH

In the case of probabilistic topology optimization random variables vector $\mathbf{r}$ can represent loads or material constants. Shape of the structure is a result of topology optimization therefore random nature of shape parameter is not taken into consideration. In the following the formulation of the elasto-plastic problem of structural analysis as well as the detailed description of the algorithm for topology optimization under reliability constraint is presented briefly. The proposed approach utilizes elastoplastic finite element analysis together with optimality criteria based on sequential removal of the least stressed elements. The whole problem is a standard stress limited topology optimization method extended by a reliability constraint which here is based on a first order reliability method. The stress constrained topology optimization works with an objective function representing the volume of the material required to safely carry the applied loading. The density of the material at a given point in three dimensional space is described by an indicator function $\chi(\mathbf{x})$, taking values 0 or 1. We are looking for a solution to the above problem within a set of kinematically admissible displacement fields V. Additionally, constraints are imposed on stresses $\sigma(\mathbf{x})$ at any the given point of the design domain Ω.

$$
\begin{aligned}
&\min_{\chi,\mathbf{u}\in V} \quad \int_{\Omega} \chi(\mathbf{x})\mathrm{d}\Omega \\
&\text{s.t.} \quad \int_{\Omega} \chi\boldsymbol{\varepsilon}(\mathbf{u}):\mathbf{D}:\boldsymbol{\varepsilon}(\mathbf{v})\mathrm{d}\Omega - \int_{\Gamma_n} \bar{\mathbf{t}}\cdot\mathbf{v}\mathrm{d}\Gamma = 0, \quad \mathbf{v}\in V \qquad (1)\\
&\qquad \frac{|\sigma(\mathbf{x})|}{\sigma_0} - 1 \le 0, \quad \mathbf{x}\in\Omega_{\mathrm{mat}}\\
&\qquad \chi(\mathbf{x}) = \begin{cases} 1, & \mathbf{x}\in\Omega_{\mathrm{mat}} \\ 0, & \mathbf{x}\in\Omega\backslash\Omega_{\mathrm{mat}} \end{cases}
\end{aligned}
$$

where $\mathbf{u}(\mathbf{x})$, $\mathbf{v}(\mathbf{x})$, $\boldsymbol{\varepsilon}(\mathbf{u})$ represent displacement, virtual displacement and strain fields, respectively. $\mathbf{D}$ is the elasto-plastic material matrix, $\bar{\mathbf{t}}$ is the traction on the boundary, σ_0 is the stress limit and finally Ω_{mat} represents the material domain defined as $\Omega_{\mathrm{mat}} := \{\mathbf{x}\in\Omega \mid \chi(\mathbf{x}) = 1\}$.

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For sake of simplicity the reliability problem is based on a first order reliability method. Probabilistic task is transformed to standard normal space $\mathbf{r} \rightarrow \mathbf{w}$. The most probable point is the closest point to the centre of the coordinate system of this space, which lies on the limit state surface. Therefore determination of this point is optimization problem formulated as follow:

$$\min \quad \|\mathbf{w}^2\| = \mathbf{w} \cdot \mathbf{w}^{\mathrm{T}}, \qquad \text{s. t.} \quad g(\mathbf{w}) = 0. \tag{2}$$

To tackle this problem Rackwitz and Fiesler [9] proposed the following gradient-based iterative scheme:

$$\mathbf{w}^{(n+1)} = \frac{1}{\|\nabla g(\mathbf{w}^{(n)})\|^2}\left(\nabla g\left(\mathbf{w}^{(n)}\right)^{\mathrm{T}}\mathbf{w}^{(n)} - g\left(\mathbf{w}^{(n)}\right)\right)\nabla g\left(\mathbf{w}^{(n)}\right). \tag{3}$$

In addition to the above formulation the load carrying capacity (P_{Limit}) of the elastoplastic structures is integrated as a first order reliability constraint. The constraint inequality is:

$$P_{\text{FORM}} > P_{\text{Limit}} \tag{4}$$

where $P_{\text{FORM}} = \Phi(-\beta)$ is cumulative distribution function. The reliability procedure above can be interchanged any appropriate formulation (second order reliability method or different simulation techniques).

NUMERICAL RESULTS

To demonstrate the above proposed algorithm the reliability assessment in topological optimization will be illustrated on a simple example shown in Figure 1. The reliability problem is based on a FORM. Regular rectangular mesh composed with four-node Lagrange finite elements is used in the example. The following parameters are used: Young's modulus $E = 71$ GPa, Poisson's ratio $\upsilon = 0.11$, yield stress $\sigma_0 = 260$MPa, thickness $t = 0.22$ units. 10% standard variation is used in case of the Poisson's ratio while the Young's modulus and the yield stress can vary by 5%. Limit state function reflects displacement condition. Unsafe state means the displacement u_c at point c (see Fig. 1.) exceeds permissible value = 0.1 units (l/200). The optimal stress distribution and the topology is shown at right.

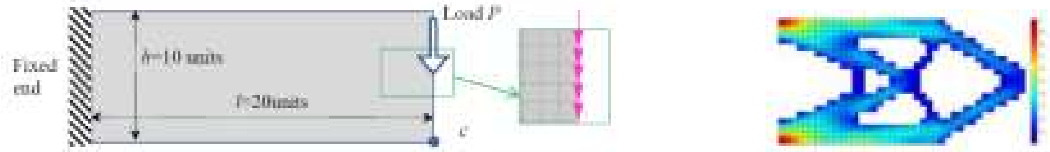

Figure 1. Test problem and the optimal topology with reliability constraint $\mathrm{P_f} = 0.005$.

CONCLUSIONS

In this paper a plasticity based topology optimization procedure with reliability constraint is presented. In the example optimal topology under reliability constraint is shown. Detailed analysis of the results will be discussed and it will be shown that the probability of failure rapidly grows above certain level of volume fraction.

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