New Shear Horizontal (SH) Surface Acoustic Waves Propagating at the Interface Between Two Elastic Half-Spaces

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Abstract*—It is commonly agreed that shear horizontal (SH) surface acoustic waves cannot exist on an elastic half-space or at the interface between two different elastic half-spaces. However, in this paper (inspired by the newly developed elastic metamaterials) we will show that SH surface elastic waves can propagate at the interface between two elastic half-spaces, providing that one of them is a metamaterial half-space with a negative elastic compliance $s_{44}(\omega)$. In addition, if $s_{44}(\omega)$ changes with frequency ω as the dielectric function $\varepsilon(\omega)$ in Drude's model of metals, then the proposed SH ultrasonic waves can be considered as an acoustic analogue of Surface Plasmon Polariton (SPP) electromagnetic waves propagating at the metal-dielectric interface. Analytical expressions for the dispersion equation, phase and group velocities of the new SH elastic surface wave were developed. The newly discovered SH elastic surface wave inherits many of extraordinary properties of SPP electromagnetic waves such as: strong subwavelength concentration of the wave field in the proximity of the guiding interface, low phase and group velocity etc. Therefore, the proposed new SH ultrasonic surface waves can potentially be used in: a) near-field subwavelength acoustic imaging, b) acoustic sensors with extremely large mass sensitivity, c) wave trapping (zero group and energy velocity) and d) non-reciprocal and topological waveguides.

Keywords—Shear Horizontal (SH) surface acoustic waves, Surface Plasmon Polaritons (SPP), Dispersion equation, Phase velocity, Group velocity

I. INTRODUCTION

The subject of this study will be a remarkable discovery of a new type of elastic surface waves propagating at the interface between two elastic half-spaces, one of which is a metamaterial.

The newly discovered Shear Horizontal (SH) elastic surface waves have only one component of mechanical displacement in the direction perpendicular to the direction of propagation and parallel to the interface.

At the beginning it will be instructive to ask the following question: "How often new types of elastic surface waves appear in the theory of elastodynamics"? The answer to this question can be discerned from Table 1 presented above [1-9].

TABLE I. MAIN TYPES OF ELASTIC SURFACE WAVES DISCOVERED IN RECENT 140 YEARS

No	Discovered by	Year of discovery	Waveguide
1	Lord Rayleigh	1885	elastic half-space
2	A.E.H. Love	1911	layered elastic half-space
3	H. Lamb	1917	free plate
4	R. Stoneley	1924	interface between two elastic half-spaces
5	J.G. Sholte	1947	interface between elastic half-space and liquid
6	J.L. Bleustein, Y. Gulyaev	1968	piezoelectric half-space
7	C. Maerfeld, P. Tournois	1971	interface between two piezoelectric half-spaces
8	P. Kiełczyński	2022	interface between metamaterial and elastic half-space

Indeed, according to Table 1 the discovery of the newest type of elastic surface waves (2022) and the precedent type of surface waves (1971) is separated by a time gap of 51 years.

The newest type of SH elastic surface waves discovered in 2022 is characterized by one crucial feature, i.e., the newly discovered elastic surface waves are exact analogue of electromagnetic surface wave of the Surface-Plasmon-Polariton type, which propagate at the metal-dielectric interface [10-12].

These two types of waves have similar: 1) distribution of wave fields, 2) dispersion equations etc. A very intriguing property of the new SH ultrasonic wave is that it slows down, i.e., the phase and group velocities tend to zero as the wave frequency approaches the surface resonant frequency $\omega \rightarrow \omega_{sp}$ (see Figs. 2 and 3). This property is crucial in potential applications of the new SH ultrasonic wave in near-field acoustic microscopy with subwavelength resolution.

The newest SH elastic surface waves discovered in 2022 have a number of unique properties, such as:

1) deeply subwavelength ($\sim\lambda/20$) penetration depth

2) high concentration of energy in the vicinity of the guiding interface

3) very low phase and group velocities tending simultaneously to zero ($\rightarrow 0$)

4) relatively simple mathematical model.

Consequently, the newly discovered (2022) elastic surface waves have a huge potential for novel applications, such as:

1. near-field subwavelength acoustic imaging
2. sensors with giant sensitivity
3. wave trapping (zero group and energy velocity)
4. non-reciprocal and topological waveguides.

II. Physical Model

The geometry of the waveguide supporting new SH elastic surface waves is sketched in Fig.1. The waveguide consists of two semi-infinite elastic half-spaces, one of which is a conventional elastic material ($x_2 \geq 0$) and the second an elastic metamaterial ($x_2 < 0$) with a negative elastic compliance $s_{44}^{(1)}(\omega) < 0$, which is a function of angular frequency ω. By contrast, the densities $(\rho_1, \rho_2) > 0$ in both half-spaces as well as the elastic compliance $s_{44}^{(2)} > 0$ in the conventional elastic material are positive and frequency independent (see Fig.1).

Two elastic half-spaces, rigidly bonded at the interface $x_2 = 0$, are uniform in the direction x_3, therefore all field quantities of the new SH elastic surface wave will vary only along the transverse direction x_2, i.e., as a function of distance from the guiding interface $x_2 = 0$. It is assumed that both half-spaces of the waveguide are linear and lossless.

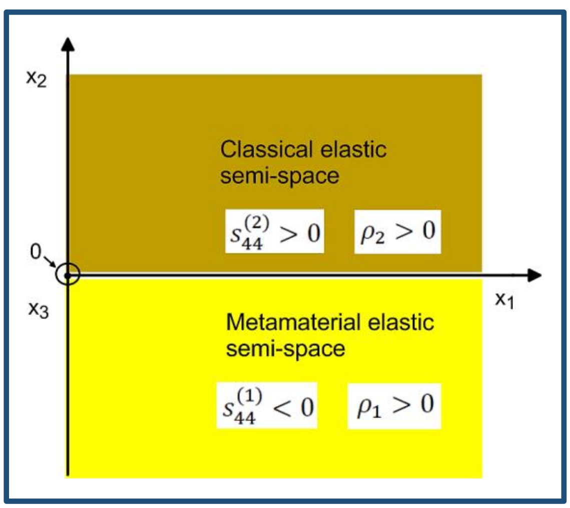

Fig. 1. Cross-section of the waveguide supporting the newly proposed SH elastic surface waves, propagating in the direction x_1, with exponentially decaying fields in the transverse direction x_2. The conventional elastic half-space ($x_2 \geq 0$) is rigidly bonded to the metamaterial elastic half-space ($x_2 < 0$) at the interface $x_2 = 0$. Mechanical displacement u_3 of the new SH elastic surface waves is polarized along x_3.

A. *Elastic compliance $s_{44}^{(1)}(\omega)$ in the metamaterial half-space ($x_2 < 0$)*

The important assumption made throughout this paper is about the elastic compliance $s_{44}^{(1)}(\omega)$ in the metamaterial half-space ($x_2 < 0$). Namely, it is assumed that $s_{44}^{(1)}(\omega)$, as a function of angular frequency ω, is given explicitly by the following formula:

$$s_{44}^{(1)}(\omega) = s_0\left(1 - \frac{\omega_p^2}{\omega^2}\right) \quad (1)$$

where: ω_p is the angular frequency of the local mechanical resonators in the metamaterial and s_0 is its reference elastic compliance for $\omega \rightarrow \infty$.

It is not difficult to notice that the elastic compliance $s_{44}^{(1)}(\omega)$ given by Eq.1, is formally identical to the dielectric function $\varepsilon(\omega)$ in Drude's model of metals [13], in which the angular frequency ω_p is named the angular frequency of bulk plasmon resonance [14].

Similarly, the density ρ_1 of the metamaterial half-space ($x_2 < 0$) corresponds to the magnetic permeability μ in Drude's model of metals.

The second elastic half-space ($x_2 > 0$) is a conventional elastic material with a positive compliance $s_{44}^{(2)} > 0$ and density ρ_2 that are both frequency independent.

III. Theory

Love surface waves have only one SH (shear-horizontal) component of the mechanical displacement u_3 polarized perpendicular to the direction of propagation x_1 and parallel to the surface (along the x_3 axis), see Fig.1.

A. *Mechanical Displacement and Shear Stresses*

The energy of the new SH surface wave is concentrated in the proximity of the interface. For the new wave to be a surface wave, its mechanical displacement u_3 should decrease exponentially with increasing distance from the interface ($x_2 = 0$), namely:

1) variation of $u_3^{(1)}$ along with the shear stress $\tau_{23}^{(1)}$ in the lower elastic metamaterial half-space ($x_2 < 0$):

$$u_3^{(1)}(x_1, x_2, t) = A \cdot exp(q_1 x_2) \cdot exp[j(Kx_1 - \omega t)] \quad (2)$$

$$\tau_{23}^{(1)} = \frac{1}{s_{44}^{(1)}} A \cdot q_1 \cdot exp(q_1 x_2) \cdot exp[j(Kx_1 - \omega t)] \quad (3)$$

2) variation of $u_3^{(2)}$ along with the shear stress $\tau_{23}^{(2)}$ in the upper conventional elastic half-space ($x_2 > 0$):

$$u_3^{(2)}(x_1, x_2, t) = B \cdot exp(-q_2 x_2) \cdot exp[j(Kx_1 - \omega t)] \quad (4)$$

$$\tau_{23}^{(2)} = \frac{1}{s_{44}^{(2)}} B \cdot (-q_2) \cdot exp(-q_2 x_2) \cdot exp[j(Kx_1 - \omega t)] \quad (5)$$

where: q_1 and q_2 are the transverse wavenumbers of the new SH elastic surface wave. They should be positive to ensure that the new SH wave is a surface wave. K is the wave number (wave vector) of the new elastic surface wave and ω is the angular frequency. A and B are constants.

B. Governing Equations

The mechanical displacements of the surface wave: $u_3^{(1)}$ in the lower metamaterial elastic half space and $u_3^{(2}$ in the upper conventional elastic half-space satisfy the following equations of motion, respectively:

$$\frac{1}{v_1^2}\frac{\partial^2 u_3^{(1)}}{\partial t^2} = \frac{\partial^2 u_3^{(1)}}{\partial x_1^2} + \frac{\partial^2 u_3^{(1)}}{\partial x_2^2} \tag{6}$$

$$\frac{1}{v_2^2}\frac{\partial^2 u_3^{(2)}}{\partial t^2} = \frac{\partial^2 u_3^{(2)}}{\partial x_1^2} + \frac{\partial^2 u_3^{(2)}}{\partial x_2^2} \tag{7}$$

C. Boundary Condidtions

The continuity of a) mechanical displacement u_3 and b) shear stress τ_{23} on the interface ($x_2 = 0$) of two half-spaces, should be provided, namely:

$$u_3^{(1)}\big|_{x_2=0} = u_3^{(2)}\big|_{x_2=0} \tag{8}$$

$$\tau_{23}^{(1)}\big|_{x_2=0} = \tau_{23}^{(2)}\big|_{x_2=0} \tag{9}$$

D. Dispersion Equation

By introducing Eq.2 into Eq.6 and Eq.4 into Eq.7, we obtain the analytical formulas for the transverse wavenumbers q_1 and q_2 as a function of the longitudinal wavenumber K. Subsequently, we substitute equations 2-5 to the boundary conditions Eqs.8 and 9. As a result, we obtain a system of two linear and homogeneous equations for the coefficients A and B. Zeroing of the determinant of the system leads to the dispersion equation of a new elastic wave.

After some algebraic manipulations, we arrive at the following dispersion equation for a new SH elastic surface wave:

$$K(\omega) = \omega \cdot \sqrt{\frac{s_{44}^{(2)} \cdot s_{44}^{(1)}(\omega)}{\left(s_{44}^{(2)} + s_{44}^{(1)}(\omega)\right)}} \cdot \sqrt{\frac{s_{44}^{(2)} \cdot \rho_1 - s_{44}^{(1)}(\omega) \cdot \rho_2}{\left(s_{44}^{(2)} - s_{44}^{(1)}(\omega)\right)}} \tag{10}$$

More details concerning the new SH wave and the derivation of the dispersion equation can be found in [15].

E. Phase Velocity

The analytical expression for the phase velocity $v_p(\omega)$ of the surface wave can be directly derived from Eq.10.

$$v_p(\omega) = \frac{\omega}{K} = \sqrt{\frac{\left(s_{44}^{(2)} + s_{44}^{(1)}(\omega)\right)}{s_{44}^{(2)} \cdot s_{44}^{(1)}(\omega)}} \cdot \sqrt{\frac{\left(s_{44}^{(2)} - s_{44}^{(1)}(\omega)\right)}{\left(s_{44}^{(2)} \cdot \rho_1 - s_{44}^{(1)}(\omega) \cdot \rho_2\right)}} \tag{11}$$

F. Group velocity

The analytical expression for the group velocity $v_{gr}(\omega)$ of the new SH elastic surface wave has been developed by differentiating the formula (10) with respect to ω.

$$v_{gr}(\omega) = \frac{d\omega}{dK} =$$

$$\frac{1}{v_p(\omega)} \frac{2 \cdot \left(\left(s_{44}^{(2)}\right)^2 - \left(s_{44}^{(1)}(\omega)\right)^2\right)}{2 s_{44}^{(2)} s_{44}^{(1)}(\omega)\left(s_{44}^{(2)} \cdot \rho_1 - s_{44}^{(1)}(\omega) \cdot \rho_2\right) - \omega s_{44}^{(2)} \frac{d s_{44}^{(1)}(\omega)}{d\omega} \left\{\left(s_{44}^{(2)} \cdot \rho_1 - 2 s_{44}^{(1)}(\omega) \cdot \rho_2\right) + 2\left(s_{44}^{(1)}(\omega)\right)^2 \frac{\left(s_{44}^{(2)} \cdot \rho_1 - s_{44}^{(1)}(\omega) \cdot \rho_2\right)}{\left(s_{44}^{(2)}\right)^2 - \left(s_{44}^{(1)}(\omega)\right)^2}\right\}} \tag{12}$$

IV. RESULTS

Numerical calculations were performed on the example of the waveguide structure in which the upper half-space is made of PMMA polimer and the lower half-space is based on ST Quartz with embedded local oscillators. We assume that, the frequency of the local oscillators equals 1 MHz. Losses in the waveguide structure are neglected. The exact values of the material parameters used in the calculations can be found in [15].

A. Phase Velocity

Using the formula (11), the graph of the phase velocity $v_p(\omega)$ as a function of frequency f was calculated and presented in Fig. 2. In our calculations the surface resonant frequency $f_{sp} = f_p / \sqrt{1 + s_{44}^{(2)}/s_0}$ is equal to 143.569 kHz.

B. Group Velocity

Figure (3) shows the relation between the group velocity $v_{gr}(\omega)$ of the new wave and the wave frequency f. The group velocity was calculated using formula (12).

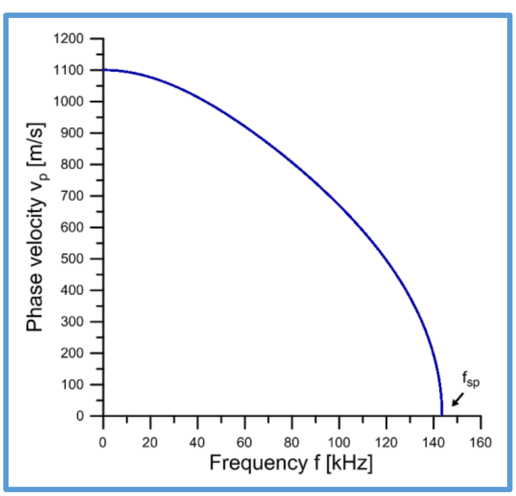

Fig. 2. Phase velocity of the new wave versus frequency.

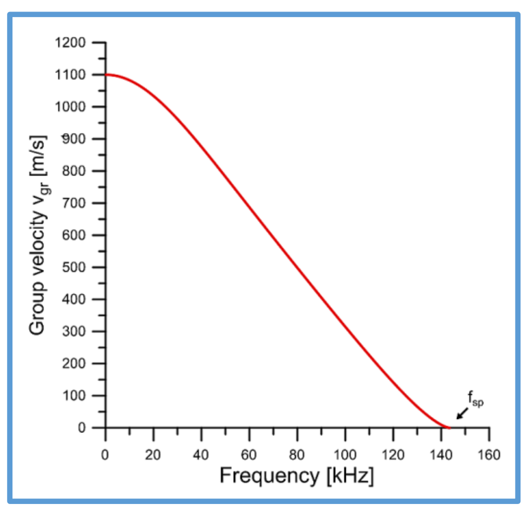

Fig. 3. Group velocity of the new wave versus frequency.

C. Dispersion Curve

The dispersion curve (i.e., wave frequency f versus wavenumber K) was evaluated using the formula (10). Figure 4 shows a plot of the dispersion curve of a new SH elastic wave.

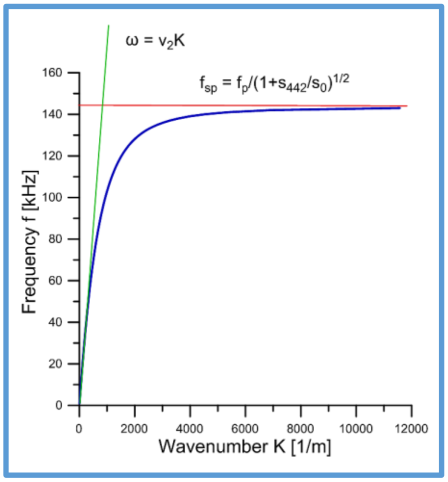

Fig. 4. Dispersion curve ($f - K$) of new SH acoustic surface waves.

B. Mechanical Displacement

Figure 5 displays the distribution of the mechanical displacement $u_3(x_2)$ of the new SH elastic surface wave as a function of the distance $|x_2|$ from the guiding interface.

V. Conclusions

Based on the results of research presented in this paper, we can draw the following conclusions:

1. We proved that the new SH elastic surface waves can exist at the interface of two elastic half-spaces one of which is an elastic metamaterial with a negative compliance $s_{44}^{(1)}(\omega) \cdot s_{44}^{(2)} < 0$

2. The new SH elastic surface waves can be considered as an elastic analogue of the electromagnetic SPP waves, due to strong formal similarities of their mathematical models

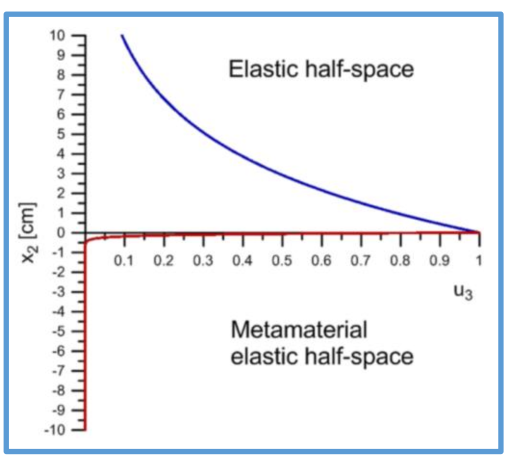

Fig. 5. Distribution of the mechanical displacement of the new elastic wave along the vertical axis x_2.

3. A very intriguing property of the new SH wave is that its phase and group velocities slow down and tend simultaneously to zero when the wave frequency approaches the surface resonant frequency f_{sp} (wave trapping)

4. The new elastic surface wave is characterized by a strong concentration of energy in the proximity of the interface. From this reason, the newly discovered elastic surface wave can be applied in sensors with extremely large mass sensitivity and near-field acoustic microscopy

5. Newly discovered SH elastic surface wave can enhance the evanescent waves, Due to this property, the new elastic waves can be applied in superlensing and subwavelength acoustic imaging.

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