Optimization Problem of the size-scale for a Foldable Chain Scissors Structure based on Stress Analysis

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Abstract. Emergency bridges are used to restore the lifeline of damaged bridges after disasters. However, the design specification of existing emergency bridges do not afford rapid bridging. Therefore, a trial deployable bridge (Scissors Bridge) using a scissors structure that folds compactly has been experimentally produced. But, the assembly process for the bridge has not been considered and there are no practical design examples or design methods for scissors bridges. In this paper, as for existing bridges, we established a design method of scissors bridges when considering the live load. In addition, a general-purpose member cross-sectional dimension optimization method, aimed at minimizing weight, was developed and proposed. Considering the problem of insufficient strength of the scissors bridge, the optimum reinforcing pattern and its cross-sectional dimensions were determined through two methods of reinforcement and optimization of the cross-sectional dimension of the member. Finally, a trial of practical design calculation was performed using the results of the study to determine whether a scissors bridge satisfying the standard of the Specifications for Highway Bridges can be designed.

1. Introduction

1.1. Background

Temporary emergency bridges are designed to be transported and assembled quickly for use in situations such as construction sites and disaster areas. Some temporary bridge structures include the truss bridge and girder bridge as module members. By adopting the module member, it is possible to adjust the function of a bridge by changing the combination according to the actual site condition and necessary performance. The truss bridge resists axial force, while the girder bridge resists shear force and bending moment as a sectional force of each other. The rigid body is a form having the structural characteristics of both these structures. The scissors bridge is a pre-assembled bridge with the main mechanism of the module structure of a pin-rigid body, in which the ramen body is connected by the pin. The application of the scissors mechanism enables collective transportation, prompt development of cross-linking at construction sites, reduced number of workers, and decreased assembly processes. With these advantages, the prototype of the scissors bridge with typical problems is being improved through research and development.

1.2. Structural optimization theory

The morphological design of a structure composed of discrete framework structural elements can be classified into a phase determination problem and a dimension determination problem. The phase determination problem is a matter of determining the necessary elements and geometrical arrangement relationship in the three-dimensional space. The dimension determination problem is a matter of determining dimensions such as length and cross sectional area in the three-dimensional space. In general, the cost of a long, large structure such as a bridge is proportional to the weight. Accordingly, bridges often involve the minimum weight design problem. Similarly, the frame structure of the scissors bridge is required to be lightweight and highly rigid. To achieve the minimum weight of the structure, the stress of each member of a certain structure (σ_i) and the displacement of the point of interest (δ) are decreased to certain constraint values (σ_a, δ_a). The formulation of this structural system is represented below;

$$
\begin{aligned}
\text{Minimize} \quad & W \\
\text{subject to} \quad & \sigma_i \leq \sigma_a \\
& \delta \leq \delta_a
\end{aligned}
\tag{1}
$$

Complex optimization concepts have the problems of uniqueness of solutions and multiple local minimum, and it is necessary to apply a metaheuristic optimization method to search for global optimal solutions or their high-accuracy approximate solutions. This class of optimization algorithms, including differential evolution (DE), has been successfully applied not only to simple truss systems but also to larger and more complex engineering problems.

1.3. Purpose of research

The scissors structure is preassembled and has been used in fields that emphasize functionality in a space structure under non-gravity. Therefore, its structural form is considered to be weak under the force of gravity. In other words, the scissors structure is a new approach as a bridge structure, and generalization of the equilibrium mechanics theory including establishment of the design calculation method, introduction of calculation examples, etc., still require further research. In addition, to efficiently improve the strength of the scissors bridge, it is necessary to construct a truss reinforcement and member sectional dimension optimization method. Therefore, in this study, we aim to develop a weight minimization program using equilibrium dynamics theory, and to propose a new scissors bridge with a new cross section according to the optimum reinforcing pattern and cross sectional design. The proposed bridge should simultaneously have design load bearing capacity and minimum weight satisfying the road bridge specification standard. In other words, we aim to solve problems such as the lack of proof strength by optimizing cross section dimensions of the main structure and strut reinforcement.

2. Sectional dimension optimization algorithm

In this optimization method, by incorporating equilibrium dynamics theory and differential evolution (DE), analysis of member forces and displacements, modification of design variables, and minimization of the objective function were performed. Furthermore, the optimization method was intended not only for the non-reinforced scissors structure but also the higher-order unstable scissors structure and the unstable truss structure considering the reinforcing member.

There are three features of this opyimization analysis. The first is the stable obtaining an optimum solution by manipulating the rate of modifying the design variable by using the return rate parameter. The second is to simplify the procedure by dividing the stress and displacement analysis into the first half and second half of the maximum iteration number (ν_{fin}). The third is to correct the variables efficiently by introducing the sensitivity analysis of each design variable to displacement analysis.

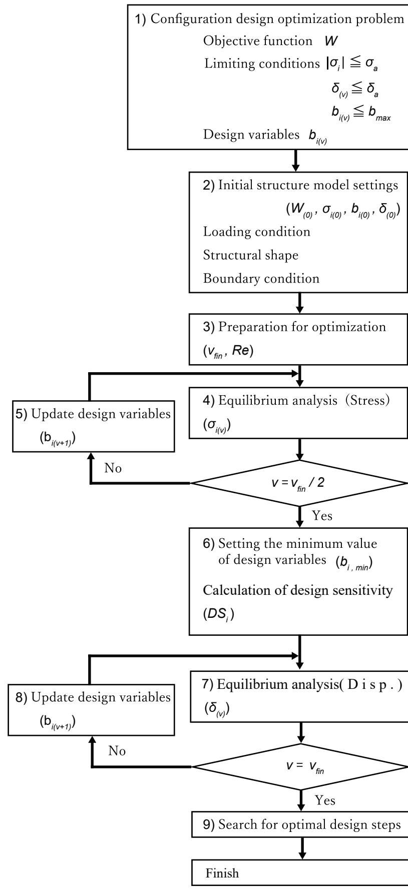


Figure 1. Example of proposed optimization procedure

2.1. Present Optimization flow

The proposed optimization flow is shown in **Fig.1**, and the procedure for this method will be described below.

1) Setting of optimization problem
   We designed a scissors structure composed of n members with minimum weight design under stress constraints and displacement constraints. By setting permissible values for each member stress, central displacement, and design variable, this optimization can be formulated from equation (2) as follows.

$$W = \sum_{i=1}^{n} \gamma_i A_{i(\nu)} l_i = \sum_{i=1}^{n} \gamma_i b_{i(\nu)}^2 L_i \quad \longrightarrow \quad Min \tag{2}$$

s.t.

$$\begin{aligned} |\sigma_i| - \sigma_a &\leq 0 \\ \delta_{(\nu)} - \delta_a &\leq 0 \\ b_{i(\nu)} &\leq b_{max} \end{aligned}$$

2) Preparation of initial model
   We set the number of iterations (ν_{fin}), maximum cross section (b_{max}), aspect ratio (c), hollow ratio (G), structural material (Young's modulus: E, specific gravity: γ_i, allowable

stress: σ_a), structure shape (member length: L_i, deployment angle: θ, allowable deflection: δ_a), load condition, and boundary condition.

3) Preparation for optimization
We set the initial variable value of the section dimension of the member ($b_{i(0)}$), reduction ratio ($Re_{1(1)}, Re_{2(1)}$), and tuning parameter ($C, r, T, \kappa_1, \kappa_2$).

4) Mechanical analysis (stress)
The stress value of each member ($\sigma_{i(\nu)}$) is calculated using the equilibrium mechanics theory. In addition, when considering buckling, calculate the allowable buckling stress.

5) Modification of design variable (stress)
The design variables are corrected according to the stress of each member and the allowable stress value is as follows;

$$b_{i(\nu+1)} \begin{cases} = (1 - Re_{1(\nu)} \cdot Rs_{i(\nu)}) \cdot b_{i(\nu)} + Re_{1(\nu)} \cdot Rs_{i(\nu)} \cdot \frac{\sigma_{i(\nu)}}{\sigma_{a,i(\nu)}} \cdot b_{i(\nu)} \\ = b_{i(\nu)} \left\{1 + Re_{1(\nu)} \cdot Rs_{i(\nu)} \left(\frac{\sigma_{i(\nu)}}{\sigma_a} - 1\right)\right\}, \quad \sigma_{i(\nu)} \leq \sigma_{a,i(\nu)} \\ \\ = (1 - Re_{1(\nu)} \cdot Rs_{i(\nu)}) \cdot b_{i(\nu)} + Re_{1(\nu)} \cdot Rs_{i(\nu)} \cdot C \cdot b_{i(\nu)} \\ = b_{i(\nu)}\{1 + Re_{1(\nu)} \cdot Rs_{i(\nu)} (C - 1)\}, \quad \sigma_{i(\nu)} > \sigma_{a,i(\nu)} \end{cases} \tag{3}$$

6) Setting of minimum value of design variable, and calculation of design sensitivity
From the ($\nu_{fin}/2$) calculation results of the first half of the design step, extract the step with the combination of the lightest variable that satisfies the stress constraint function. From the variable value, set to the minimum value of the second half of the design step, and calculate the sensitivity parameter (DS_i) to be incorporated in the correction equation of the design variable.

7) Mechanical analysis (displacement)
Using equilibrium mechanics theory, calculate the central displacement of the main structure ($\delta_{i(\nu)}$).

8) Modification of design variable (displacement)
The design variables are corrected by the following formula based on the stress of each member and the allowable stress value;

$$b_{i(\nu+1)} \begin{cases} = (1 - Re_{2(\nu)}) \cdot b_{i(\nu)} + Re_{2(\nu)} \cdot (1 + T \cdot DS_i) \cdot b_{i(\nu)} \\ = b_{i(\nu)}(1 + T \cdot DS_i \cdot Re_{2(\nu)}) \qquad , \quad \delta_{(\nu)} > \delta_a \\ \\ = (1 - Re_{2(\nu)}) \cdot b_{i(\nu)} + Re_{2(\nu)} \cdot (1 - T \cdot DS_i) \cdot b_{i(\nu)} \\ = b_{i(\nu)}(1 - T \cdot DS_i \cdot Re_{2(\nu)}) \qquad , \quad \delta_{(\nu)} \leq \delta_a \end{cases} \tag{4}$$

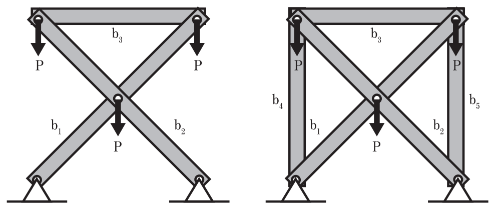
9) Search for optimum design steps
From the calculation results of (ν_{fin}) in all design steps, extract the step with the combination of the lightest variable that satisfies both the stress and displacement constraint functions, and set it as the optimal solution.

2.2. Change in optimal solution by correction

In this section, the effect and role of each parameter will be described.

2.2.1. Presence or absence of design sensitivity The displacement of the point of interest of the scissors model with n design variables is expressed by the following equation;

$$\delta_c(b_i) = \sum_{i=1}^{n} \left(\int_0^{L_i} \frac{\overline{N_i}(b_i) N_i(b_i)}{EA_i} d\chi_i + \int_0^{L_i} \frac{\overline{M_i}(b_i, \chi_i) M_i(b_i, \chi_i)}{EI_i} d\chi_i \right) \tag{5}$$



Model (b) Model (c)

Figure 2. Analytical model of a scissors unit

Design sensitivity (ds_i) is calculated by partially differentiating Equation(5) with each design variable (b_i).

$$ds_i = \frac{\partial \delta_c(b_i)}{\partial b_i} \tag{6}$$

$$DS_i = \frac{ds_i}{\sum_{j=1}^{n} ds_j} \tag{7}$$

The value of $b_{i(\nu)}$ to be substituted into Equation(6) is used as the value of the design variable in the step with the lightest variable satisfying the stress constraint condition in the first half of the design steps. This correction equation is as shown in Equation(7). The unit scissors models of the upper string reinforced with two types of members (b) and (c) are shown in Figure 2. A comparison of the analysis values of these two models is shown in **Table 1**.

2.2.2. Correction of reduction rate A vibration phenomenon periodically appears, in which the reduction rate varies up and down near the tolerance value as long as it is constant, regardless of the course of the design step. Consequently, it is not always possible to pick up the optimal solution. Therefore, we aim to converge design variables by gradually changing the reduction rate by the number of iterations. The new reduction rate is gradually decreased by the following equation;

$$Re_{1(\nu+1)} = Re_{1(\nu)} \cdot \left(\frac{\nu_{fin}/2 - \nu}{\nu_{fin}/2} \right)^{\kappa} \tag{8}$$

$$Re_{2(\nu+1)} = Re_{2(\nu)} \cdot \left(\frac{\nu_{fin} - \nu}{\nu_{fin}} \right)^{\kappa} \tag{9}$$

Fig.3 and **Fig.4** show the transition of the weight (W) and centre displacement(δ) for each design step in models (b) and (c) when $\kappa = 0.1$. The graphs indicate that the up and down cycling behaviour disappeared, and the reduction ratio converged to an arbitrary value. Therefore, it was possible to obtain the optimum solution of the reduction ratio.

3. Numerical analysis result

3.1. Analysis of scissors structure with 5 units

We analysed the results of a scissors structure with five units to optimize the sectional dimension by the optimization algorithm described in the previous section. A load of $P = 1000(N)$ was imposed vertically downward at all nodes. **Fig.5** shows the cross-sectional dimension layout diagram at the optimum step of models (d) - (g) under the presence/absence of reinforcement

Table 1. Comparison of optimized weights of each scissors model

Model	Weight(kg)
(b)	1.01
(c)	0.40
(d)	1222.9
(e)	389.3
(f)	46.6
(g)	46.5

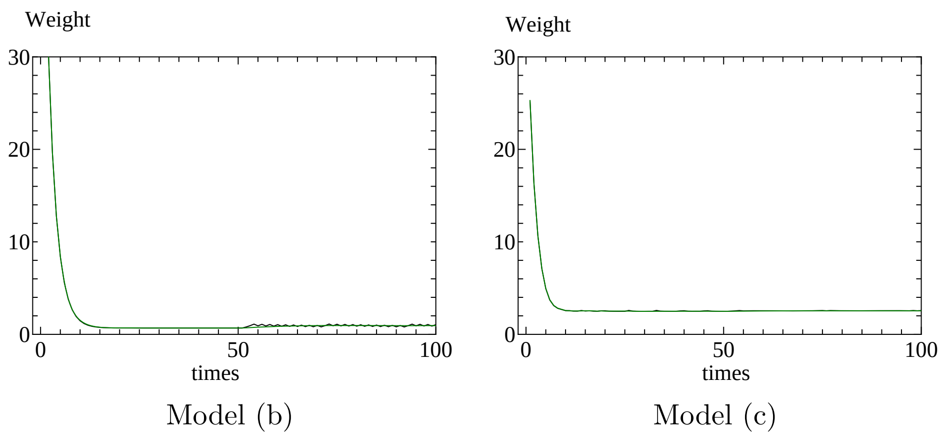


Figure 3. A scissors unit weight

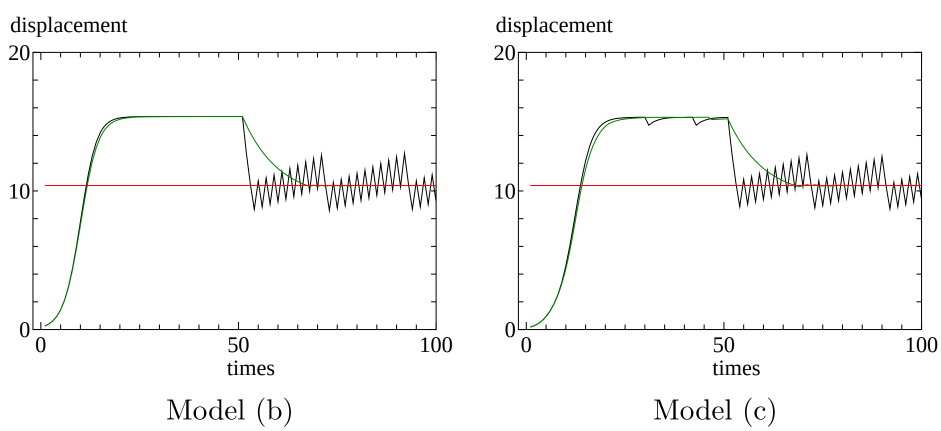
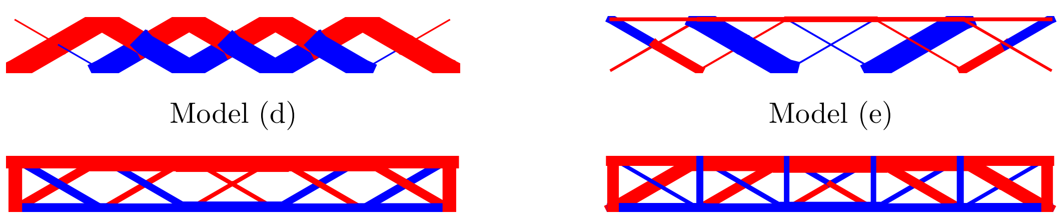


Figure 4. A scissors unit central displacement

members and placement differences. Model (d) features a layout in which the bending moment and the axial dimensions of the whole structure are maximised such that the member dimensions of the middle case become thicker. Particularly in model (e), the right descending member of the second rank and the right ascending member of the fourth rank located at symmetrical positions greatly influence displacement control. The layout diagram also indicates that the scissors member becomes remarkably thick. In addition, the upper chord member shows resistance as a compression shaft member, plays a role of connecting the fulcrums in an arch shape, and greatly contributes to the improvement of load bearing capacity. Next, model (f) shows that the scissors member becomes significantly thinner. This is because the bending moment of the scissors member was rapidly reduced by inserting the reinforcing member. Furthermore, considering that the compression axial force member and the lower chord member are tension members, the same mechanical state as the truss structure can be confirmed. Because the member resisting the shear force of the entire structure is a scissors member, and the size of the scissors member at the base is larger than the central case, the member shows resistance from the centre towards the fulcrum as the characteristic of the structure. It is understood that shearing takes place with increased force. Finally, model (g) resists as a tensile axial force member, in which four newly inserted vertical members pull up the load, as compared to model (f). As a result, the size of the lower chord material decreases, and the dimension of the upper chord material increases. **Table 1** shows the comparison results of the optimized weights of the abovementioned four models. Weight reductions of about 68% in model (e) and about 96% in model (f) and (g) could be achieved, compared with model (d).

3.2. Analysis of double Warren truss structure with 5 units

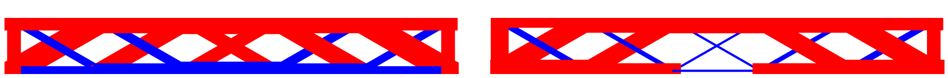
To verify the versatility of the proposed optimization algorithm, we analysed the model of two double Warren truss structures (hereinafter referred to as DWT) with different loading strengths equivalent to the scissors model with different boundary conditions. In the case of a static truss, since the axial force (N_i) does not depend on the cross-sectional dimension, an optimal solution can be obtained with one feedback. However, because DWT has an intrinsic and unstable constant order number of 1, it can be iteratively optimized by this method. The analysis result is shown in **Fig. 6**. In DWT model (a), the lower chord material acted as a tension member and the upper chord material acted as a compression member. In addition, the diagonal member in the shape of character “ˆ” and the diagonal member in the shape of character “v” showed tendencies of resisting compression force and tensile axial force, respectively, as in the scissors structure. In DWT model (b), since the horizontal displacement of the lower chord member is suppressed as a boundary condition, the lower chord member becomes a compression axial force member except for the middle case. The other members exhibit the same characteristics as those of DWT (a). A comparison of the results of the optimized weights under different boundary conditions is presented in **Table 2**. DWT (b) with both pin-support boundary conditions could achieve a reduction of 10% for the optimized weight. This can be said to be reasonable because increasing the constraint condition generally improves the yield strength of the structure.



Model (d) Model (e)

Model (f) (Scale up to 5 times for thickness) Model (g) (Scale up to 5 times for thickness)

Figure 5. Layout of the cross-sectional dimension for each scissors model



DWT (a) DWT (b)

Figure 6. Layout of the cross-sectional dimension for different supports of DWT

Table 2. Comparison of the optimized weight of each DWT model

Model	Weight(kg)
DWT (a)	74.2
DWT (b)	66.4

4. Conclusion

In this study, the optimum stiffness of the scissors members and the reinforcing members could be determined in a simple supported state after expansion without reinforcement and in a state using a reinforcing member with optimum arrangement and optimum dimension. We could also reduce the weight compared with the initial design. The main merit of the newly proposed optimal design theory is that it can easily predict the improved bridge design (including various reinforcing patterns and boundary conditions) by combining the versatile equilibrium mechanics theory and DE method. This method is very effective for actual engineering work, allowing rapid construction of lighter and more powerful deployable scissors bridges with or without reinforcing members. We addressed the weight optimization problem of the beams and truss structures in addition to the scissors structure under stress and displacement constraint conditions. In this problem, after reviewing buckling, all constraints could be finally satisfied, and its cross-sectional dimensions could be decreased.

Acknowledgments

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References

[1] Mele, T. V., Mollaert, M., Temmerman, N. D., Laet, L. D. (2006), Design of Scissor Structures for Retractable Roofs, Proceedings of Adaptables2006, International Conference On Adaptable Building Structures, 68-73.

[2] Hoberman, C. (1990), Reversibly expandable doubly-curved truss structures, United States Patent 4,942,700.

[3] Hoberman, C. (1991), Radial expansion retraction truss structure, United States Patent 5,024,031.

[4] Holnicki-Szulc, J. (2008), Smart Technologies for Safety Engineering, Wiley.

[5] Ario, I., Nakazawa, M., Tanaka, Y., Tanikura I., Ono, S. (2013), Development of a prototype deployable bridge based on origami skill, Automation in Construction, 32, 104-111.

[6] Oda et al. (1989), Optimum Design of Structures and Materials, ed. by The Japan Society of Mechanical Engineers, Gihodo Publishing Co. (in Japanese)

[7] Bureerat, S. and Pholdee, N. (2015), Optimal truss sizing using an adaptive differential evolution algorithm, Journal of Computing in Civil Engineering, 30(2), 04015019.

[8] Ho-Huu, V., Vo-Duy, T., Luu-Van, T., Le-Anh, L., Ngu-yen-Thoi, T. (2016), Optimal design of truss structures with frequency constraints using improved differential evolution algorithm based on an adaptive mutation scheme, Automation in Construction, 68, 81-94.

[9] Adeli, H. and Cheng, N.-T. (1994), Concurrent Genetic Algorithms for Optimization of Large Structures, J ournal of Aerospace Engineering, ASCE, 7(3), 276-296.

[10] Kociecki, M., & Adeli, H. (2013), Two-phase genetic algorithm for size optimization of free-form steel space-frame roof structures. Journal of Constructional Steel Research, 90, 283-296.

[11] Kociecki, M., & Adeli, H. (2014), Two-phase genetic algorithm for topology optimization of free-form steel space-frame roof structures with complex curvatures. Engineering Applications of Artificial Intelligence, 32, 218-227.

[12] Li, W., Pu, H., Schonfeld, P., Yang, J., Zhang, H., Wang, L., and Xiong, J. (2017), Mountain railway alignment optimization with bidirectional distance transform and genetic algorithm, Computer-Aided Civil and Infrastructure Engineering, 32(8), 691-709.

[13] Tusar, T., Filipi., B. (2007), Differential Evolution versus Genetic Algorithms in Multi-objective Optimization, In In-ternational Conference on Evolutionary Multi-Criterion Optimization, Springer, Berlin, Heidelberg, 257-271.

[14] Dong, X. L., Liu, S. Q., Tao, T., Li, S. P. and Xin, K. L. (2012), A comparative study of differential evolution and genetic algorithms for optimizing the design of water dis-tribution systems, Journal of Zhejiang University SCIENCE A, 13(9), 674-686.

[15] C. Khongkham, I. Ario, Y. Chikahiro: Smart design of coupling scissors-type bridge, Structures, (2021), 206-216, 30.

[16] Chikahiro, Y., Ario, I., Holnicki-Szulc, J., Pawlowski, P., Graczykowski, C. (2015), Study on the optimization of the reinforced scissor type bridge, Proc. of PCMCMM 2015, 319-320.

[17] Chikahiro, Y., Ario, I., Nakazawa, M. (2016), Theory and Design Study of a Full- Scale Scissors-Type Bridge, Journal of Bridge Engineering, 21(9).

[18] Chikahiro, Y., Ario, I., Nakazawa, M., Ono, S., Holnicki-Szulc, J., Pawlowski, P., Graczykowski, C., Watson, A. (2016), Experimental and numerical study of full-scale scissor type bridge, Automation in Construction, vol.71(2), 171-180.